CSCI 301 - Lecture 18: Relations

or: Everything in math is a set, Part K

$$(a, b) \to \{a, \{a, b\}\}$$

$$\cancel{\{a, b\}}$$

$1 = 1 \qquad 5 \leq 9 \qquad \mathbb{Z} \subseteq \mathbb{R}$

$6 \equiv 2 \pmod 4$

Relations!

Definition 11.1 A **relation** on a set A is a subset $R \subseteq A \times A$. We often abbreviate the statement $(x, y) \in R$ as xRy. The statement $(x, y) \notin R$ is abbreviated as $x \not R\, y$.

$x R y \qquad x \not R\, y$

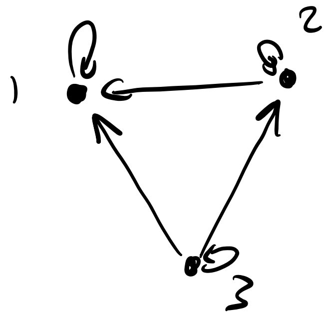
$x \leq y \qquad x \not\leq y$

Example

$A = \{1, 2, 3\}$

$R = \{(1,1), (2,1), (2,2), (3,3), (3,2), (3,1)\}$

Notice: $R \subseteq A \times A$ $\boxed{R \text{ is } \geq}$

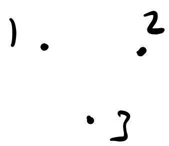


A.1 $R = \{(1,1),(2,2),(3,3)\}$

$A = \{1,2,3\}$

A.3. $R = \{(1,1),(1,3),(3,1) \dots\}$

A.4.



Properties of Relations

If R is a relation on set A,

R is reflexive if xRx for all $x \in A$

R is symmetric if $xRy \Rightarrow yRx$ for all $x,y \in A$

R is transitive if $(xRy \text{ and } yRz) \Rightarrow xRz$ for all $x,y,z \in A$

$R = \{(1,1),(2,1),(2,2),(3,3),(3,2),(3,1)\}$

Reflexive? yes!

Symmetric? No! $(2,1)$ is $\in R$, but $(1,2)$ is not

Transitive? Yes!

B. 2.4 $R = \emptyset$ $\quad xRy \Rightarrow yRx$ ✓ Symmetric

B. 3. 1 $\{(a,b),(b,a),(a,c),(c,a),(a,a),(b,b),(c,c)\}$

X not transitive

Definition 11.2 Suppose R is a relation on a set A.

1. Relation R is **reflexive** if xRx for every $x \in A$.
   That is, R is reflexive if $\forall x \in A, xRx$.
2. Relation R is **symmetric** if xRy implies yRx for all $x, y \in A$.
   That is, R is symmetric if $\forall x, y \in A, xRy \Rightarrow yRx$.
3. Relation R is **transitive** if whenever xRy and yRz, then also xRz.
   That is, R is transitive if $\forall x, y, z \in A, ((xRy) \wedge (yRz)) \Rightarrow xRz$.

Definition 11.3 A relation R on a set A is an **equivalence relation** if it is reflexive, symmetric and transitive.

Definition 11.4 Suppose R is an equivalence relation on a set A. Given any element $a \in A$, the **equivalence class containing** a is the subset $\{x \in A : xRa\}$ of A consisting of all the elements of A that relate to a. This set is denoted as $[a]$. Thus the equivalence class containing a is the set $[a] = \{x \in A : xRa\}$.