CSCI 301 - Lecture 5: Sets 2, Logic 1

Sometimes it's natural to define a universal set U for a set S.

This is informal and context-dependent: The only requirement is $S \subseteq U$

Examples:

$\{x : x \text{ is even}\}$ might have universal set: $\mathbb{Z}$

Definition: For a set S with universal set U, the complement of S, written $\overline{S}$ is $U - S$: everything not in S

Example:

$A = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$

$B = \{11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$

$U = \{x : x \in \mathbb{N} \text{ and } x \leq 20\}$

$\overline{A} = \{x : x \text{ is even and } x \in U\}$

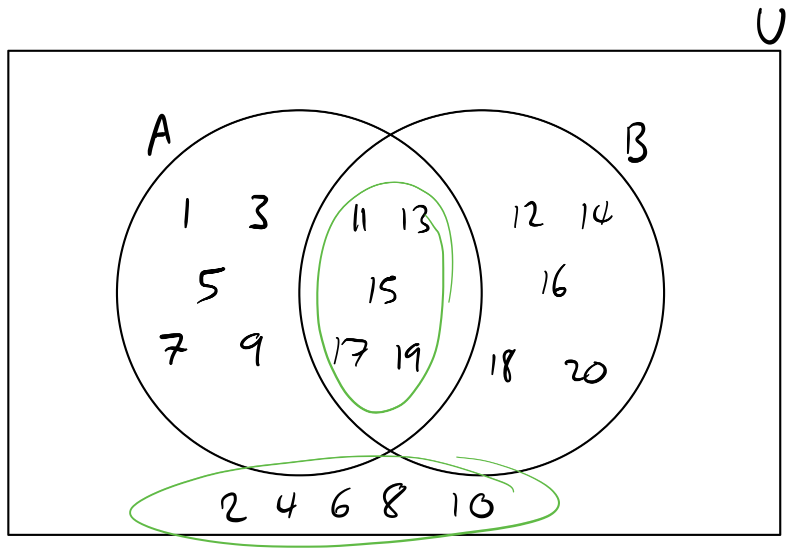
$\overline{B} = \{x : x \leq 10 \text{ and } x \in U\}$

Venn Diagrams

$A = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$

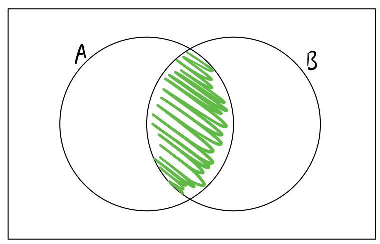
$B = \{11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$

$U = \{x : x \in \mathbb{N} \text{ and } x \leq 20\}$



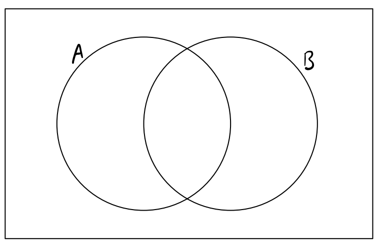
Can help visualize set operations. For example:

$A \cap B$ is everything in **both** circles.

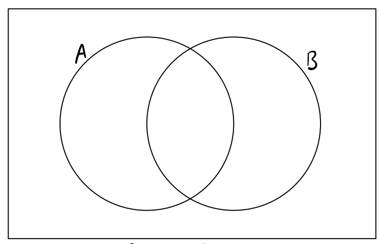


Do Exercises Pt. A

(Exercises):

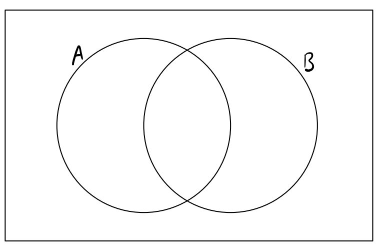


$A \cup B$

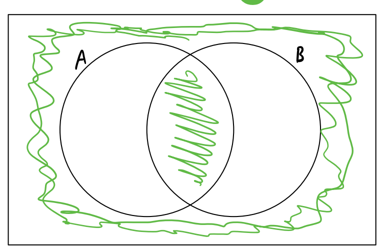


$A - B$

$\{x : x \in U ; (x \text{ is even and } \leq 10) \text{ or } (x \text{ is odd and } x > 10)\}$

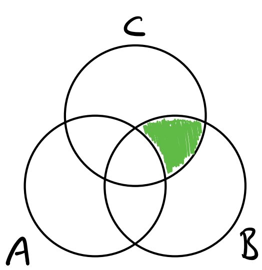


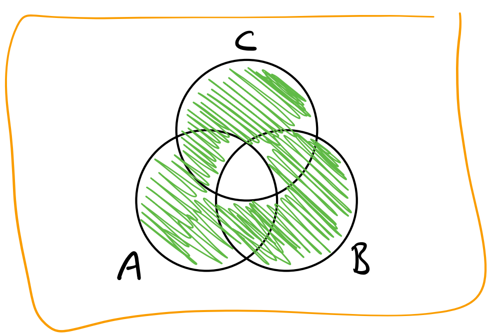
$\bar{B}$



$\overline{(A \cup B) - (A \cap B)}$

$(B \cap C) - A$





$(A \cup B \cup C) - (A \cap B \cap C)$

Logic 1

A statement is a sentence or expression that is either definitely true or definitely false.

✗ - Add 5 to both sides

✗ - $\mathbb{Z}$

✓ - Adding 5 to both sides of $x-5=37$ gives $x=42$

✓ - $42 \in \mathbb{Z}$

✗ - 42

✗ - What is the solution of $2x=84$?

✓ - The solution of $2x=84$ is 41

✓ - 42 is not a number

Do Exercises Pt B

x is a multiple of 8: open sentence

Logical Operators: A Logic Puzzle

Truth table:

P	
T	
F	

P	Q	
T	T	
T	F	
F	T	
F	F	

Three important truth tables:

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

P	$\neg P$
T	F
F	T

Given the above truth tables, describe the meaning of the three operators: $\wedge$, $\vee$, $\neg$

$\wedge$: and : $P \wedge Q$ is true if both P, Q are true, false otherwise

$\vee$: or : $P \vee Q$ is true if at least one of P, Q is true, false otherwise

$\neg$: not : $\neg P$ is true if P is false, false if P is true

— End of Lecture 5 —

Do Exercises Pt. C

P	Q	$P \Rightarrow Q$