CSCI 241

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Binary Search Trees: Removal

Goals

Be able to remove a node from a BST on paper.

Be prepared to implement BST removal.

Warm-up

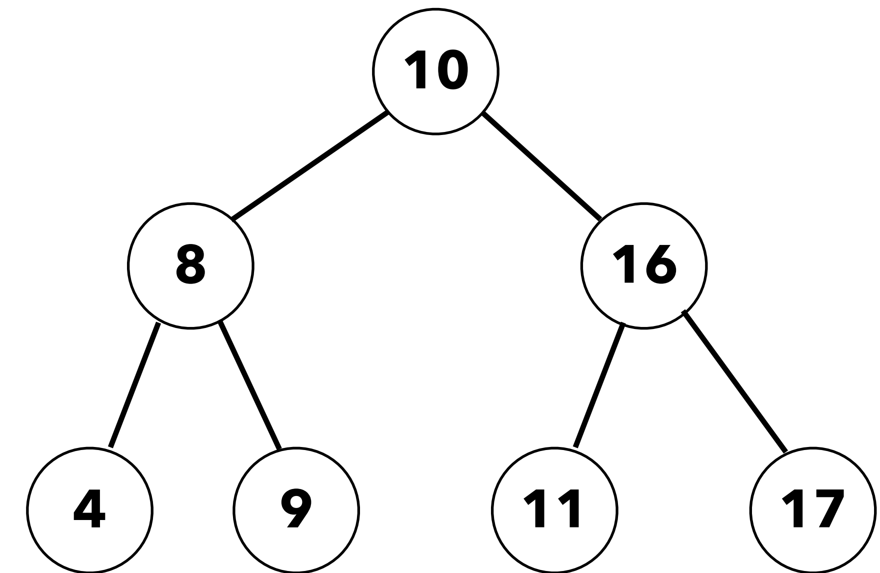
1. Spec ✓
2. Base case
3. Recursive definition
4. Implement 3 with recursive calls.

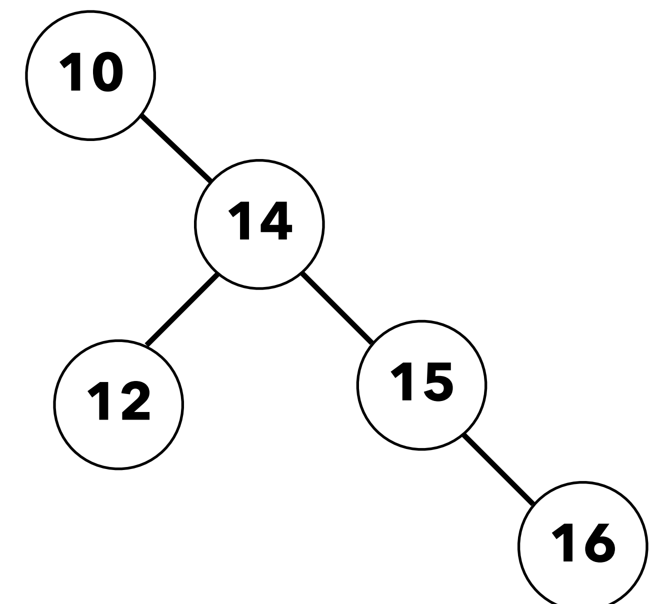
Write a method to find the smallest value in a BST:

```
/** Returns min value in BST n.
  * pre: n is not null */
public int minimum(Node n) {



}
```





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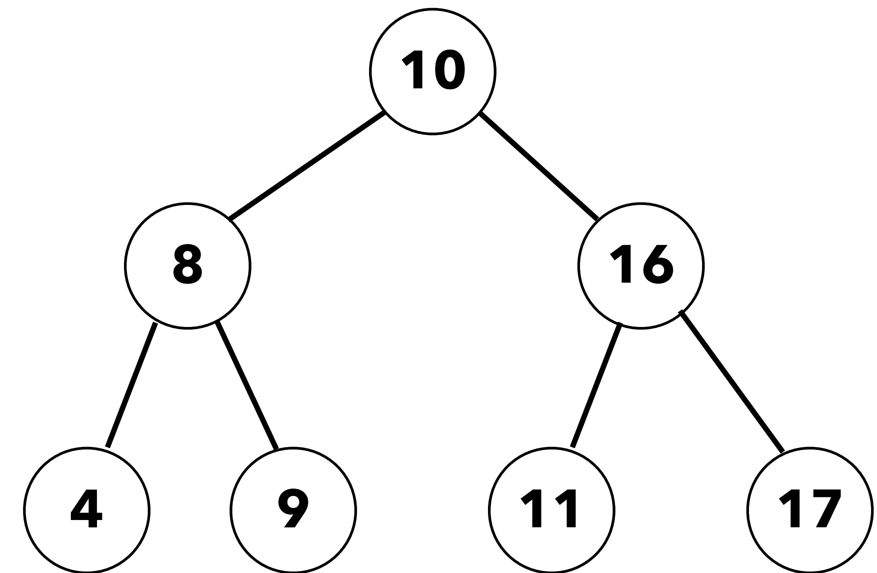
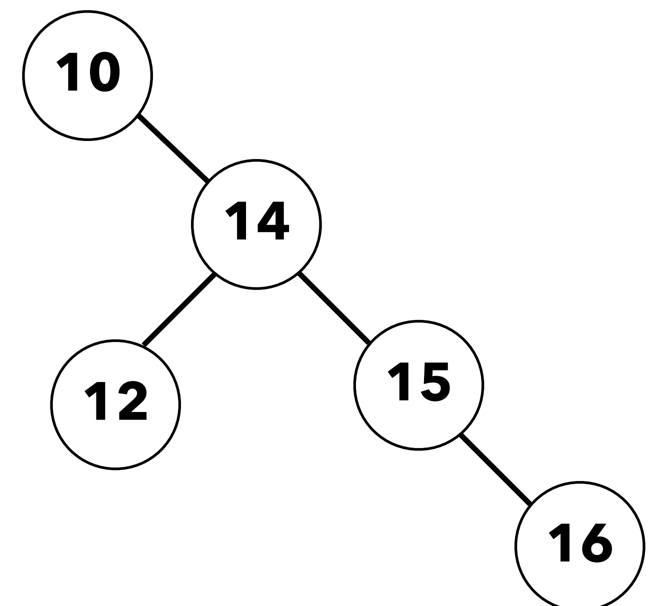
Warm-up

Write a method to find the smallest value in a BST:

```
/** Returns min value in BST n.
  * pre: n is not null */
public int minimum(Node n) {
  if (n.left == null)
    return n.value;

}
```





