Sets

A set is an unordered collection of unique items.

The items are called its elements

Membership is denoted:

$$a \in S$$

Non-membership:

$$a \notin S$$

We can write a set using Roster notation/method such as:

$$\{1, 2, 3, 4\}$$

You can use "..." if the meaning is obvious: $\{a, b, c, \ldots z\}$

Sets are unordered: $\{1, 2, 3\} = \{2, 1, 3\}$

Elements are unique: $\{1, 2, 2, 3, 3\} = \{1, 2, 3\}$

Famous Sets - Such a big deal they have a symbol!

- $\mathbb{N}$ natural numbers $\{1, 2, 3, \dots\}$
- $\mathbb{Z}$ integers $\{\dots, -2, -1, 0, 1, 2, \dots\}$
- $\mathbb{Q}$ rational numbers
- $\mathbb{R}$ real numbers
- $\mathbb{C}$ Complex numbers
- $\emptyset, \mathbb{0}, \{\}$ empty set

(do Ex. A)

Set Builder Notation

(Roster: $\{1, 2, 3\}$ $\{True, False\}$)

$\{$ expression : membership test $\}$

Example: $S = \{1, 3, 5, 7, 9\}$

$S = \{x \in \mathbb{N} : x \text{ is odd and } x < 10\}$

$S = \{x \in \mathbb{Z}^+ : x \text{ is odd and less than } 10\}$

$S = \{2k-1 : k \in \mathbb{N} \text{ and } k \leq 5\}$

(do Ex. B)

Perfect Squares:

$\{x \in \mathbb{N} : \sqrt{x} \in \mathbb{N}\}$

$\{k^2 : k \in \mathbb{N}\}$

$\mathbb{Q}^+$ $\{x : x \in \mathbb{Q} \text{ and } x > 0\}$

$\{n/m : n \in \mathbb{N}, m \in \mathbb{N}, n, m > 0\}$

Cardinality, Sets of Sets

If a set S has n elements and $n \in \mathbb{Z}$, then S is finite.

Otherwise, S is infinite.

Def The cardinality of a finite set A, written $|A|$ is the number of distinct elements in A.

Examples:

$\mathbb{N}$ is infinite

$|\{1,2,3\}|$ is 3

$|\{a : a \text{ is a letter in the English alphabet}\}| = 26$

Fact: Sets can contain sets.

Example: $S = \{\{1,2\}, \{2,3\}, \emptyset\}$

Facts: $\{1,2\} \in S$ $\quad |S| = 3$

$1 \notin S$ $\quad \emptyset \in S$

(do Ex. C)

Ex. C: 1. $\{n^2 : n \in \mathbb{Z}\}$ is infinite

2. 4

3. $|\emptyset| = 0$

4. $|\{\emptyset\}| = 1$

5. $|\{a, \{\{\emptyset\}\}\}| =$