Sets 2

Definition: Let A and B be sets. A is a <u>subset</u> of B if every element of A is in B. This is written $A \subseteq B$

Examples:

Let $A = \{1, 2, 3\}$
$B = \{2, 3, 1\}$
$C = \{1, 2\}$

Facts:

$A \subseteq B$ $\quad C \not\subseteq A$

$B \subseteq A$ $\quad A \not\subseteq C$

$C \subseteq A$ $\quad \{n^2 : n \in \mathbb{Z}\} \subseteq \mathbb{Z}$

Aside: If $A \subseteq B$ and $B \subseteq A$, then $A = B$.

If $A \subseteq B$ and $A \neq B$, then A is a <u>proper</u> subset of B.

written: $A \subset B$

Examples: $A \not\subset B$

$C \subset A$

$\{n^2 : n \in \mathbb{Z}\} \subset \mathbb{Z}$

Definition: The <u>power set</u> of a set A, written $\mathcal{P}(A)$, is the set of all subsets of A.

occasionally also written as: 2^A

$$\mathcal{P}(A) = \{ S : S \subseteq A \}$$

$\boxed{\text{Do Ex. A}}$

$\{\} \subseteq \{a, \emptyset\}$

$\emptyset \subseteq A$ T

$\{\emptyset\} \subseteq A$ T

$\{\emptyset\} \in A$ F

$\{\emptyset, \{1\}, \{2\}, \{1,2\}\} = \mathcal{P}(\{1,2\})$

$\{\emptyset\}$

Cartesian Product

Terminology/Notation:

- An ordered pair (also called a 2-tuple, or just pair), is written (a,b).
- A tuple generalizes this to more than 2. $(a,4,4)$ is a 3-tuple.
- Unlike sets, these are ordered and items are not necessarily unique.

Definition: The cartesian product of two sets A, B, written $A \times B$, is the set of ordered pairs (a,b) where $a \in A$, $b \in B$.

$$A \times B = \{ (a,b) : a \in A, b \in B \}$$

Example: $A = \{a, b\}$, $B = \{1, 2, 3\}$

$$A \times B = \{ (a,1), (a,2), (a,3), (b,1), (b,2), (b,3) \}$$

Generalization: $A \times B \times C$ is a set of 3-tuples with $a \in A$, $b \in B$, $c \in C$

$\boxed{\text{Do Ex. B}}$

$A = \{a, b\}$
$B = \{1, 2, 3\}$
$C = \{4\}$
$D = \emptyset$

$C \times A = \{(4, a), (4, b)\}$

$D \times B = \emptyset$

$|A \times B| = |A| \cdot |B|$

$B \times C \times D = \emptyset$

Set Operations

Given sets A, B, their:

Union	$A \cup B$	is	$\{x : x \in A \text{ or } x \in B\}$
intersection	$A \cap B$	is	$\{x : x \in A \text{ and } x \in B\}$
difference	$A - B$ (or) $A \setminus B$	is	$\{x : x \in A \text{ and } x \notin B\}$

Examples: Let $S = \{1, 2, 3\}$ and
$T = \{2, 4\}$

- $S \cup T = \{1, 2, 3, 4\}$
- $S \cap T = \{2\}$
- $S - T = \{1, 3\}$
- $T - S = \{$

$\boxed{\text{Do Ex. C}}$

Complement, Venn Diagrams

Sometimes it's natural to define a universal set U for a set S.

This is informal and context-dependent: The only requirement is $S \subseteq U$

Examples:

$\{x : x \text{ is even}\}$ might have universal set $\mathbb{Z}$

Definition: For a set S with universal set U, the complement of S, written $\overline{S}$ is $U - S$: everything not in S

Example:

$A = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$

$B = \{11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$

$U = \{x : x \in \mathbb{N} \text{ and } x \leq 20\}$

$\bar{A} = \{x \in \mathbb{N} : x \text{ is even and} \in U\}$

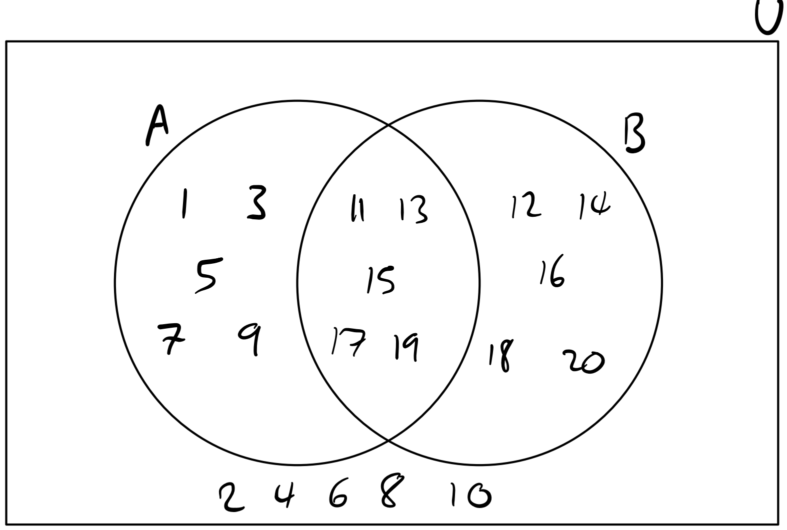
$\bar{B} = \{x \in \mathbb{N} : x \leq 10 \text{ and} \in U\}$

Venn Diagrams

$A = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$

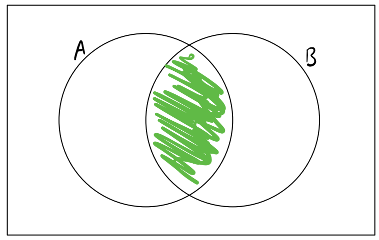
$B = \{11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$

$U = \{x : x \in \mathbb{N} \text{ and } x \leq 20\}$

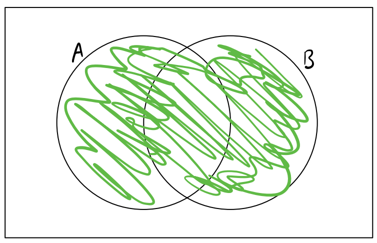


Can help visualize set operations. For example:

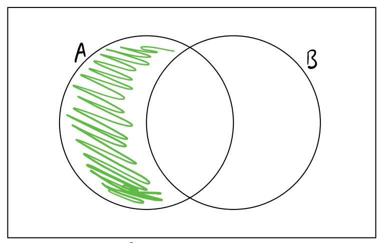
$A \cap B$ is everything in both circles.



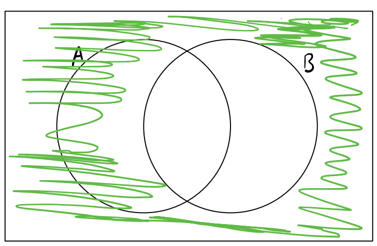
Do Ex. D



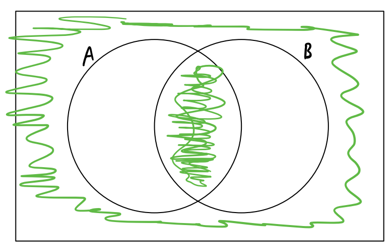
$A \cup B$



$A - B$



$\bar{B}$



$\overline{(A \cup B) - (A \cap B)}$