1. ✓ Spec
2. ✓ Base case
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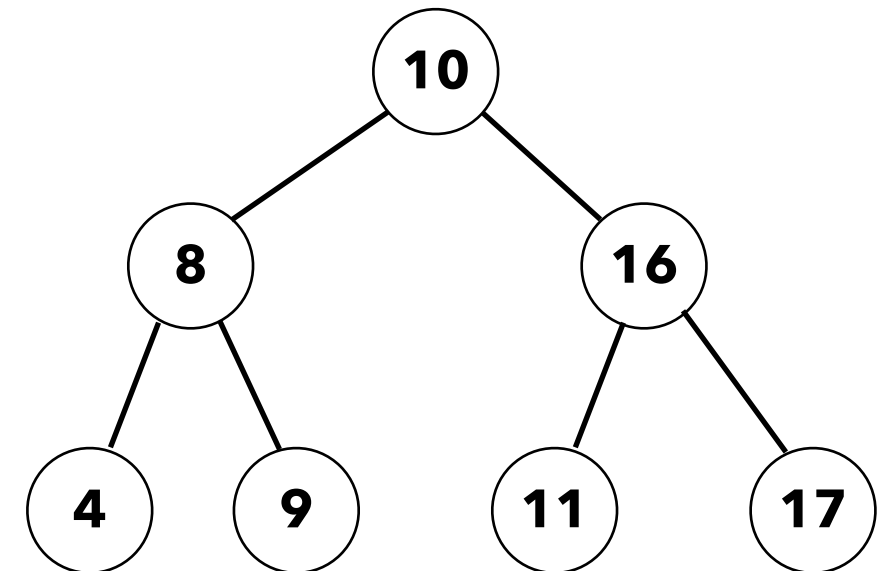
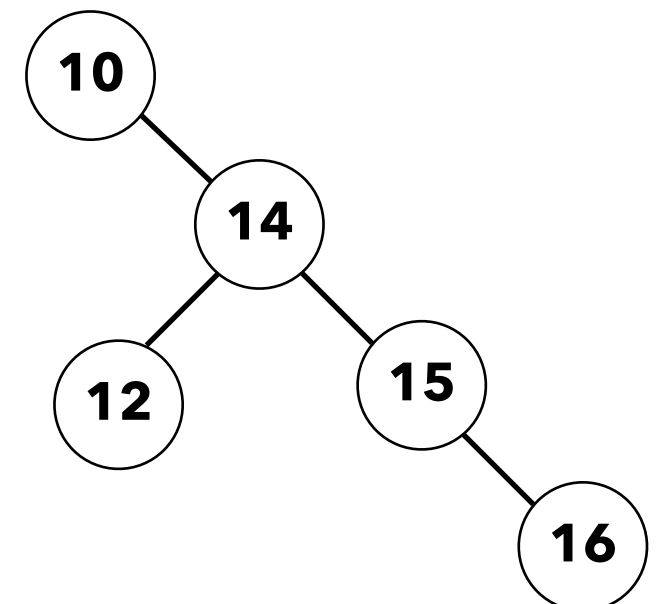


3. Recursive definition:

Smallest(n) is:

- the smallest value in the left subtree, or
- n.value if no left subtree exists.

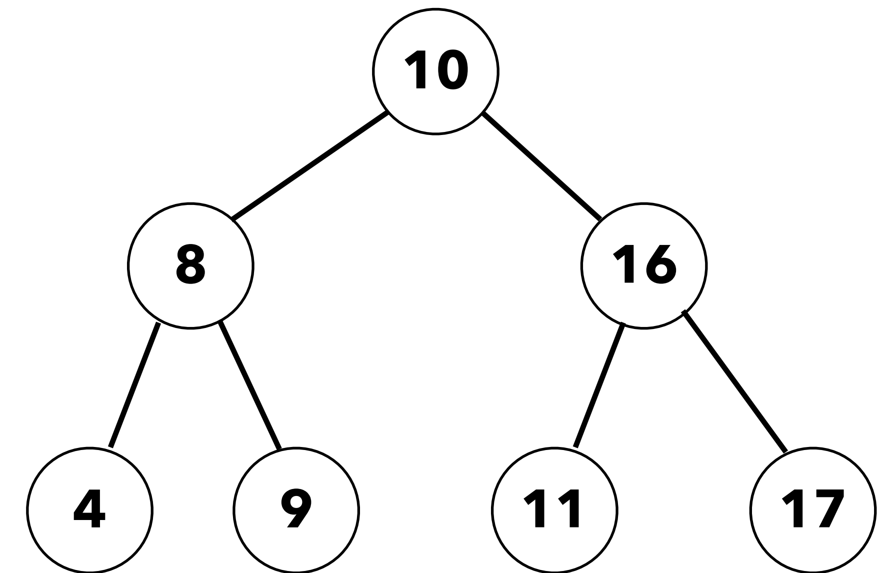


1. Spec ✓
2. Base case ✓
3. Recursive definition ✓
4. Implement 3 with recursive calls. ✓

Warm-up

Write a method to find the smallest value in a BST:

```
/** Returns min value in BST n.
  * pre: n is not null */
public int minimum(Node n) {
  if (n.left == null)
    return n.value;
  return minimum(n.left);
}
```



