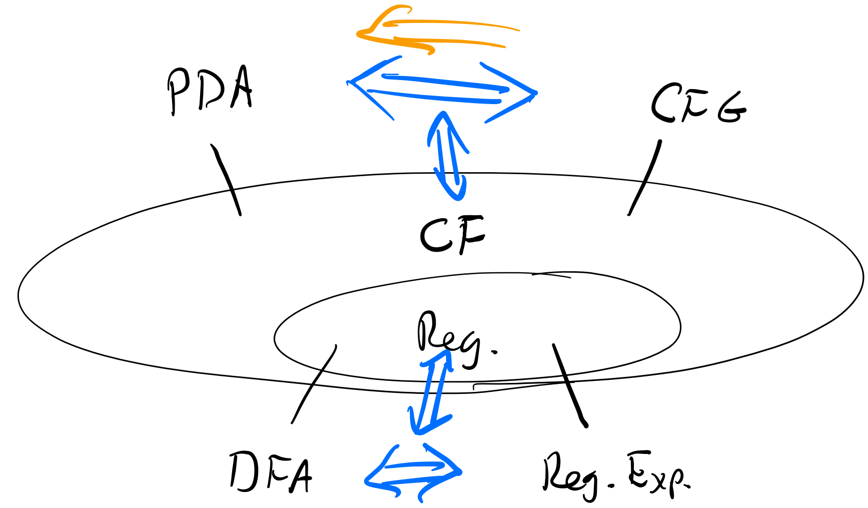
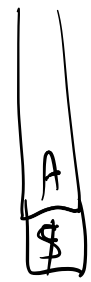
CSCI 301 - Lecture 33: CFG to PDA



$A \to \underline{xAB}c$



Aside: Chomsky Normal Form

Grammar with <u>only</u> rules like:

1. $\underline{A} \to \underline{BC}$ 2 variables, could be A, but not S
2. $A \to \underline{a}$ <u>single</u> terminal
3. $S \to \varepsilon$ only start

Thm: Any context-free grammar can be converted to CNF!

Let G be the grammar:

$S \to ASA \mid aB$

$A \to B \mid S$

$B \to b \mid \varepsilon$

Derive

bba

$S \to ASA$

BSA

bSA

$bASAA$

$bBSAA$

$bbSAA$

$bbaBAA$

bba

CNF Conversion:

1. Eliminate S from RHS
2. Eliminate ε-rules (except $S \to \varepsilon$)
3. Eliminate unit rules $A \to B$
4. Eliminate rules w/ too many things on RHS $A \to BCD$, $B \to xy$

Step 0:

$S \to ASA \mid aB$

$A \to B \mid S$

$B \to b \mid \varepsilon$

Step 1:

$S_0 \to S$

$S \to ASA \mid aB$

$A \to B \mid S$

$B \to b \mid \varepsilon$

Step 2a: eliminate $B \to \varepsilon$

$S_0 \to S$

$S \to ASA \mid aB \mid a$

$A \to B \mid S \mid \varepsilon$

$B \to b$

Step 2b: eliminate $A \to \varepsilon$

$S_0 \to S$

$S \to ASA \mid SA \mid AS \mid aB \mid a$

$A \to B \mid S$

$B \to b$

Step 3a:

eliminate $A \to B$

$S_0 \to S$
$S \to ASA \mid SA \mid AS \mid aB \mid a$
$A \to S \mid b$
$B \to b$

Step 3b: eliminate $A \to S$

$S_0 \to S$
$S \to ASA \mid SA \mid AS \mid aB \mid a$
$A \to ASA \mid SA \mid AS \mid aB \mid a \mid b$
$B \to b$

Step 3c: elim. $S_0 \to S$

$S_0 \to ASA \mid SA \mid AS \mid aB \mid a$
$S \to ASA \mid SA \mid AS \mid aB \mid a$
$A \to ASA \mid SA \mid AS \mid aB \mid a \mid b$
$B \to b$

Step 4: elim. big RHS

$S_0 \to AM \mid SA \mid AS \mid NB \mid a$
$S \to AM \mid SA \mid AS \mid NB \mid a$
$A \to AM \mid SA \mid AS \mid NB \mid a \mid b$
$B \to b$

$M \to SA$
$N \to a$

$S_0 \to AS$
bS
bAS
bbS
bba

bba

S_0

AS

bS

bAS

bbS

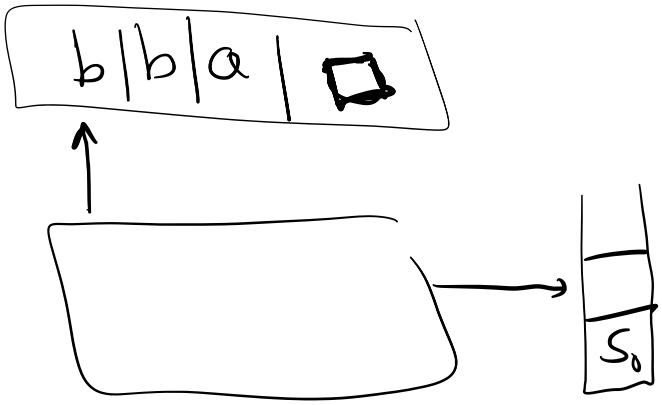
bba

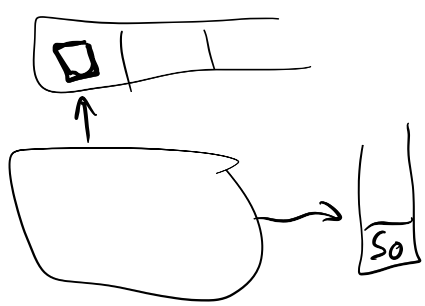
$a_1 a_2 \ldots a_k \mid A_1 A_2 \ldots A_m$

$a_{k+1} \mid A_2 A_3 \ldots$

$\mid A_{1a} A_{1b} A_2 \ldots$

Construct a PDA that accepts L(G): Sketch





Rules:

$S_0 \to AS$ $\quad q\,a\,S_0$, $q\,b\,S_0 \to q\,N\,AS$ — For each rule $A \to BC$

$S_0 \to AM$ $\quad q\,a\,S_0$, $q\,a\,S_0 \to q\,N\,AM$

⋮

$A \to a$ $\quad q\,a\,A \to q\,R\,\varepsilon$ — For each rule $A \to a$

⋮

$S_0 \to \varepsilon$ $\quad q\,\square\,S_0 \to q\,N\,\varepsilon$ — If $S_0 \to \varepsilon$ is a rule