CSCI 301 - Lecture 16: Induction!!!

$S_1 \quad 1 = 1^2$

$S_2 \quad 1+3 = 2^2$

$S_3 \quad 1+3+5 = 3^2$

$S_4 \quad 1+3+5+7 = 4^2$

S_i : The first i odd natural numbers sum to i^2.

Proof By Induction

Base case → S_1 is true because the first 1 odd natural number (1), sums to 1^2.

(Show:) If S_k is true, then S_{k+1} is true

Suppose S_k is true. Then

Inductive Hypothesis

By the I.H., $1+3+5+\cdots+2k-1 = k^2$

$$1+3+5+\cdots+(2k-1)+(2k+1) = k^2+2k+1$$
$$= (k+1)(k+1)$$
$$= (k+1)^2$$

$\boxed{S_{k+1}} \quad \underbrace{1+3+5+\cdots+(2k+1)}_{\text{The first } k+1 \text{ odd } \mathbb{N}\text{'s}}$

Inductive case (step)

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Strong Induction

Inductive Hypothesis is now:

$$S_0 \wedge S_1 \wedge S_2 \wedge \cdots \wedge S_k$$

Show that $\Rightarrow S_{k+1}$

Fibonacci Sequence

F_0 F_1 2 3 4 5 6

1 1 2 3 5 8 13 ...

$$F_0 = 1$$
$$F_1 = 1$$
$$F_n = F_{n-1} + F_{n-2}$$

A.1 Base: (...)

I.H.: $1 + 2 + \cdots + n = \frac{n^2+n}{2}$

$$1 + 2 + \cdots + n + n+1 = \frac{n^2+n}{2} + n+1$$
$$= \frac{n^2+n}{2} + \frac{2n+2}{2}$$
$$= \frac{n^2+3n+2}{2}$$
$$(n^2+2n+1) + (n+1)$$

want: $\frac{(n+1)^2 + (n+1)}{2} \longleftrightarrow \frac{(n+1)^2 + (n+1)}{2}$

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