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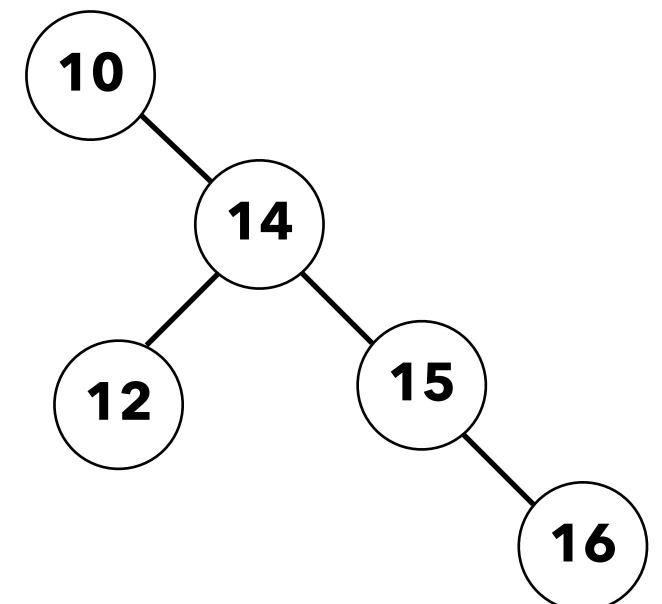
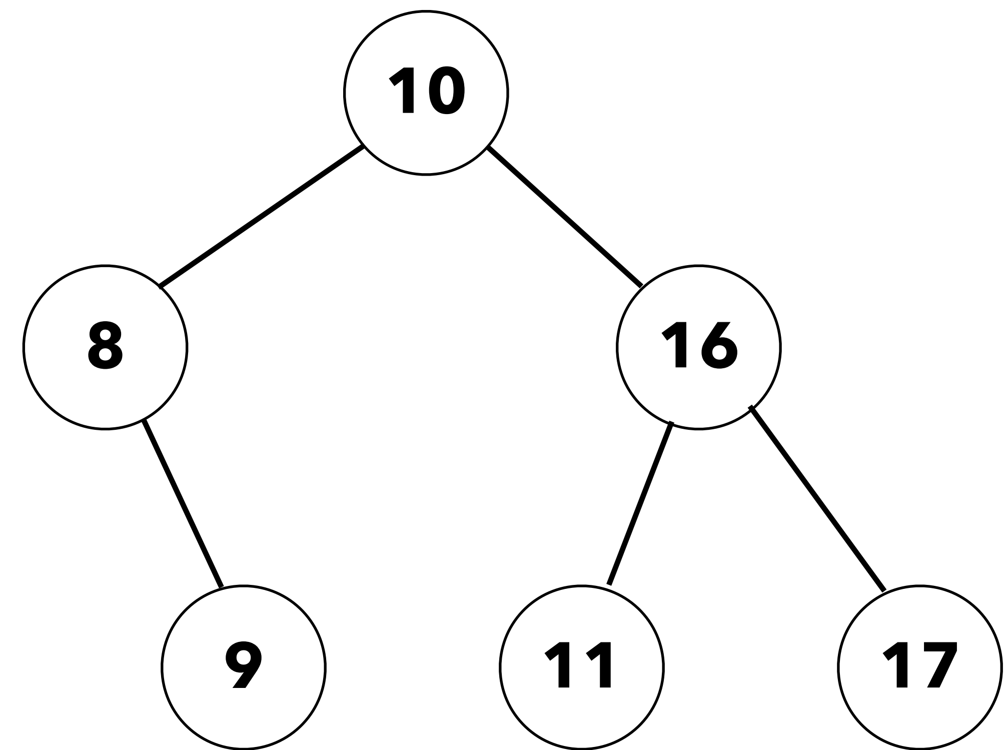


Deleting a node from a BST

Three possible cases:

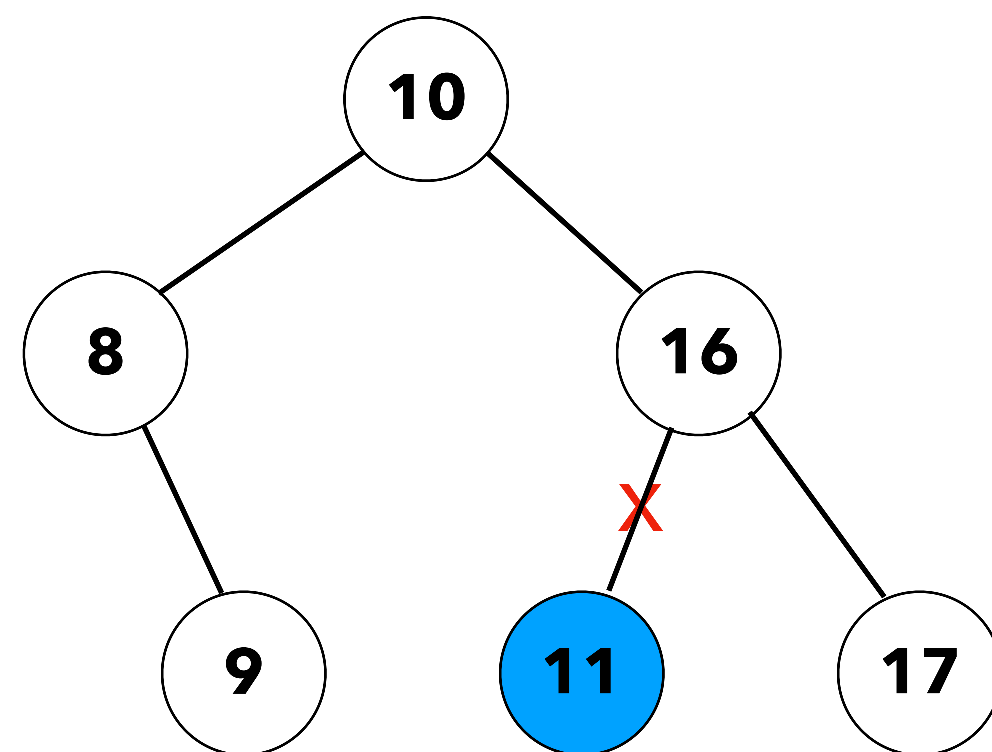
1. n has no children (is a leaf)
2. n has one child
3. n has two children



Deleting a node from a BST: Case 1

Three possible cases:

1. **n has no children (is a leaf)**
2. n has one child
3. n has two children

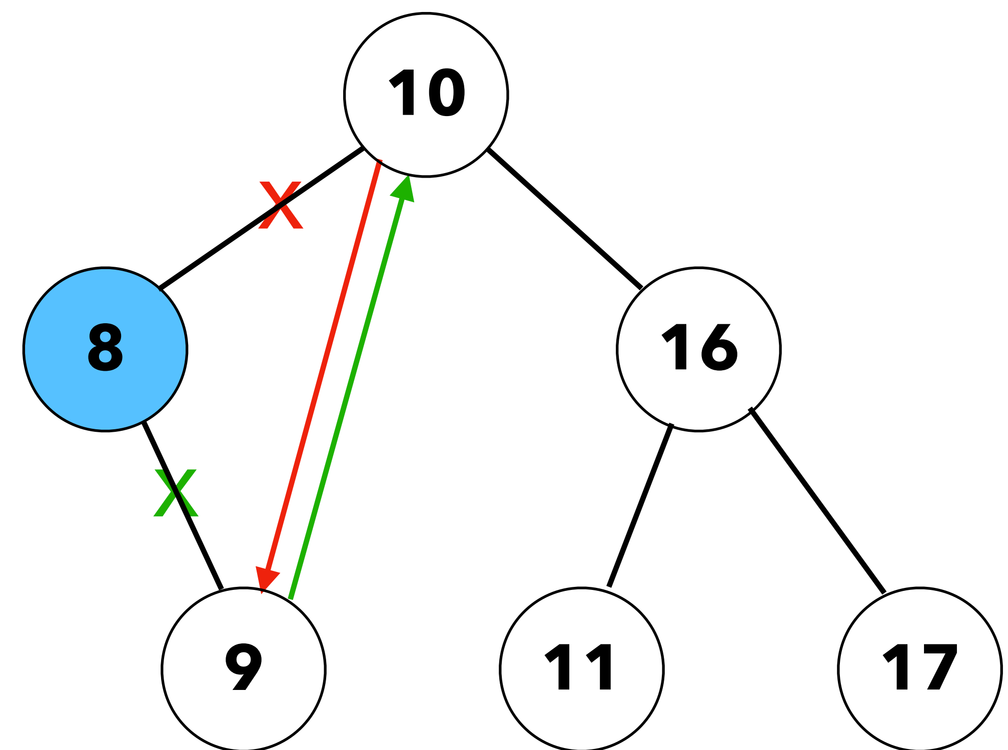



```
if (n is a leaf)
   replace parent’s child with null
```

