Logical Operators: A Logic Puzzle

Truth table:

P	
T	
F	

P	Q	
T	T	
T	F	
F	T	
F	F	

Three important truth tables:

P	Q	P ∧ Q
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	P ∨ Q
T	T	T
T	F	T
F	T	T
F	F	F

P	¬P
T	F
F	T

Given the above truth tables, describe the meaning of the three operators: ∧, ∨, ¬

∧: and: P ∧ Q is true if both P, Q are true, false otherwise

∨: or: P ∨ Q is true if at least one of P, Q is true, false otherwise

~ ¬: not: ¬P is true if P is false, false if P is true

\neg

Do Exercises Pt. A

8. $(N \vee P) \wedge \neg(Q \wedge S)$

$(T \vee T) \wedge \neg(F \wedge T)$

$T \wedge \neg(F)$

$T \wedge T$

T

Logical Operator Logic Puzzle - Pt 4: Conditional operator

"if P, then Q"

P	Q	$P \Rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

Example: "if $x = y$, then $x^2 = y^2$" is a true statement

Notice: this does not commute!

for all x, y if $x^2 = y^2$, then $x = y$" is a false statement

Weirdness #1: False Implies Anything

"False $\Rightarrow$ True" is true

False $\Rightarrow$ False" is true

P	Q	P $\Rightarrow$ Q
T	T	T
T	F	F
F	T	T
F	F	T

If P, then Q

Sane Example:

P: I am healthy

Q: I will come to class

Don't Panic! Tips for staying sane:

- Apply the truth table; don't think too hard.
- Think of P $\Rightarrow$ Q as a promise that Q is true if P is true
- The first 2 rows are the useful ones!

Do Exercises Part B

Insane example:

"if 1=2, then 30=10"

"if 1=2, then 30=30"

Weirdness #2: Natural Language

P	Q	P ⇒ Q
T	T	T
T	F	F
F	T	T
F	F	T

Stylistic Variants of Conditional Statements

- If P then Q
- Q if P
- Q whenever P
- Q, provided that P
- Whenever P, then also Q
- P is a sufficient condition for Q
- For Q it is sufficient that P
- For P it is necessary that Q
- P only if Q

$P \Rightarrow Q$

Definition (aside): $Q \Rightarrow P$ is the converse of $P \Rightarrow Q$

Do Exercises Pt. C

you fail (P) only if you stop writing (Q)

$P \Rightarrow Q$

If P then Q

If you fail, you stop writing

you fail ↓ you stop writing ↓

P	Q	P ⇒ Q
T	T	T
T	F	F
F	T	T
F	F	T

End of Lecture 6

Biconditional Statements

"P if and only if Q"

P ___ Q

Do Exercises Pt D

Logical Equivalence

Truth Table for $P \Rightarrow Q \wedge Q \Rightarrow P$