

# Some Useful Equivalences - A Cheat Sheet

## Fairly intuitive:

Commutative:

$$\begin{aligned}P \wedge Q &\equiv Q \wedge P \\P \vee Q &\equiv Q \vee P\end{aligned}$$

Distributive:

$$\begin{aligned}P \wedge (Q \vee R) &\equiv (P \wedge Q) \vee (P \wedge R) \\P \vee (Q \wedge R) &\equiv (P \vee Q) \wedge (P \vee R)\end{aligned}$$

Associative:

$$\begin{aligned}P \wedge (Q \wedge R) &\equiv (P \wedge Q) \wedge R \\P \vee (Q \vee R) &\equiv (P \vee Q) \vee R\end{aligned}$$

## Less intuitive:

Contrapositive:

$$P \Rightarrow Q \equiv (\neg Q) \Rightarrow (\neg P) \tag{1}$$

DeMorgan's Laws:

$$\begin{aligned}\neg(P \wedge Q) &\equiv (\neg P) \vee (\neg Q) \\ \neg(P \vee Q) &\equiv (\neg P) \wedge (\neg Q)\end{aligned}$$

Negating Quantifiers:

$$\begin{aligned}\neg(\forall x \in X, P(x)) &\equiv \exists x \in X, \neg P(x) \\ \neg(\exists x \in X, P(x)) &\equiv \forall x \in X, \neg P(x)\end{aligned}$$

Negating conditionals:

$$\neg(P \Rightarrow Q) \equiv P \wedge \neg Q \tag{2}$$