Deleting a node from a BST: Case 2

Three possible cases:

1. n has no children (is a leaf)
2. **n has one child**
3. n has two children

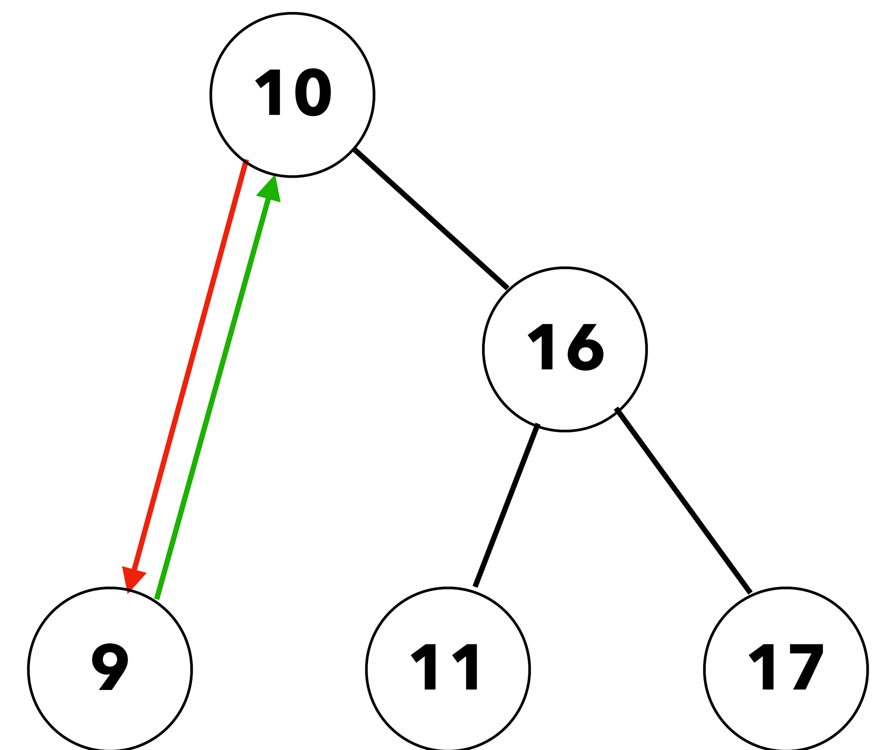



```
if (n has exactly one child)
   replace parent's child with n's child
   replace n's child's parent with n's parent
```

Deleting a node from a BST: Case 2

Three possible cases:

1. n has no children (is a leaf)
2. **n has one child**
3. n has two children

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if (n has exactly one child)
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Deleting a node from a BST: Case 2

Three possible cases:

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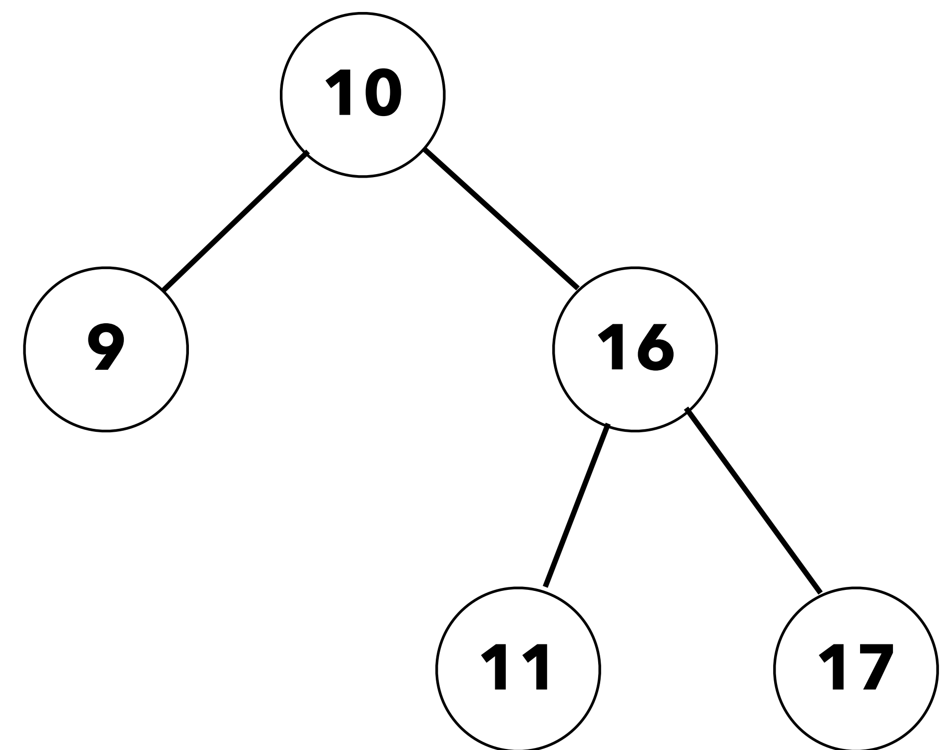


