CSCI 301 Lecture 10 - Direct Proof

Definition: An integer n is <u>even</u> if $n = 2a$ for some integer a.

if and only if

$\Leftrightarrow$

Definition: An integer n is <u>odd</u> if $n = 2a+1$ for some integer a.

Definition: Suppose a and b are integers. We say a <u>divides</u> b, $a \mid b$, if $b = ac$ for some $c \in \mathbb{Z}$. a is a <u>divisor</u> of b, and b is a <u>multiple</u> of a.

Definition: Two integers have <u>the same parity</u> if they are both even or both odd. Otherwise they have <u>opposite parity</u>

Proof: if P, then Q

$$P \Rightarrow Q$$

Suppose P ... show Q

Some circles

Outline for Direct Proof

Proposition If P, then Q.

Proof. Suppose P.

$\vdots$ the rest of the owl

Therefore Q. $\blacksquare$

Example: If x is odd, x^2 is odd.

Proof: Suppose x is odd.

Then $x = 2a+1$ for some $a \in \mathbb{Z}$.

$$x^2 = (2a+1)^2 = 4a^2 + 4a + 1$$
$$2(2a^2+2a)+1$$

$x^2 = 2b+1$ for $b = 2a^2+2a$.

Therefore x^2 is odd. $\blacksquare$ QED

Cases

If x is an integer, x^2 has the same parity as x.

Case 1 (even): Suppose $x \in \mathbb{Z}$ and x is even.

(proof)

Case 2 (odd) Suppose $x \in \mathbb{Z}$ and x is odd.

(proof)

$\blacksquare$

WLOG

If two integers have opposite parity, then their sum is odd.

Case 1	Case 2
a is even b is odd	a is odd b is even

Without loss of generality, suppose a is even and b is odd.