

CSC 301 Lecture 10 - Direct Proof

Definition: An integer n is even if $n = 2a$ for some integer a .

↑
if and only if
↔

Definition: An integer n is odd if $n = 2a + 1$ for some integer a .

Definition: Suppose a and b are integers. We say a divides b , $a \mid b$, if $b = ac$ for some $c \in \mathbb{Z}$. a is a divisor of b , and b is a multiple of a .

Definition: Two integers have the same parity if they are both even or both odd. Otherwise they have opposite parity.

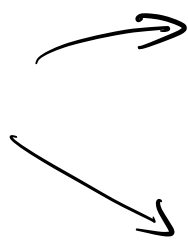
Proof: if P , then Q

$$P \Rightarrow Q$$

Suppose P ... show Q

Outline for Direct Proof

Some
circles



Proposition If P , then Q .
Proof. Suppose P .
 \vdots
Therefore Q . ■

the rest of
the owl

Example: If x is odd, x^2 is odd.

Proof: Suppose x is odd.

Then $x = 2a+1$ for some $a \in \mathbb{Z}$.

$$x^2 = (2a+1)^2 = 4a^2 + 4a + 1$$

$$2(2a^2 + 2a) + 1$$

$$x^2 = 2b+1 \text{ for } b = 2a^2 + 2a.$$

Therefore x^2 is odd. ■ \leftarrow QED

Cases

If x is an integer, x^2 has the same parity as x .

Case 1 (even): Suppose $x \in \mathbb{Z}$ and x is even.
(proof)

Case 2 (odd) Suppose $x \in \mathbb{Z}$ and x is odd.
(proof)



WLOG

If two integers have opposite parity,

Then their sum is odd.

a is even
b is odd

Case 1

a is odd
b is even

Case 2

Without loss of generality, suppose

a is even and b is odd.