CSCI 301 - Lecture 14

- Proving Nonconditional Stmts
- Disproof

Proving iff statements.

To prove $P \Leftrightarrow Q$, show $P \Rightarrow Q$ and show $Q \Rightarrow P$

Proving Equivalences

- P x is even
- Q $2 \mid x$
- R x is not odd

$$\begin{tikzcd} & P \arrow[dr, Rightarrow] & \\ R \arrow[ur, Rightarrow] & & Q \arrow[ll] \end{tikzcd}$$

$P \Leftrightarrow Q$

$P \Leftrightarrow R$

Ex. A.1 Suppose $a \in \mathbb{Z}$. Then $14 \mid a$ iff $7 \mid a$ and $2 \mid a$.

$\Rightarrow$ Suppose $14 \mid a$. Then $a = 14b$ for some $b \in \mathbb{Z}$

So $a = 14b = 7 \cdot 2b$.

This shows that $7 \mid a$ because $a = 7(2b)$, $2b \in \mathbb{Z}$.

and $2 \mid a$ because $a = 2(7b)$, $7b \in \mathbb{Z}$.

$\Leftarrow$ Suppose $7 \mid a$ and $2 \mid a$.

Then $a = 7c$ for some integer c, and $a = 2d$ for some integer d.

$a = 7c = 2d$, meaning ~~a must be even~~, and $7c$ is even. If $7c$ is even, then c is even.

$2 \mid 7c$, meaning $7c = 7 \cdot 2e$ for some e.

$a = 7c = 7 \cdot 2e = 14e$ for some $e \in \mathbb{Z}$.

So $14 \mid a$.

Proving $\exists x : P(x)$:

Give an x and show $P(x)$!

Disproof

Some things are not true.

How to disprove P: Prove $\sim P$.

How to disprove $\forall x \in S, P(x)$.
Produce an example of an $x \in S$ that makes $P(x)$ false.

$\forall x \in S, P(x)$

$\exists x \in S, \neg P(x)$

How to disprove $P(x) \Rightarrow Q(x)$.
Produce an example of an x that makes $P(x)$ true and $Q(x)$ false.

$P(x) \Rightarrow Q(x)$

$\exists x, P(x) \wedge \neg Q(x)$

How to disprove P with contradiction:
Assume P is true, and deduce a contradiction.

$\exists x \in S, P(x)$

$\forall x \in S, \neg P(x)$