

CSCI 301 - Assignment 4, Fall 2024

Your name here

Modify the .tex source file for this document, replacing the placeholders with your solutions. This is an individual assignment. See the syllabus for the collaboration policy.

1. (5 points) Let $a, b \in \mathbb{R}$ and $a \neq 0$. Prove that if a is rational and ab is irrational, then b is irrational.

Proof: Replace this text with your proof.

2. (5 points) Prove or disprove the following:

Proposition: There exist two prime numbers p and q for which $p - q = 97$.

This statement is (*replace this text with true or false*).

Proof: Replace this text with your proof.

3. (5 points) Prove that $|P(A)| = 2^{|A|}$. *Hint:* Think back to Lab 2.

Proof: Replace this text with your proof.

4. (5 points) **Definition:** A number n is **prime** if it has exactly two positive divisors, 1 and n . If it has more than two positive divisors, it is **composite**.

Here's an important theorem you may have seen before:

Fundamental Theorem of Arithmetic: Any integer greater than 1 has a **unique** prime factorization.

Prove the following *part* of the Fundamental Theorem of Arithmetic: Any integer n greater than 1 has a prime factorization (you need not prove its uniqueness). In other words, n can be written as a product of prime factors, as in $n = p_1 p_2 \cdots p_k$, where p_i is prime for all $1 \leq i \leq k$.

Proof: Replace this text with your proof.