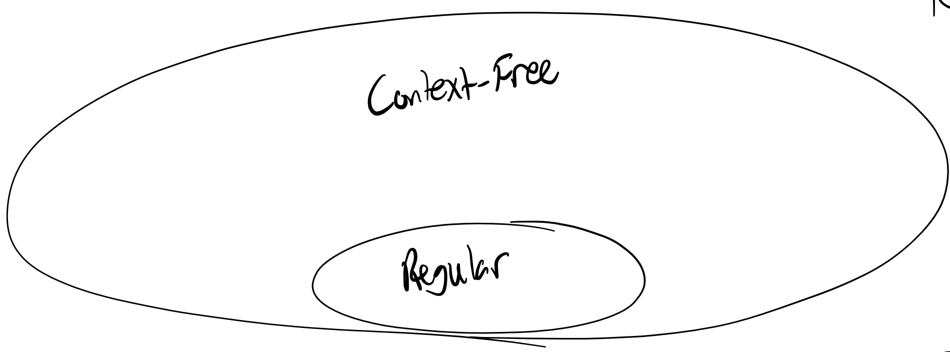
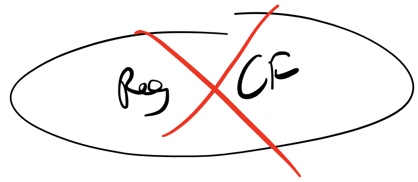
CSCI 301 - Lecture 32: The Pumping Lemma
(Regular and Context-free)

✓ Reg $\subseteq$ CF?

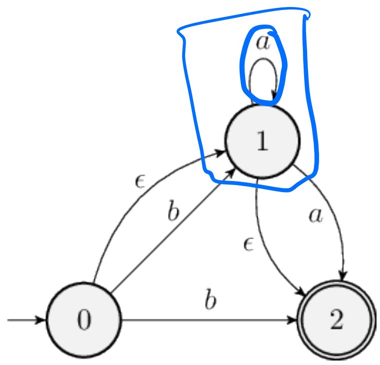
CF $\overset{?}{\subseteq}$ Reg??



$a^n b^n$



NFA:



What is true of any string $w \in L(M)$ with $|w| > 2$?

The Pumping Lemma for Reg. Languages

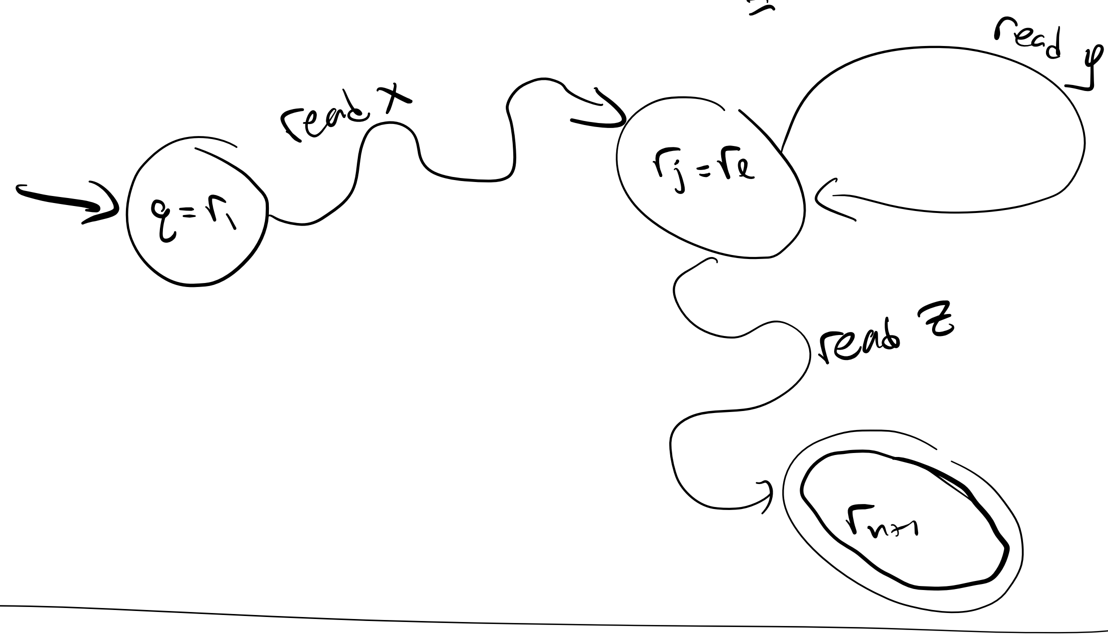
Let A be a regular language. There exist an integer p, called the pumping length, such that:

For every string $s \in A$ with $|s| \geq p$,

s can be written as $s = xyz$, where:

- $y \neq \varepsilon$, i.e. $|y| > 0$
- $|xy| < p$
- $\boxed{xy^iz} \in A$ for any $i \geq 0$

$s = s_1 s_2 s_3 \cdots s_n \qquad r = r_1 r_2 r_3 \cdots r_{n+1}$



Prove that $A = \{0^n 1^n : n \geq 0\}$ is not regular.

<u>Proof</u>: By contradiction. Suppose A is regular and let p be the pumping length.

Consider the string $s = 0^p 1^p$. $|s| = 2p \geq p$.

Then by the pumping lemma, $s = xyz$ where

- $y \neq \varepsilon$
- $|xy| < p$
- $xy^i z \in A$

$$\overbrace{\underbrace{00}_{x}\ \underbrace{\cdots 0}_{y}}^{p}\ \underbrace{11 \cdots 1}_{z}$$

$x y^{2} z$ adds $|y|$ zeros, to form

$|x| \quad 2|y| \quad |z|$

$0 \quad 0 \quad 1$

Therefore A is not regular.

Ex. 2 ...

Let $s = 1^{p^2}$

$= xyz$

$|xy^2z| = |x| + 2|y| + |z|$

$= \underline{p^2 + |y|} < p^2 + p$

$|s| > p^2$

$\left|1^{(p+1)^2}\right| = \boxed{p^2 + 2p + 1}$

$\cancel{x}\ y \neq \varepsilon$

$|xy| < p$

$- xy^iz \in A$

$0 < |y| < p$

$p^2 + p < p^2 + 2p + 1$

Lemma (The Pumping Lemma for Context-Free Languages): Let L be a context-free language. Then there exists an integer $p \geq 1$, called the pumping length, such that every string s in L with $|s| \geq p$ can be written as $s = uvxyz$, where

- $|vy| \geq 1$ (i.e., v and y are not both empty)
- $|vxy| \leq p$, and
- $uv^ixy^iz \in L$ for all $i \geq 0$.