

Logical Operators: A Logic Puzzle

Truth table:

P	
T	
F	

P	Q	
T	T	
T	F	
F	T	
F	F	

Three important truth tables:

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

P	$\neg P$
T	F
F	T

Given the above truth tables, describe the meaning of the three operators: \wedge , \vee , \neg

\wedge : and : $P \wedge Q$ is true if both P, Q are true, false otherwise

\vee : or : $P \vee Q$ is true if at least one of P, Q is true, false otherwise

\neg : not : $\neg P$ is true if P is false, false if P is true

neg

Do Exercises Pt. A

$$8. (N \vee P) \wedge \neg(Q \wedge S)$$

$$(T \vee T) \wedge \neg(F \wedge T)$$

$$\frac{\quad}{T \wedge \neg(F)}$$

$$\frac{T \wedge T}{T}$$

Logical Operator Logic Puzzle - Pt 4: Conditional operator

"if P, then Q"

P	Q	<u>$P \Rightarrow Q$</u>
T	T	T
T	F	F
F	T	T
F	F	T

Example: "if $x = y$, then $x^2 = y^2$ " is a true statement

Notice: this does not commute!

for all x, y if $x^2 = y^2$, then $x = y$ is a false statement

Weirdness # 1: False Implies Anything

"False \Rightarrow True" is true

False \Rightarrow False" is true

P	Q	<u>$P \Rightarrow Q$</u>
T	T	T
T	F	F
F	T	T
F	F	T

Sane Example:

P: I am healthy

Q: I will come to class

If P, then Q

Don't Panic! Tips for staying sane:

- Apply the truth table; don't think too hard.
- Think of $P \Rightarrow Q$ as a promise that Q is true if P is true
- The first 2 rows are the useful ones!

Do Exercises Part B

Insane example:

"if $1=2$, then $30=10$ "

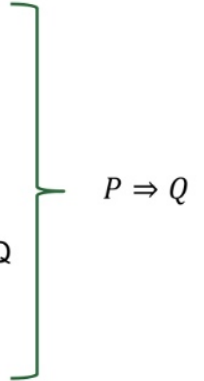
"if $1=2$, then $30=30$ "

Weirdness #2: Natural Language

P	Q	<u>$P \Rightarrow Q$</u>
T	T	T
T	F	F
F	T	T
F	F	T

Stylistic Variants of Conditional Statements

- If P then Q
- Q if P
- Q whenever P
- Q, provided that P
- Whenever P, then also Q
- P is a sufficient condition for Q
- For Q it is sufficient that P
- For P it is necessary that Q
- **P only if Q**



Definition: $Q \Rightarrow P$ is the converse of $P \Rightarrow Q$ (aside)

Do Exercises Pt. C

you fail only if you stop writing
 P Q

$P \Rightarrow Q$

If P then Q

If you fail, you stop writing

you fail ✓ you stop writing

P	Q	<u>$P \Rightarrow Q$</u>
T	T	T
T	F	F
F	T	T
F	F	T

— End of Lecture 6 —

Biconditional Statements

"P if and only if Q"

P Q'

Do Exercises Pt D

Logical Equivalence

Truth Table for $P \Rightarrow Q \wedge Q \Rightarrow P$