CSCI 241

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Heaps:
Definition, `add`, Storage

Goals

Know the definition and properties of a heap.

Understand how heaps are stored in practice.

Be prepared to implement `add` and execute it on paper

Heap implements PriorityQueue

A heap is a **concrete** data structure that can be used to **implement** a Priority Queue

Better runtime complexity than list, BST, AVL implementation:

- **peek()** is O(1)
- **poll()** is O(log n)
- **add()** is O(log n)

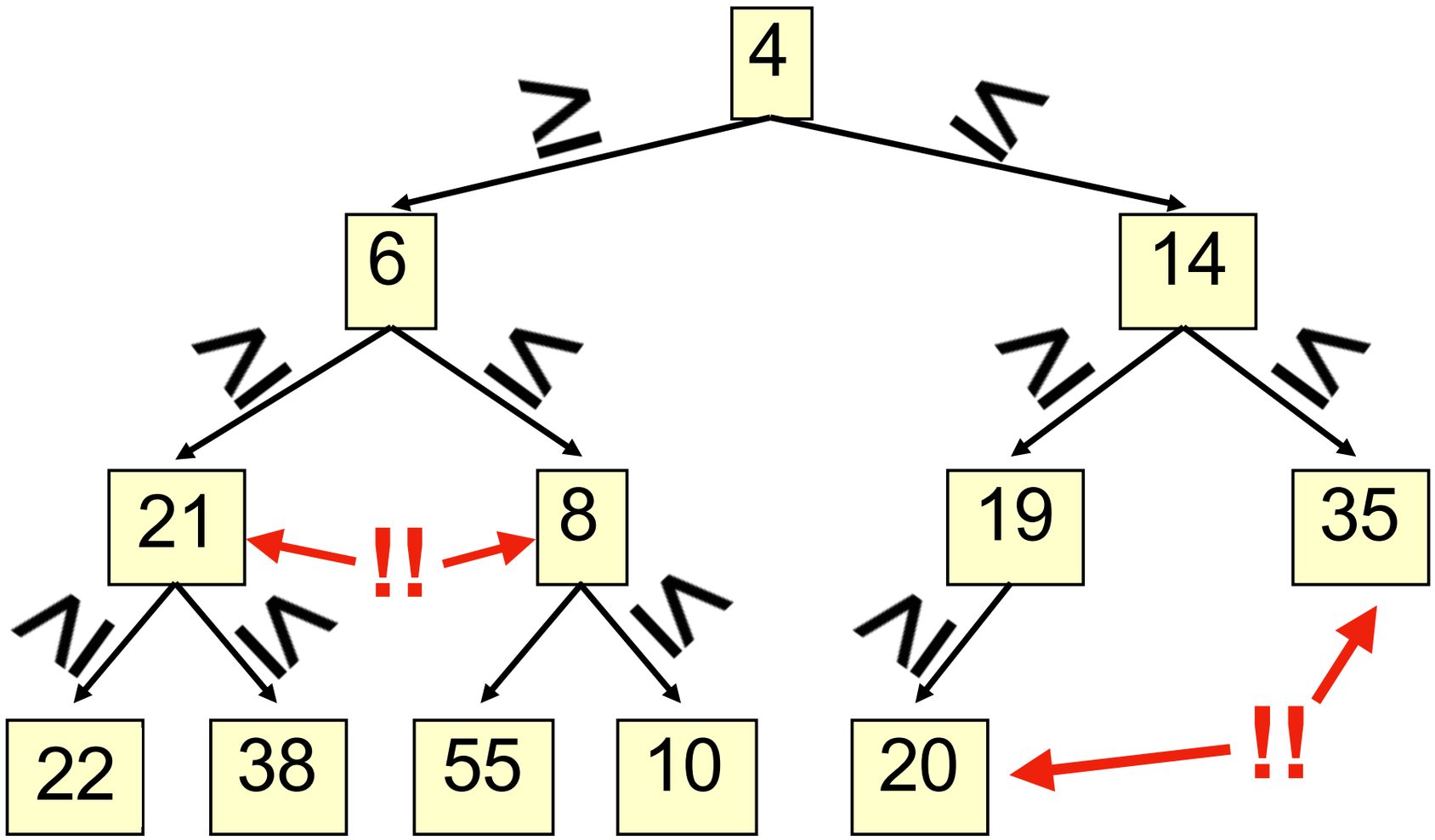
Not to be confused with *heap memory*, where the Java virtual machine allocates space for objects – different usage of the word heap.

