Logic 1

A **Statement** is a sentence or expression that is either definitely true or definitely false.

Sentence/Expression	Statement?
Add 5 to both sides	No
$\mathbb{Z}$	No
Adding 5 to both sides of $x - 5 = 37$ gives $x = 42$	Yes
$42 \in \mathbb{Z}$	Yes
42	No
What is the solution of $2x = 84$?	No
The solution of $2x = 84$ is 41	Yes
42 is not a number	Yes
x is a multiple of 8	No

↳ **open sentence** - depends on some variable

Aside: statements that have a truth value that nobody knows.

Nonexample: it is cloudy

Not a statement!

Example: Collatz conjecture:

Start with a positive integer x.
Repeat: if x is even, divide by 2
if x is odd, multiply by 3 and add 1.

Statement, but no one knows whether it's T or F!

Given any starting x, this goes to 1 in a finite number of steps.

Do Ex. A

Logical Operators: A Logic Puzzle

Truth table:

P	
T	
F	

P	Q	
T	T	
T	F	
F	T	
F	F	

Three important truth tables:

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

P	$\neg P$
T	F
F	T

Given the above truth tables, describe the meaning of the three operators: $\wedge$, $\vee$, $\neg$

$\wedge$: and : $P \wedge Q$ is true if P and Q are true

$\vee$: or : $P \vee Q$ is true if at least one of P and Q are true

$\neg$: not : $\neg P$ is true if P is false, false if P is true

Do Ex. B

$$P \wedge (Q \vee S)$$

$$T \wedge (F \vee T)$$

$$(T \wedge T)$$

$$T$$

$$N \wedge P \wedge Q$$

Logical Operator Logic Puzzle - Pt 4: Conditional

P	Q	P ⇒ Q
T	T	T
T	F	F
F	T	T
F	F	T

"if P then Q"

Example: "if $x = y$, then $x^2 = y^2$" is a true statement

Notice: this does not commute!

"for all x, y if $x^2 = y^2$, then $x = y$" is a false statement

Weirdness #1: False Implies Anything

"False ⇒ True" is true

"False ⇒ False" is true

P	Q	P ⇒ Q
T	T	T
T	F	F
F	T	T
F	F	T

If P, then Q

Sane Example:

P: I am healthy

Q: I will come to class

Insane example:

"if 1=2, then 30=10"

"if 1=2, then 30=30"

Don't Panic! Tips for staying sane:

- Apply the truth table (and don't think)
- $P \Rightarrow Q$ is a promise
- The first 2 rows are where it's at.

Do Ex. C

Weirdness #2: Natural Language

P	Q	P ⇒ Q
T	T	T
T	F	F
F	T	T
F	F	T

Stylistic Variants of Conditional Statements

- If P then Q
- Q if P
- Q whenever P
- Q, provided that P
- Whenever P, then also Q
- P is a sufficient condition for Q
- For Q it is sufficient that P
- For P it is necessary that Q
- P only if Q

$P \Rightarrow Q$

Definition: $Q \Rightarrow P$ is the converse of $P \Rightarrow Q$
(aside)

Do Ex. D

P: You fail

Q: only if you stop writing

P only if Q ≡ if P then Q

if you fail, then you stop(ped) writing

fail ⇒ stop means:

(row 1) if you fail, then you also stopped

(rows 3-4) if you didn't fail, you may or may not have stopped

row 2 isn't possible because fail ⇒ stop is true

fail ↓ stop ↓

P	Q	P ⇒ Q
T	T	T
T	F	F
F	T	T
F	F	T

Biconditional Statements

"P if and only if Q"

P ___ Q

P	Q	P ⇒ Q
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	P ⇔ Q
T	T	
T	F	
F	T	
F	F	