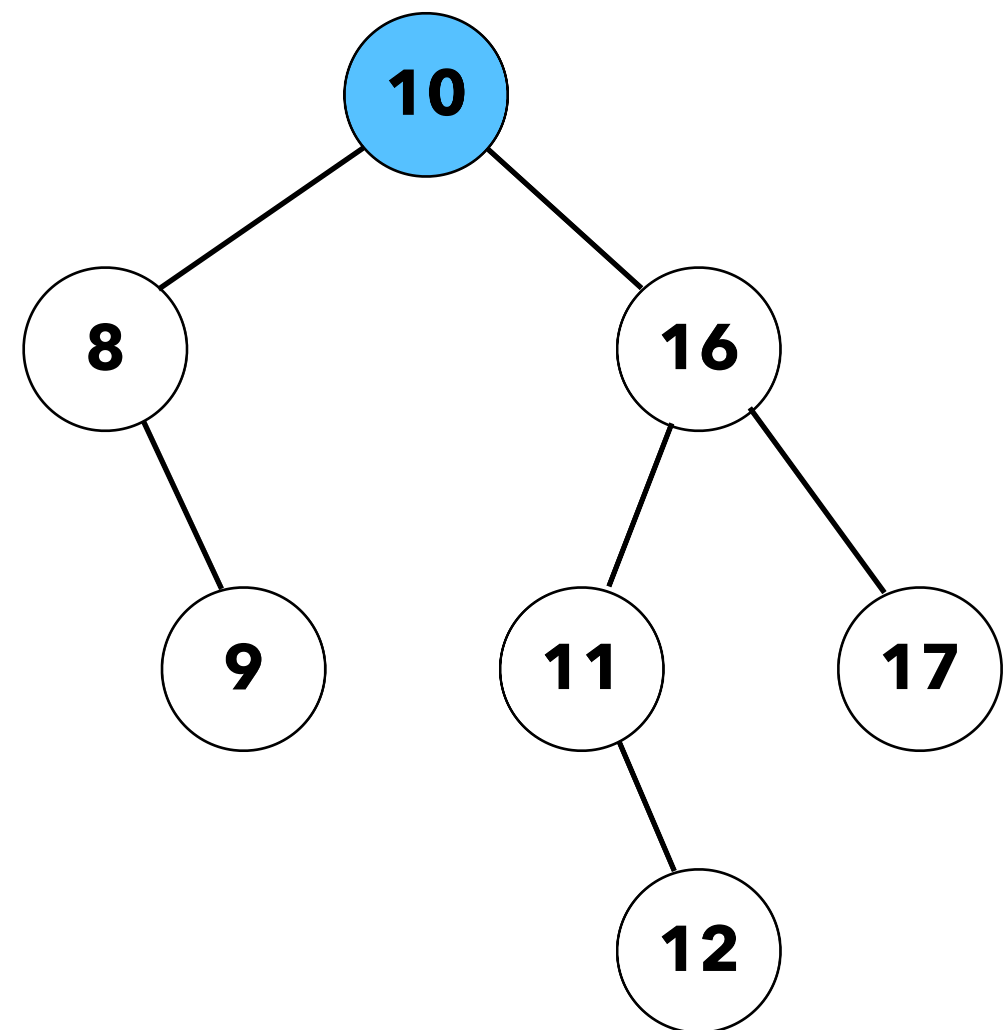

```
if (n has exactly one child)
  replace parent's child with n's child
  replace n's child's parent with n's parent
```

Deleting a node from a BST: Case 3

Three possible cases:

1. n has no children (is a leaf)
2. n has one child
3. **n has two children**

```
if (n has two children)
```

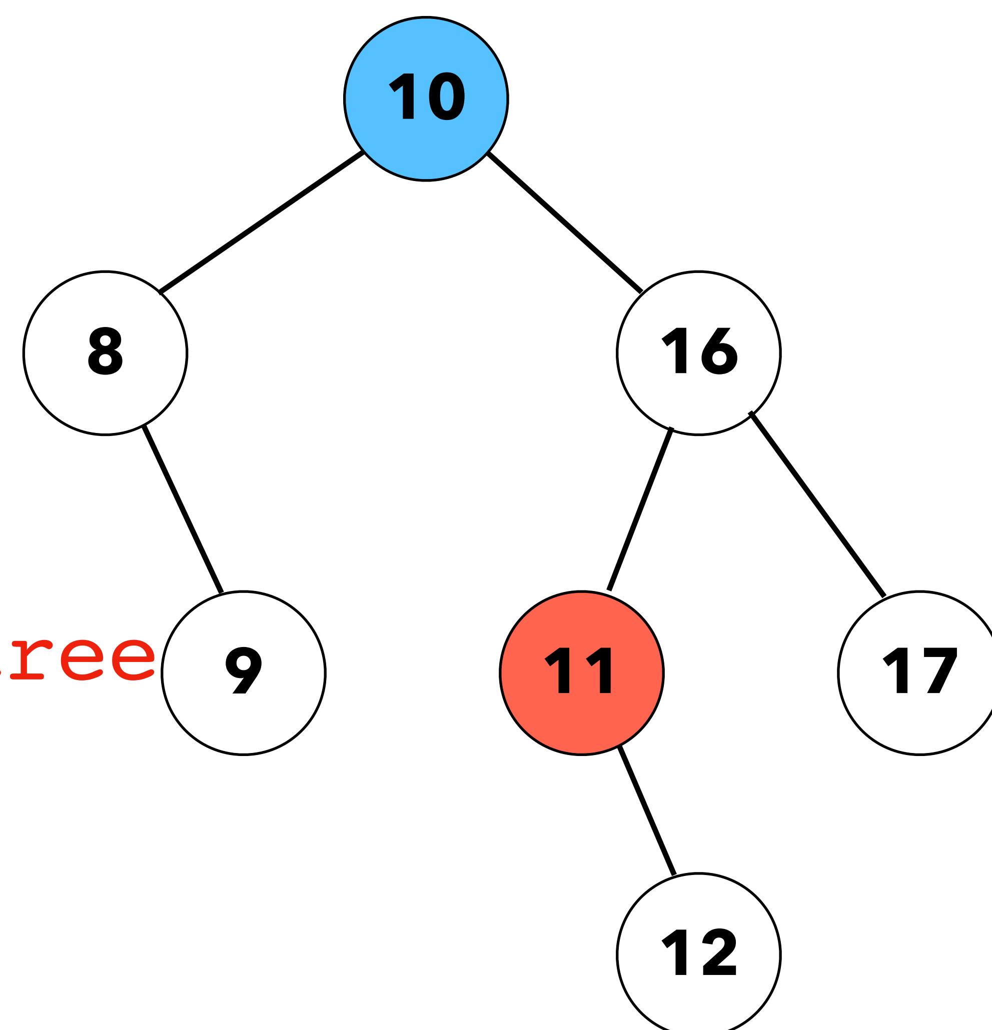


Deleting a node from a BST: Case 3

Three possible cases:

1. n has no children (is a leaf)
2. n has one child
3. **n has two children**

```
if (n has two children)
  let k = min node in right subtree
```



Deleting a node from a BST: Case 3

Three possible cases:

