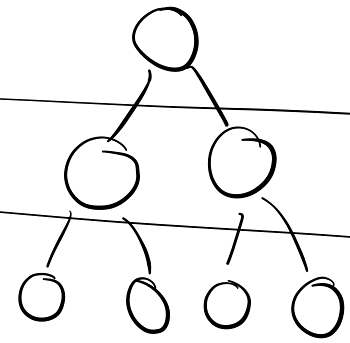
CSCI 301 - Lecture 17

$2^0 = 1$

$2^1 = 2$

$2^2 = 4$



$7 = 2^3 - 1$

$$\boxed{\sum_{i=0}^{n} 2^i = 2^{n+1} - 1}$$

Proof: By induction.

Base case: let $n=0$, $\sum_{i=0}^{0} 2^i = 2^0 = 1 = 2^1 - 1$

$2 - 1$

$= 1$

Inductive step: Suppose

$$2^0 + 2^1 + \cdots + 2^n = 2^{n+1} - 1$$

Want: $2^{n+2} - 1 = \underline{2^0 + 2^1 + \cdots + 2^n + 2^{n+1}} = 2^{n+1} - 1 + 2^{n+1}$

$2^{n+1} + 2^{n+1} - 1$

$2(2^{n+1}) - 1$

$= 2^{n+2} - 1$

We have shown that

$2^0 + 2^1 + \cdots + 2^{n+1} = 2^{n+2} - 1$, so

The claim is true for $n+1$ if it's true for n.
This completes the proof by induction. $\blacksquare$