CSCI 301 - Lecture 20 (!) : More Functions

Definition(s): If $f : A \to B$, then

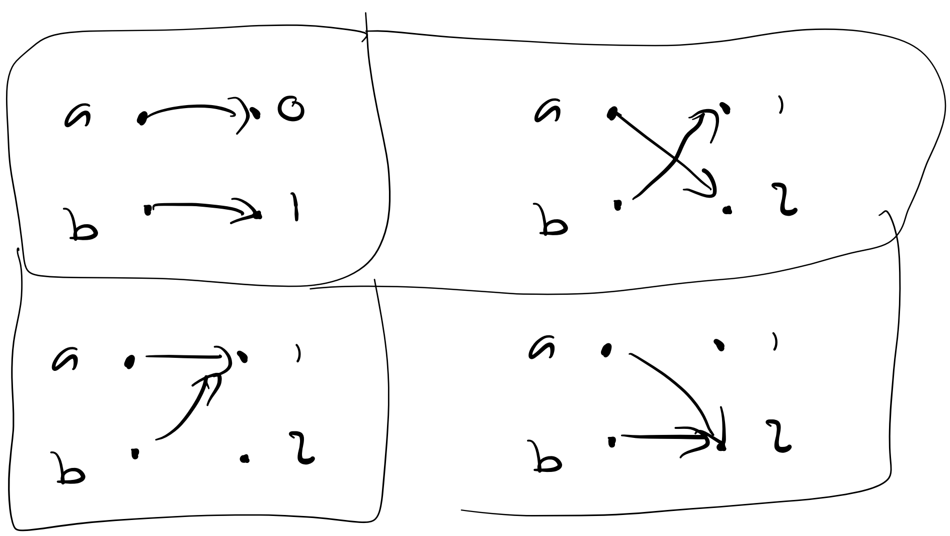
- A is the domain of f (the set of possible inputs)
- B is the codomain of f (the set of things f <u>might</u> map elements of A to)
- $\{f(a) : a \in A\}$ is the range of f (the set of things f actually maps to)

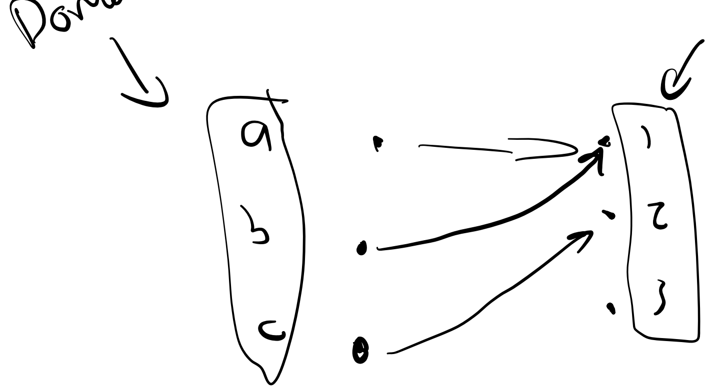
Example: $f : \mathbb{Z} \to \mathbb{N} = \{(n, |n|+2) : n \in \mathbb{Z}\}$

$$f(n) = |n| + 2$$

- Domain: $\mathbb{Z}$
- Codomain: $\mathbb{N}$
- Range: $\{n \in \mathbb{N} : n \geq 2\}$

A.1

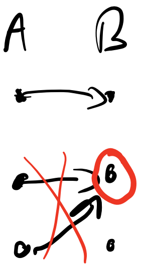




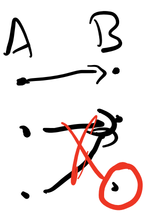
Range: $\{1, 2\}$

Definition(s):

A function $f: A \to B$ is



- injective (one-to-one) if forall $a, a' \in A$,
$$a \neq a' \Rightarrow f(a) \neq f(a')$$
intuition: no 2 elements of A map to the same $b \in B$.



- surjective (onto) if for all $b \in B$, there is some $a \in A$ where $f(a) = b$.

intuition: every $b \in B$ is mapped to by some $a \in A$
equiv: Codomain is the range

- bijective if it is injective and surjective

How to show a function $f: A \to B$ is injective:

Direct approach:	**Contrapositive approach:**
Suppose $a, a' \in A$ and $a \neq a'$.	Suppose $a, a' \in A$ and $f(a) = f(a')$.
$\vdots$	$\vdots$
Therefore $f(a) \neq f(a')$.	Therefore $a = a'$.

How to show a function $f : A \to B$ is surjective:

Suppose $b \in B$.
[Prove there exists $a \in A$ for which $f(a) = b$.]

$f : \mathbb{Z} \to \mathbb{Z}$

D.3.1 $\quad f(n) = 2n+1$

✓ Injective? Suppose $a, a' \in A$ and $a \neq a'$.

$$f(a) = 2a+1$$
$$f(a') = 2a'+1$$

X Surjective? $b = 4$

$$f(a) = 2a+1 = b$$

$$a = \frac{b-1}{2}$$

Inverse functions

Definition: Given a relation R, the inverse of R (R^{-1}) is

$$R^{-1} = \{(y,x) : (x,y) \in R\}$$

Fact: If $f : A \to B$, the inverse relation f^{-1} is a function $f^{-1} : B \to A$ iff f is bijective.

Induction:

$S_0 = 3 \mid (0^2+5\cdot 0+6)$ $\quad 3 \mid (n^2+5n+6) \ \forall n \geq 0$

$S_1 : 3 \mid (1^2+5\cdot 1+6)$

$S_2 = 3 \mid (2^2+5\cdot 2+6)$

S_3

$\vdots$

1. S_0 is true

2. Suppose S_k is true.
   Show S_{k+1} must be true.

Strong induction:

2. Suppose $S_0 \cdots S_k$ are all true
   Show S_{k+1} must be true

Converse: $P \Rightarrow Q$

The converse is $Q \Rightarrow P$

Contrapositive of $P \Rightarrow Q$ is $\neg Q \Rightarrow \neg P$

logically equivalent

negation of P is $\neg P$