1. n has no children (is a leaf)
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if (n has two children)
   let k = min node in right subtree
   replace n's value with k's value
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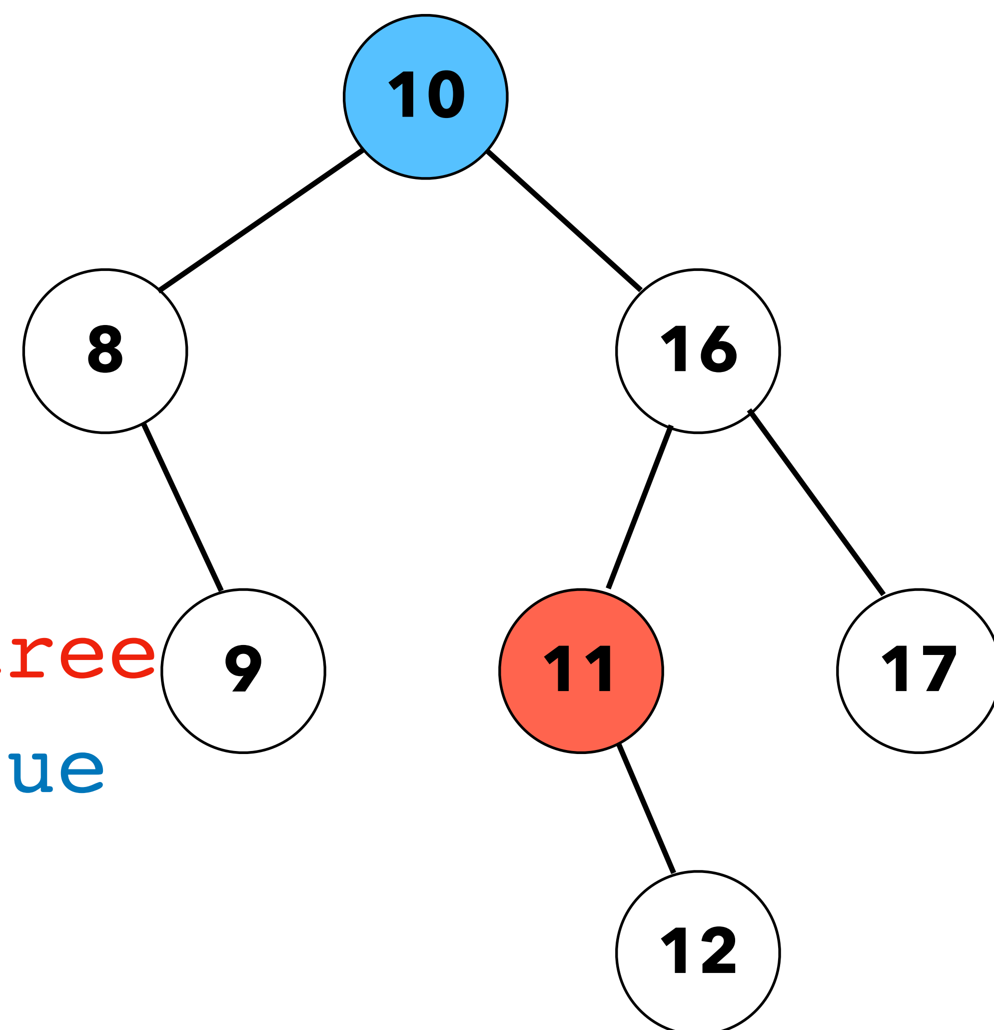

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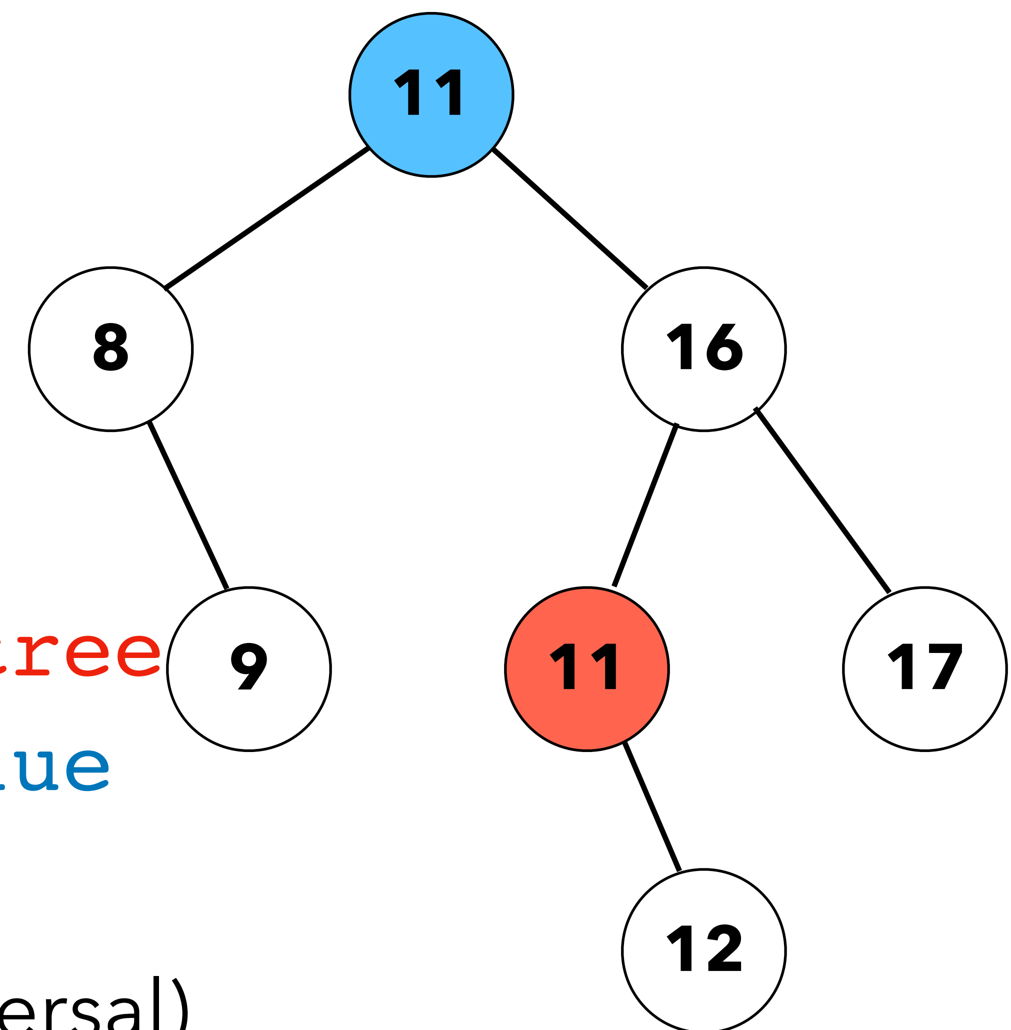



```
if (n has two children)
  let k = min node in right subtree
  replace n's value with k's value
```

Can we do that?

