Lecture 9 - Exercises

Part A - Equivalences

1. Verify using a truth table that $P \Leftrightarrow Q$ is logically equivalent to $(P \wedge Q) \vee (\neg P \wedge \neg Q)$

2. Verify that $P \Rightarrow Q$ is equivalent to $(\neg Q) \Rightarrow (\neg P)$. This one is important enough to have a name: it is called the **contrapositive**.

Part B - Quantifiers

1. Translate each of the following English statements into symbolic form.
    1. All natural numbers are integers.

    2. Every integer that is not even is odd.

    3. For every integer x, there is an integer y for which $x + y = 0$.

2. Translate each of the following into English, and say whether they are true or false.
   1. $\forall x \in \mathbb{R}, x^2 > 0$
   2. $\exists a \in \mathbb{R}, \forall x \in \mathbb{R}, ax = x$
   3. $\forall n \in \mathbb{N}, \exists X \in \mathcal{P}(\mathbb{N}), |X| < n$

Part C - Negating Statements

For each of the following, convert it to symbols, negate it, then translate it back into English.

1. x is positive, but y is not positive

2. Every even integer greater than 2 is the sum of two primes.

Let $P = \{p : p \text{ is prime}\}$

$$\forall x \in \mathbb{Z}, (\underbrace{x \text{ is even}}_{A} \wedge \underbrace{x > 2}_{B}) \Rightarrow \underbrace{\exists p, q \in P, p + q = x}_{C}$$

$$\neg(\forall x \in \mathbb{Z}, A \wedge B \Rightarrow C)$$

$$\exists x \in \mathbb{Z}, \neg((A \wedge B) \Rightarrow C)$$

$$\neg(P \Rightarrow Q) \equiv P \wedge \neg Q$$

3. $2a$ is even if and only if a is an integer

$$\exists x \in \mathbb{Z} \; ((A \wedge B) \wedge \neg C)$$