CSCI 301 - Lecture 19: Relations, cont. Functions

Definition 11.2 Suppose R is a relation on a set A.

1. Relation R is **reflexive** if xRx for every $x \in A$.
   That is, R is reflexive if $\forall x \in A, xRx$.
2. Relation R is **symmetric** if xRy implies yRx for all $x, y \in A$.
   That is, R is symmetric if $\forall x, y \in A, xRy \Rightarrow yRx$.
3. Relation R is **transitive** if whenever xRy and yRz, then also xRz.
   That is, R is transitive if $\forall x, y, z \in A, ((xRy) \land (yRz)) \Rightarrow xRz$.

Definition 11.3 A relation R on a set A is an **equivalence relation** if it is reflexive, symmetric and transitive.

Show that $\equiv \pmod 3$ is an equivalence relation.

✓ • Reflexive: $a \equiv a \pmod 3$?

$3 \mid (a-a)$ because $0 = 3 \cdot \underset{\text{integer}}{0}$

✓ • Symmetric: if $a \equiv b \pmod 3$, then $b \equiv a \pmod 3$

$3 \mid (a-b)$, so $(a-b) = 3k$ for some $k \in \mathbb{Z}$

$-(a-b) = -3k$

$b - a = 3(-k)$

✓ • Transitive: suppose $a \equiv b \pmod 3$ and $b \equiv c \pmod 3$

then show $a \equiv c \pmod 3$

$3 \mid (a-b)$ $\quad$ $3 \mid (b-c)$

$a - b = 3p$

$b - c = 3q$

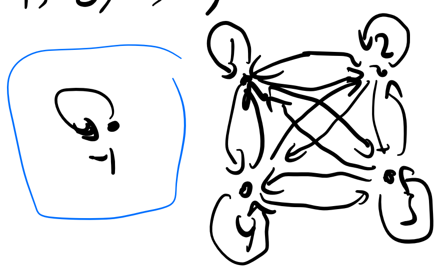
$$(a-b)+(b-c) = 3p+3q$$
$$a-c = 3(p+q)$$
$$3 \mid (a-c)$$

Definition 11.4 Suppose R is an equivalence relation on a set A. Given any element $a \in A$, the **equivalence class containing** a is the subset $\{x \in A : xRa\}$ of A consisting of all the elements of A that relate to a. This set is denoted as $[a]$. Thus the equivalence class containing a is the set $[a] = \{x \in A : xRa\}$.

$$[a] = \{x \in A : xRa\}$$

Example

$A = \{-1, 1, 2, 3, 4\}$ $R =$ "has the same sign as"



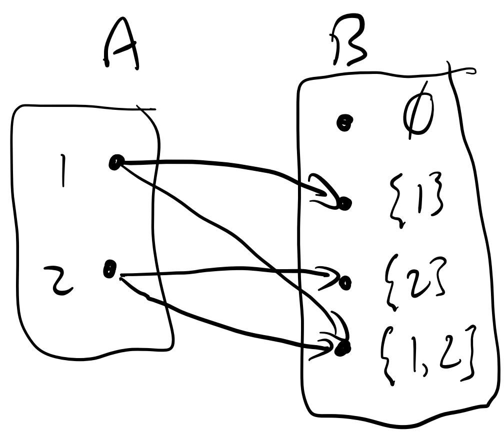
$$[-1] = \{-1\}$$
$$[1] = \{1, 2, 3, 4\} = [4]$$

Relation Between 2 sets: $R \subseteq A \times B$

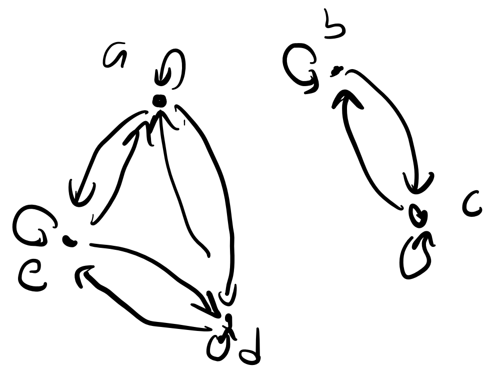
Example: $A = \{1, 2\}$, $B = P(A)$

$R = \{(1, \{1\}), (2, \{2\}), (1, \{1,2\}), (2, \{1,2\})\}$

$\in$



Ex. A.4: $A = \{a, b, c, d, e\}$



Functions (but more formally) $\quad f(x) = x^2$

Definition: Suppose A, B are sets. A function from A to B, written $f: A \to B$ is a relation $f \subseteq A \times B$ with the property:

f has exactly one element (a, b) for all $a \in A$.

Intuition: all elements of A map to some element of B.

Example: $f: \mathbb{Z} \to \mathbb{N}$ defined as $\{(n, |n|+2) : n \in \mathbb{Z}\}$