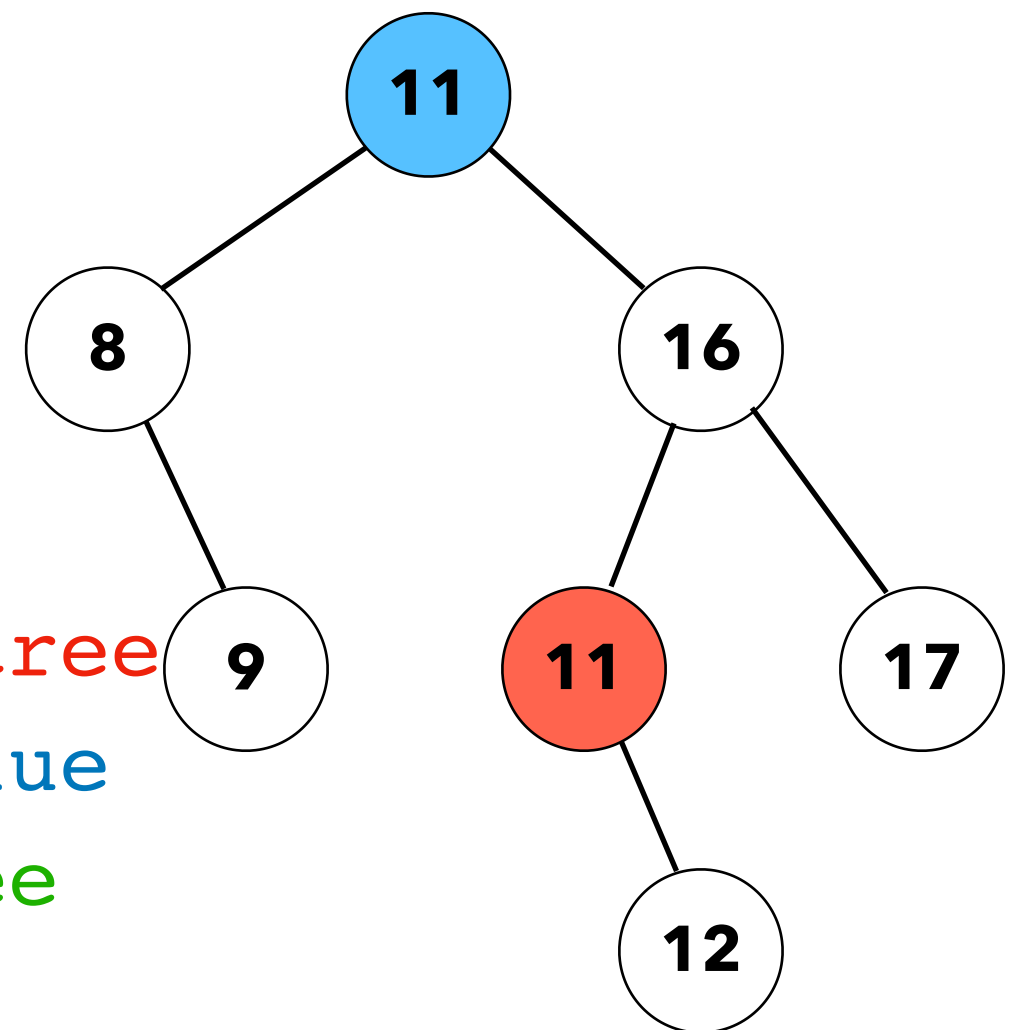
- **k** is **n**'s **successor** (next in an in-order traversal)
- Everything *else* in **n**'s right subtree is bigger than it
- Everything in **n**'s left subtree is smaller than it
- **k**'s value can safely replace **n**'s…but now we have a duplicate.

Deleting a node from a BST: Case 3

Three possible cases:

1. n has no children (is a leaf)
2. n has one child
3. **n has two children**

```
if (n has two children)
   let k = min node in right subtree
   replace n's value with k's value
   remove k from n's right subtree
```



Deleting a node from a BST: Case 3

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if (n has two children)
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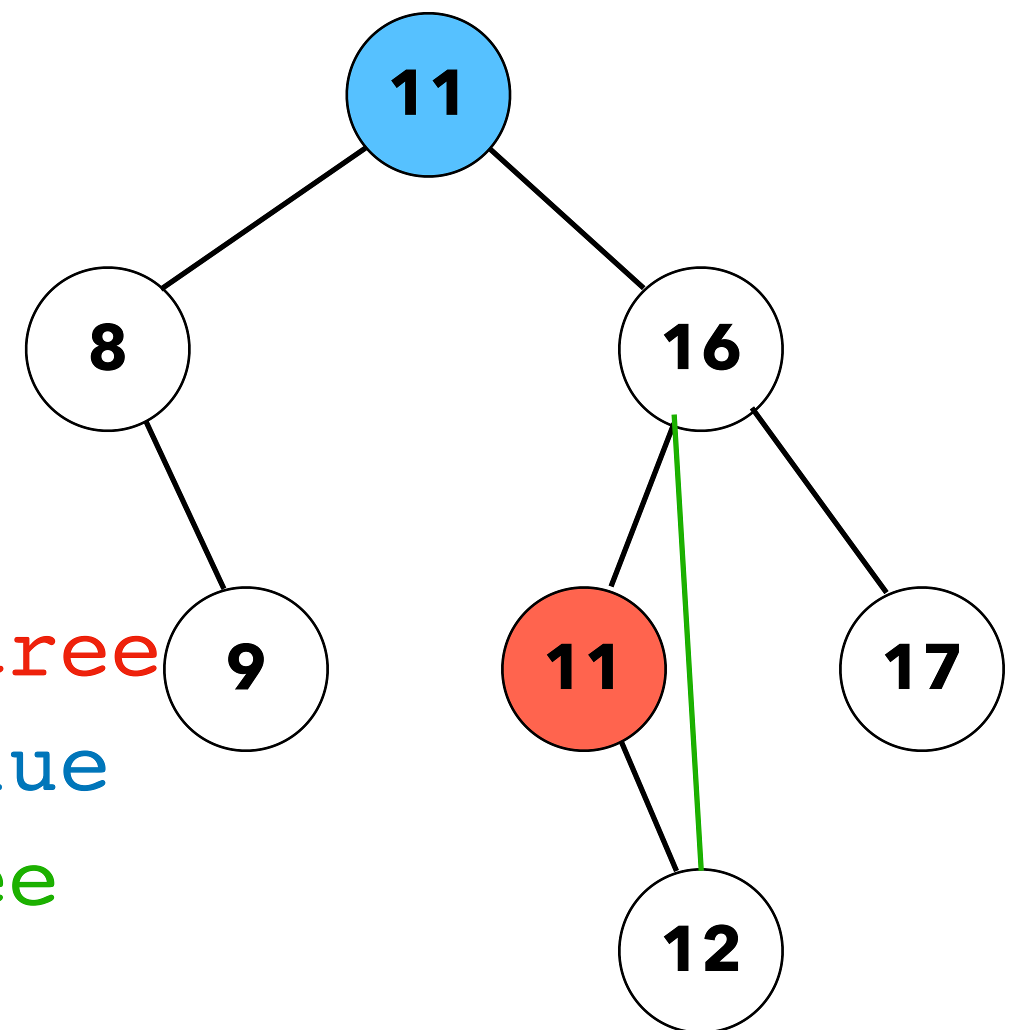

