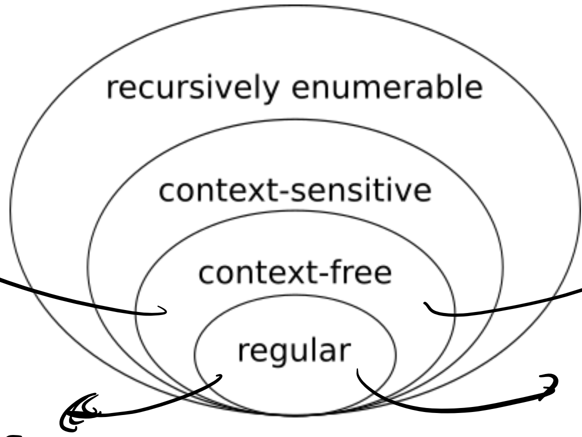
CSCI 301 - Lecture 27: Context-Free Grammars

Chomsky Hierarchy

described by context free grammars

Pushdown automata



described using regular expressions

accepted by DFAs $\overset{\text{III}}{}$ NFAs

start symbol

rules
productions

CFG:

$$S \to AB$$
$$A \to a$$
$$A \to aA$$
$$B \to b$$
$$B \to bB$$

terminals $\in \Sigma$

Variables
nonterminals $\in V$

$$S \Rightarrow AB \Rightarrow aAB \Rightarrow aAbB \Rightarrow aaAbB \Rightarrow aaabB \Rightarrow aaabb \in \Sigma^*$$

Rule:

$$V \ni A \to w \in (\Sigma \cup V)^* \cup \varepsilon$$

Derivation:

$$S \Rightarrow AB$$

↳ derives in one step

"AB is derived in one step from S."

$S \overset{*}{\Rightarrow} aaabb$ "aaabb can be derived from S"

(in 0 or more steps)

$$L(G) = \{ w \in \Sigma^* : S \overset{*}{\Rightarrow} w \}$$

A language L is context free if there is a CFG G s.t. $L(G) = L$

Context free but not regular:

$$\{ a^n b^n : n \in \{0, 1, 2 \ldots\} \}$$

$S \to \varepsilon$

$S \to aSb$

$$\boxed{\begin{aligned} S &\to AB \\ A &\to a \\ A &\to aA \\ B &\to b \\ B &\to bB \end{aligned}}$$

$S \Rightarrow AB$

$\Rightarrow aAB$

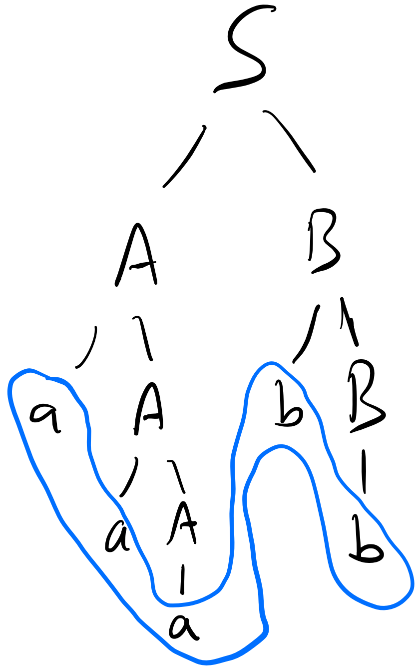
$\Rightarrow aAbB$

$\Rightarrow aaAbB$

$\Rightarrow aaabB$

$\Rightarrow aaabb$

Parse Tree:



$S \to \varepsilon \mid aSbS \mid bSaS$

Ex. A

G_1:

$S \to \varepsilon$

$S \to aSa$

$S \to bSb$

G_2:

notation

$S \to \varepsilon$

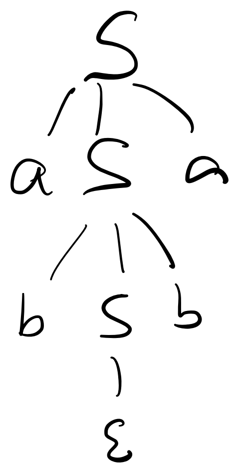
$\mid aSbS$

$\mid bSaS$

$S \Rightarrow aSa$

$\Rightarrow abSba$

$\Rightarrow abba$



$$E \to E + E$$
$$E \to E - E$$
$$E \to E * E$$
$$E \to E / E$$
$$E \to (E)$$
$$E \to 0 \mid 1 \mid 2 \mid \dots \mid 9$$