CSCI 241

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Dijkstra's Algorithm:
Detailed Pseudocode
The Path Itself

Goals

Know how to augment Dijkstra's algorithm to keep backpointers in order to reconstruct the sequence of nodes in a shortest path.

Dijkstra's Shortest Paths: Pseudocode

```
S = { }; F = {v};  v.d = 0;
while  (F ≠ {})  {
    f = node in F with min d value;
    Remove f from F, add it to S;
    for each neighbor w of f {
        if (w not in S or F) {
            w.d =  f.d + weight(f, w);
            add w to F;
        } else if (f.d+weight(f,w) < w.d) {
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Initialize Settled to empty
Initialize Frontier to the start node
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   move node f with smallest d
    from F to S
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}
```

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Initialize Settled to empty
Initialize Frontier to the start node

While the frontier isn't empty:
   move node f with smallest d
    from F to S

   For each neighbor w of f:
      if we've never seen w before:
         set its path length
         add it to frontier

      else if path to w via f is shorter:
          update w's shortest path length
```

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What if we want to know the shortest path?

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At termination: for each reachable node n, n.d stores the **length** of the shortest path from v to n.

We didn't keep track of **how** to get from v to n!

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Each node could store the full path, but that would be expensive to keep updated.

Strategy: maintain a **backpointer** at each node pointing to the previous node in the shortest path.

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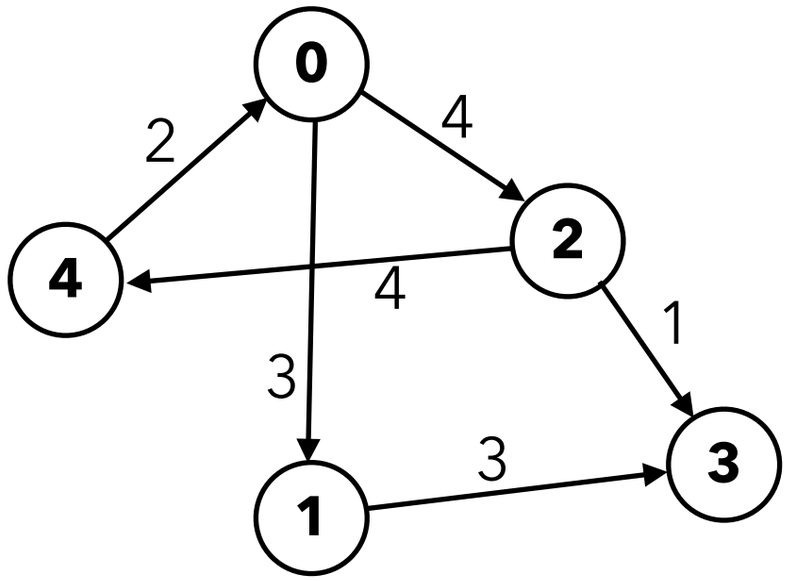

`shortest-paths(4)`

Node	d	bp
0	2	4
1	5	0
2	6	0
3	7	2
4	0	null

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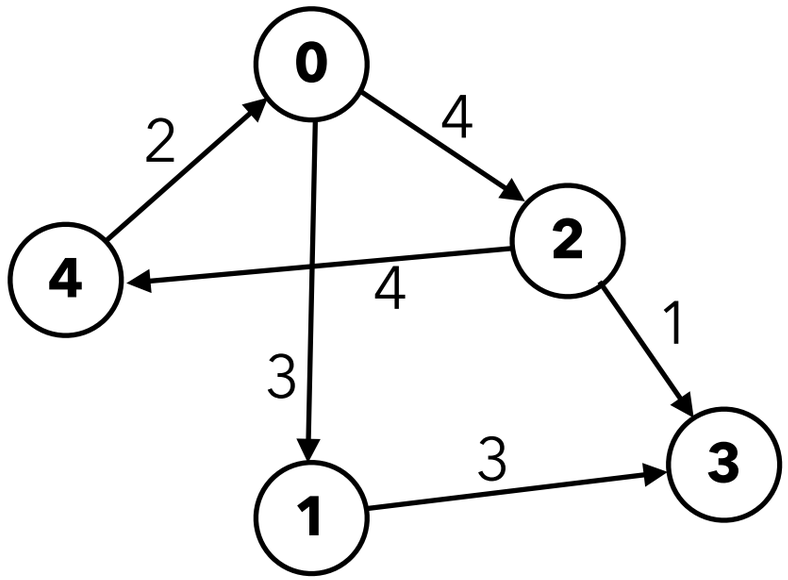

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Shortest path to Node 3:

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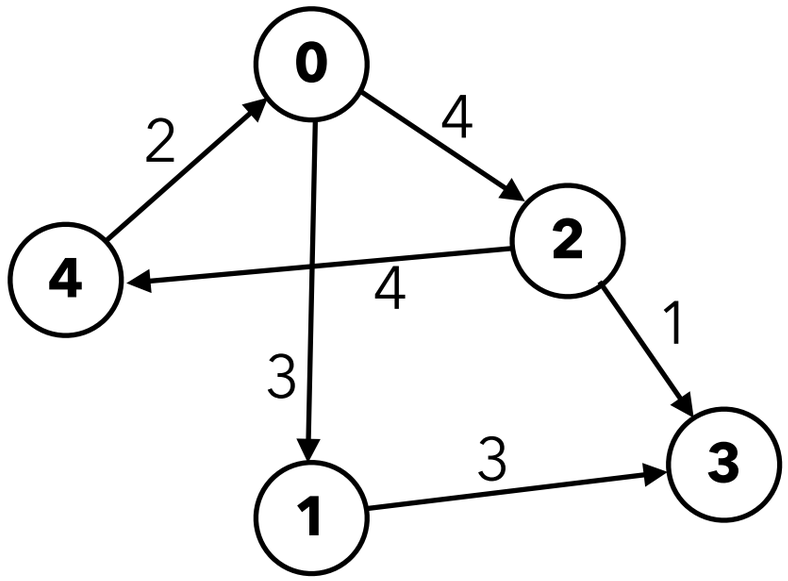

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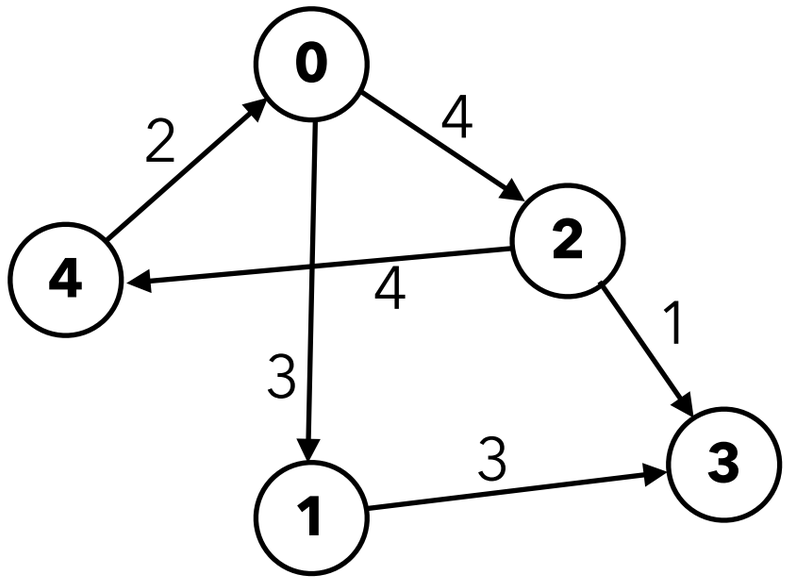

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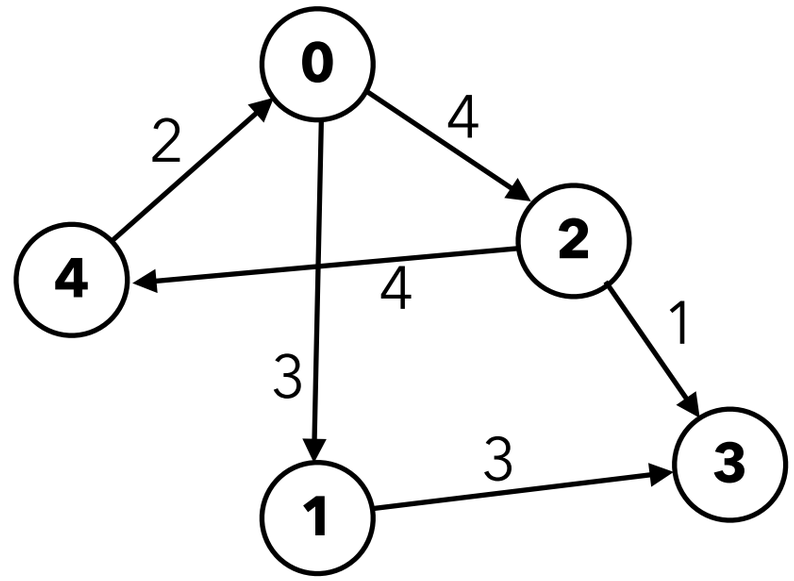

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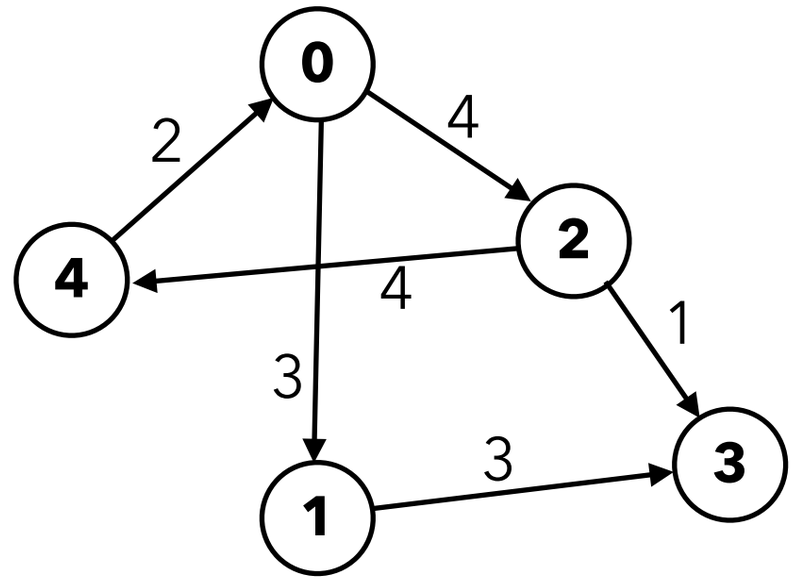

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