Deleting a node from a BST: Case 3

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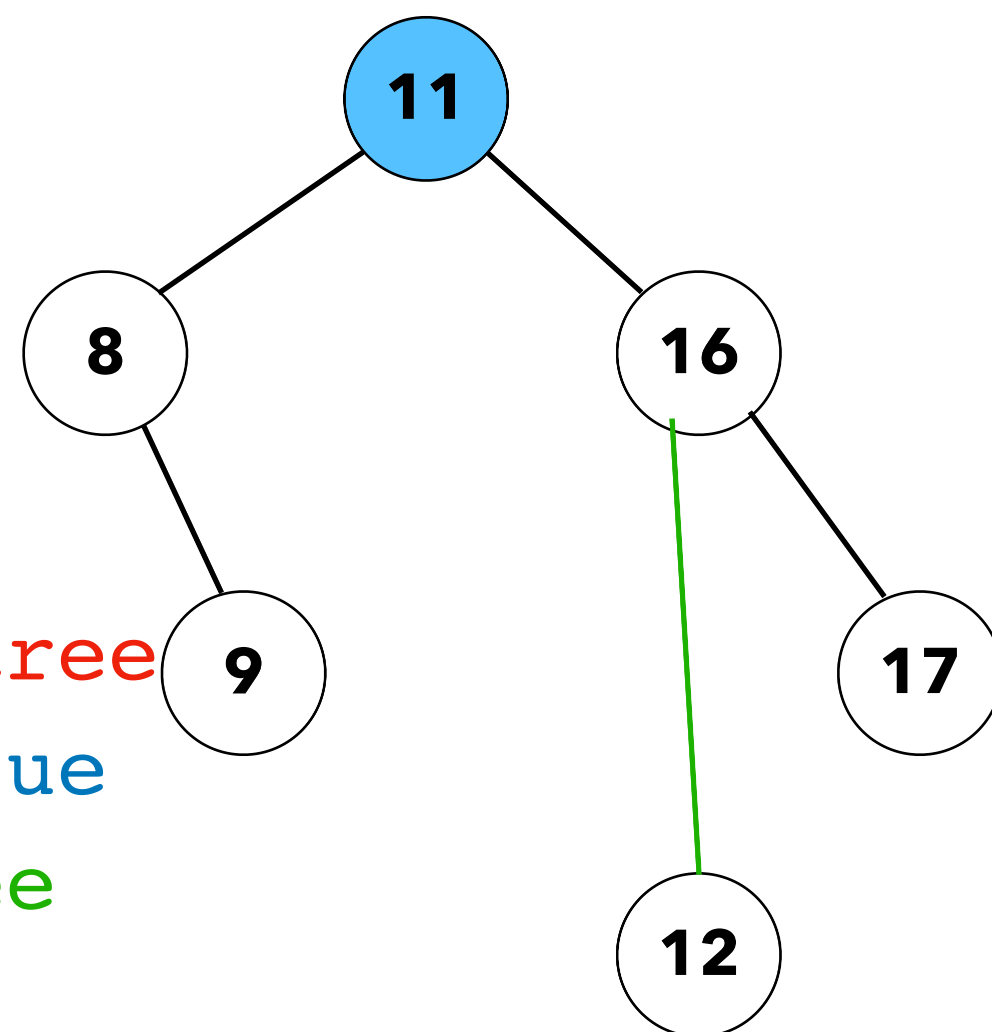

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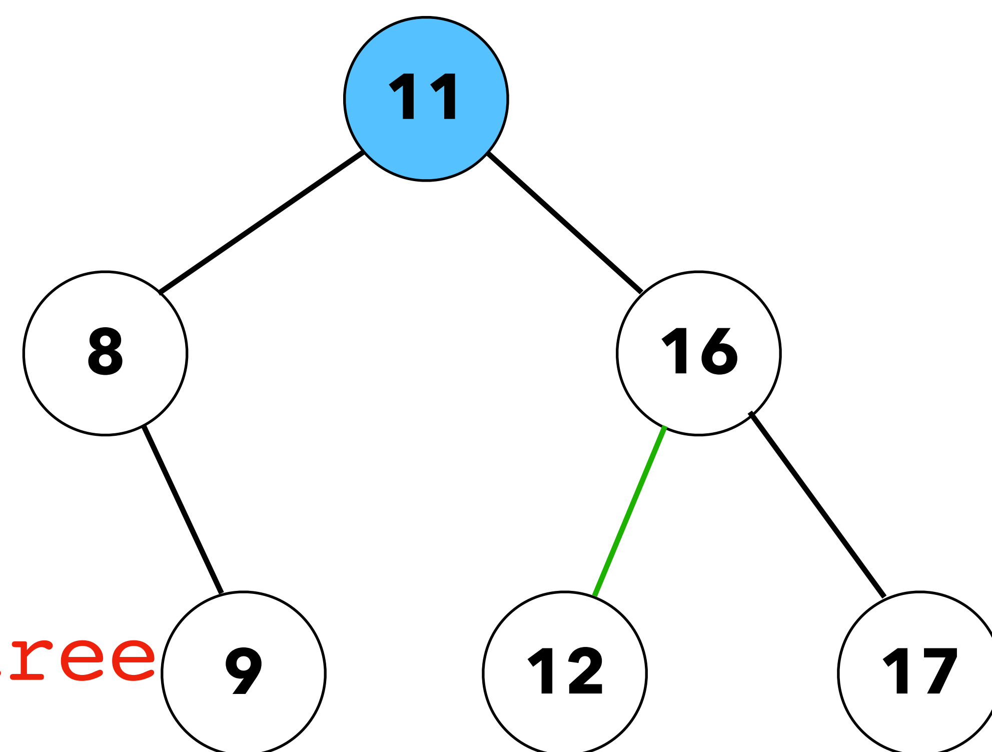


```
if (n has two children)
  let k = min node in right subtree
  replace n's value with k's value
  remove k from n's right subtree (recursively!)
```

Question: does this always make progress towards the base case?

Details

- Handle the root:
  - Update root pointer if root is removed.
  - Can't assume `n.parent` is non-null
- To update parent's child pointer, you need to know which (L or R) child pointer to update.
- The approach presented differs from that in CLRS and some other resources.