CSCI 301 - Lecture 24:

NFAs, NFA-DFA equivalence

Reminder:

Def. A language A is regular if there is a DFA M such that $L(M) = A$

Regular Operations

- The **union** of two languages A and B is defined as $A \cup B = \{w : w \in A \text{ or } w \in B\}$.
- The **concatenation** or **product** of two languages A and B is defined as $AB = \{ww' : w \in A \text{ and } w' \in B\}$.
- The **closure** or **star** (or Kleene closure) of a language A is defined as: $A^* = \{u_1 u_2 \dots u_k : k \geq 0 \text{ and } u_i \in A \text{ for all } i = 1, 2, \dots, k\}$

Thm: Regular languages are closed under regular operations.

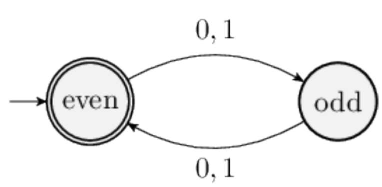
Proof: not yet. But let's do an example for union

$\Sigma = \{0, 1\}$

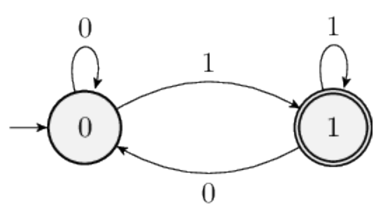
$L_A = \{w : |w| \text{ is even}\}$

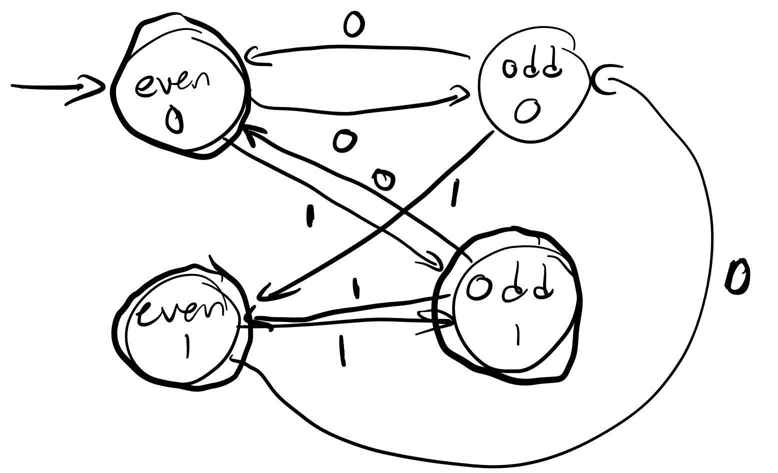
$L_B = \{w : w \text{ ends with } 1\}$

M_A :



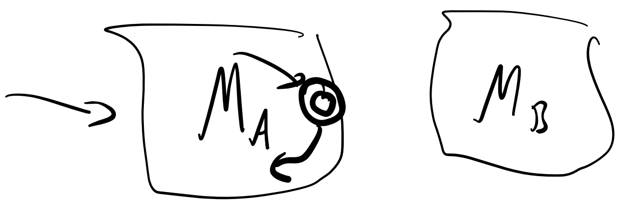
M_B



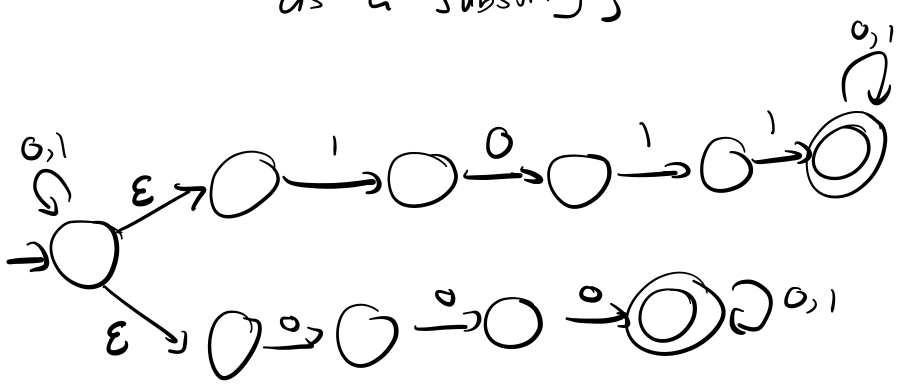


The general procedure is this:

- The set of states is $Q_A \times Q_b$
- The start state is the state (q_A, q_B)
- The accept states are $\{(s_A, s_B) : s_A \in F_A \text{ or } s_B \in F_B\}$
- The transition function is defined by:
    - $\delta_{A \cup B}((s_A, s_B), x) = (\delta_A(s_A, x), \delta_B(s_B, x))$
      In other words, the transition function takes you from the pair of states you're at to the pair of states the two individual machines would be in after they each saw the next symbol.



$L = \{ w : w$ contains either 1011 or 000 as a substring $\}$



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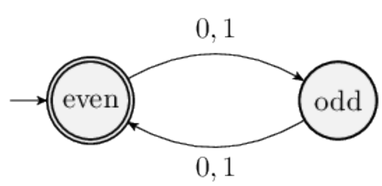
Nondeterministic FA

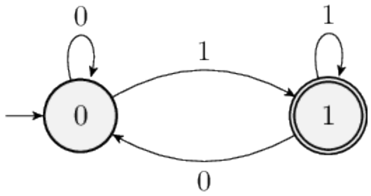
DFA, but:

$$\Sigma' = \Sigma \cup \{\varepsilon\}$$

instead of: $\delta : (Q \times \Sigma) \to Q$

do: $\delta : (Q \times \Sigma') \to \mathcal{P}(Q)$

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