

# CSCI 301 - Lecture 4: Sets 2

Definition: Let  $A$  and  $B$  be sets.  $A$  is a **subset** of  $B$  if every element of  $A$  is in  $B$ .

Notation:  $A \subseteq B$

Examples:

$$A = \{1, 2, 3\}$$

$$B = \{2, 3, 1\}$$

$$C = \{1, 2\}$$

True Facts:

$$A \subseteq B$$

$$B \subseteq A$$

$$C \subseteq A$$

$$C \not\subseteq A$$

$$A \not\subseteq C$$

Aside:

If  $A \subseteq B$  and  $A \neq B$ , then

$A \subset B$   $A$  is a "proper subset" of  $B$ .

$$\{n^2 : n \in \mathbb{Z}\} \subseteq \mathbb{Z}$$

Definition: The **power set** of a set  $A$  is the set of all subsets of  $A$ .

$$\mathcal{P}(A) = \{S : S \subseteq A\}$$

(Sometimes also written  $2^A$ )

$$A = \{a, \emptyset\}$$

**T** 1.  $a \in A$

**F** 2.  $\{a\} \in A$

**F** 3.  $a \subseteq A$

**T** 4.  $\{a\} \subseteq A$

**T** 5.  $\emptyset \subseteq A$

**T** 6.  $\{\emptyset\} \subseteq A$

**F** 7.  $\{\emptyset\} \in A$

$$\mathcal{P}(\{1, 2\}) = \{\{1, 2\}, \{1\}, \{2\}, \emptyset\}$$

$$\mathcal{P}(\{1\}) =$$

$$3. \mathcal{P}(\emptyset) = \{\emptyset\} \quad \emptyset = \{\}$$

$$\mathcal{P}(\{1, \{2\}\}) =$$

$$5. \mathcal{P}(\{\emptyset\}) = \{\emptyset, \{\emptyset\}\}$$

Ordered pair (aka 2-tuple, or just pair)

Written  $(a, b)$

↳ tuple generalizes this to any # of elements

$(a, 4, 4)$  is a 3-tuple

Definition: The Cartesian product of sets  $A$  and  $B$

$(A \times B)$  is the set of ordered pairs

$(a, b)$  where  $a \in A$ ,  $b \in B$ .

$$A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$$

$$A = \{a, b\}$$

$$B = \{1, 2, 3\}$$

$$A \times B = \{(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3)\}$$

Generalization:  $A \times B \times C$  - 3-tuples

$$\emptyset \times B = \{\} = \emptyset$$

Union  $A \cup B$   $\{x: x \in A \text{ or } x \in B\}$   
Intersection  $A \cap B$   $\{x: x \in A \text{ and } x \in B\}$   
difference  $A - B$   $\{x: x \in A \text{ and } x \notin B\}$   
or  $A \setminus B$

$$S = \{1, 2, 3\}$$
$$T = \{2, 4\}$$
$$S \cup T = \{1, 2, 3, 4\}$$
$$S \cap T = \{2\}$$
$$S - T = \{1, 3\}$$
$$T - S = \{4\}$$