A heap is a special binary tree with two additional properties.

A heap is a special binary tree.

1. **Heap Order Invariant:**

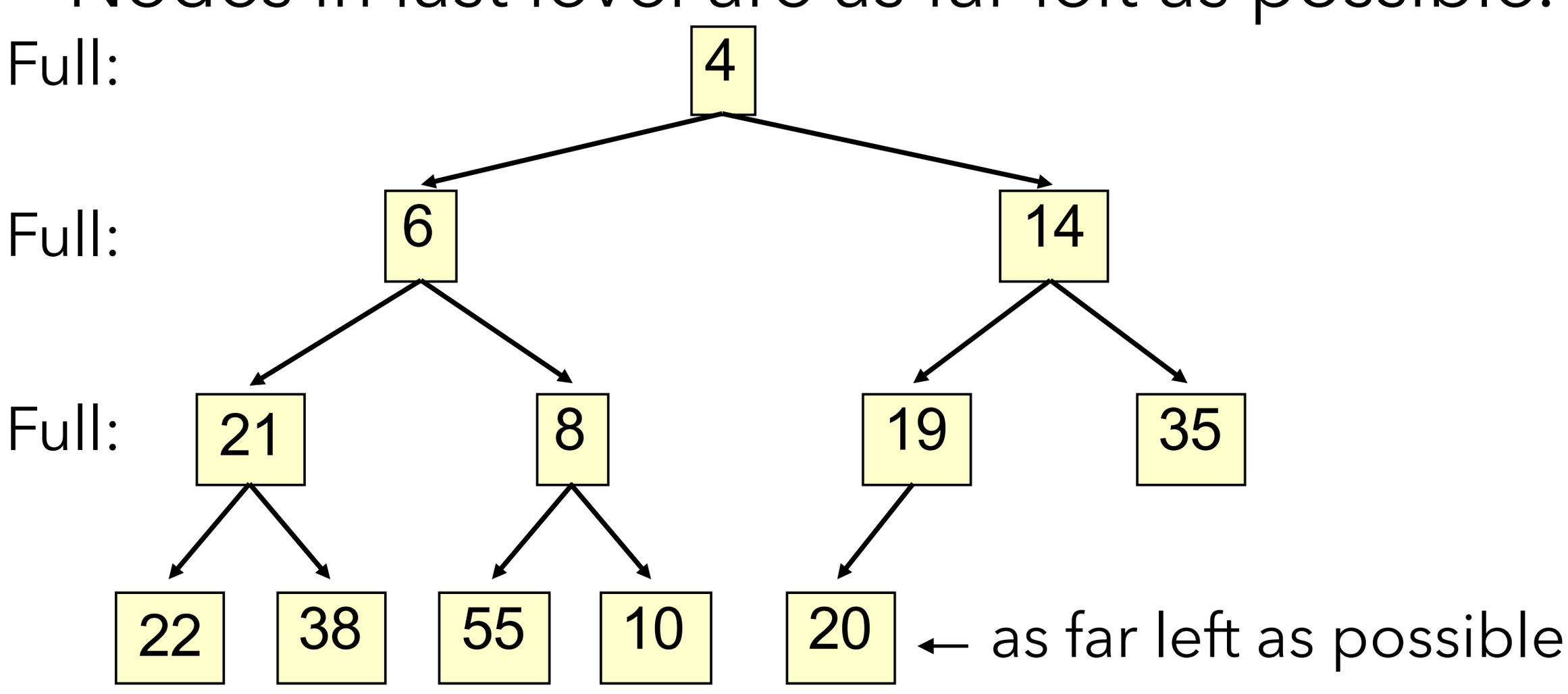
Each element >= its parent.



A heap is a special binary tree.

2. **Complete:** no holes!

- All levels except the last are **full**.
- Nodes in last level are as far left as possible.



