

CSCI 301 - Assignment 3, Spring 2025

Your name here

Modify the .tex source file for this document, replacing the placeholders with your solutions. This is an individual assignment. See the syllabus for the collaboration policy.

In the Direct Proof lecture Exercises, we proved the following two propositions, which we number here for ease of reference:

Proposition 1: If two integers have the same parity, then their sum is even.

Proposition 2: If two integers have opposite parity, then their sum is odd.

In Problems 1 and 2, you will prove the corresponding theorems for the *product* of two integers. In Problem 3, you are then free to make use of these four propositions in support of your proof.

Prove each of the following propositions. Consider using direct proof, contrapositive proof, or proof by contradiction; try to use the approach that makes for the simplest proof.

1. (5 points) **Proposition 3:** If a and b have the same parity, then ab also has the same parity.

Proof: Replace this text with your proof.

2. (5 points) **Proposition 4:** If a and b have opposite parity, then ab is even.

Proof: Replace this text with your proof.

3. (10 points) Suppose a and b are integers. Then $a^2(b + 3)$ is even if and only if a is even or b is odd. Follow the outline for If-and-Only-If Proof from the Book of Proof Chapter 7.1, page 147. *Hint:* Given that this proof consists of two parts, you may find it helpful to consider different proof strategies for each part.

Proof: Replace this text with your proof.

4. **Bonus Proposition** (extra credit, up to 2 points): In any right triangle with integer side lengths, at least one of the non-hypotenuse edges has even length.

Proof: Replace this text with your proof.