Heap Operations

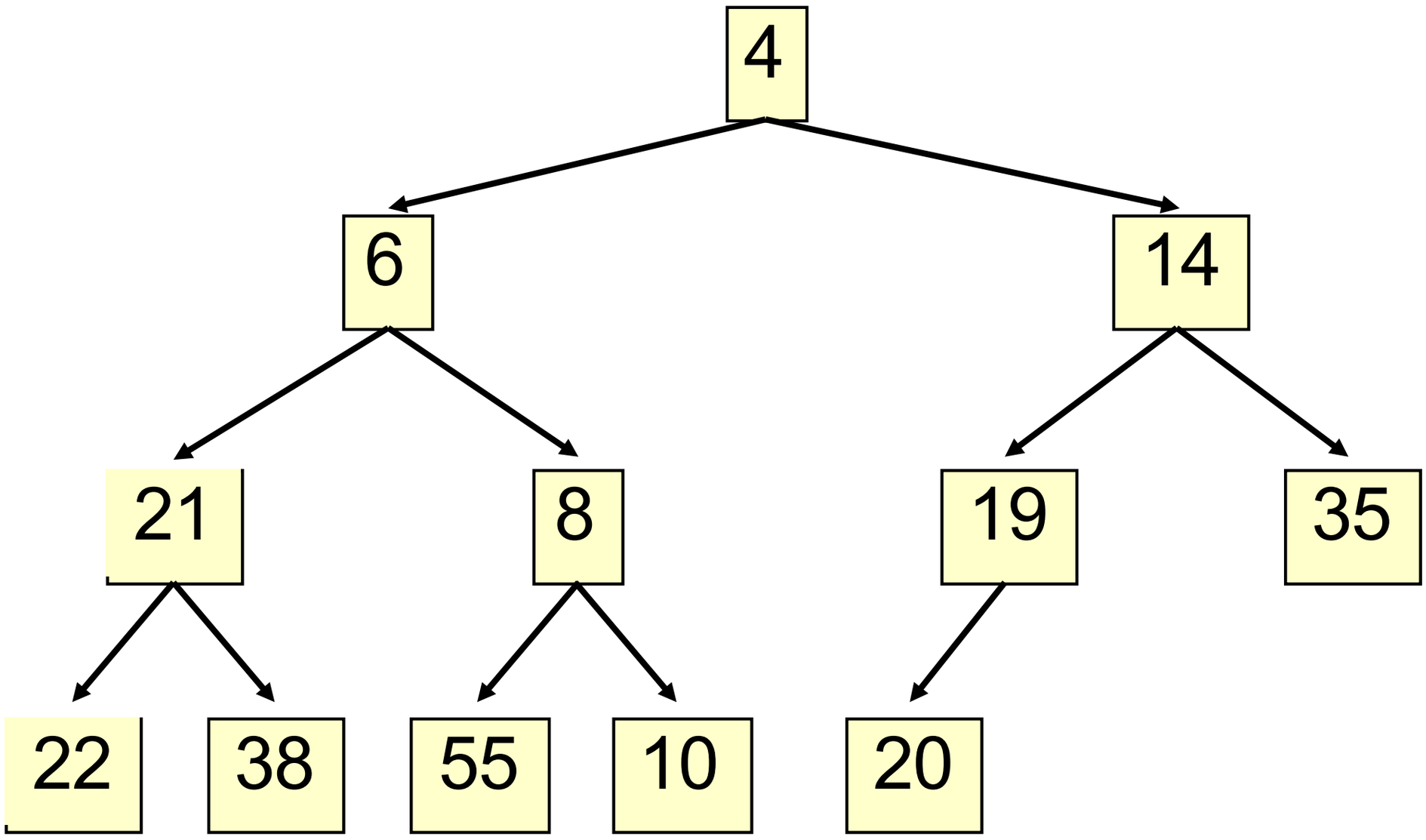
```
interface PriorityQueue<V v, P p> {
 // insert value v with priority p
 void add(V v, P p);

 // return value with min priority
 V peek();

 // remove/return value with min priority
 V poll();

 // more methods…
}
```

```
void add(V v, P p);
```

Algorithm:

- Add v in the wrong place
- While v is in the wrong place
  - move v towards the right place

```
void add(V v, P p);
```



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void add(V v, P p);
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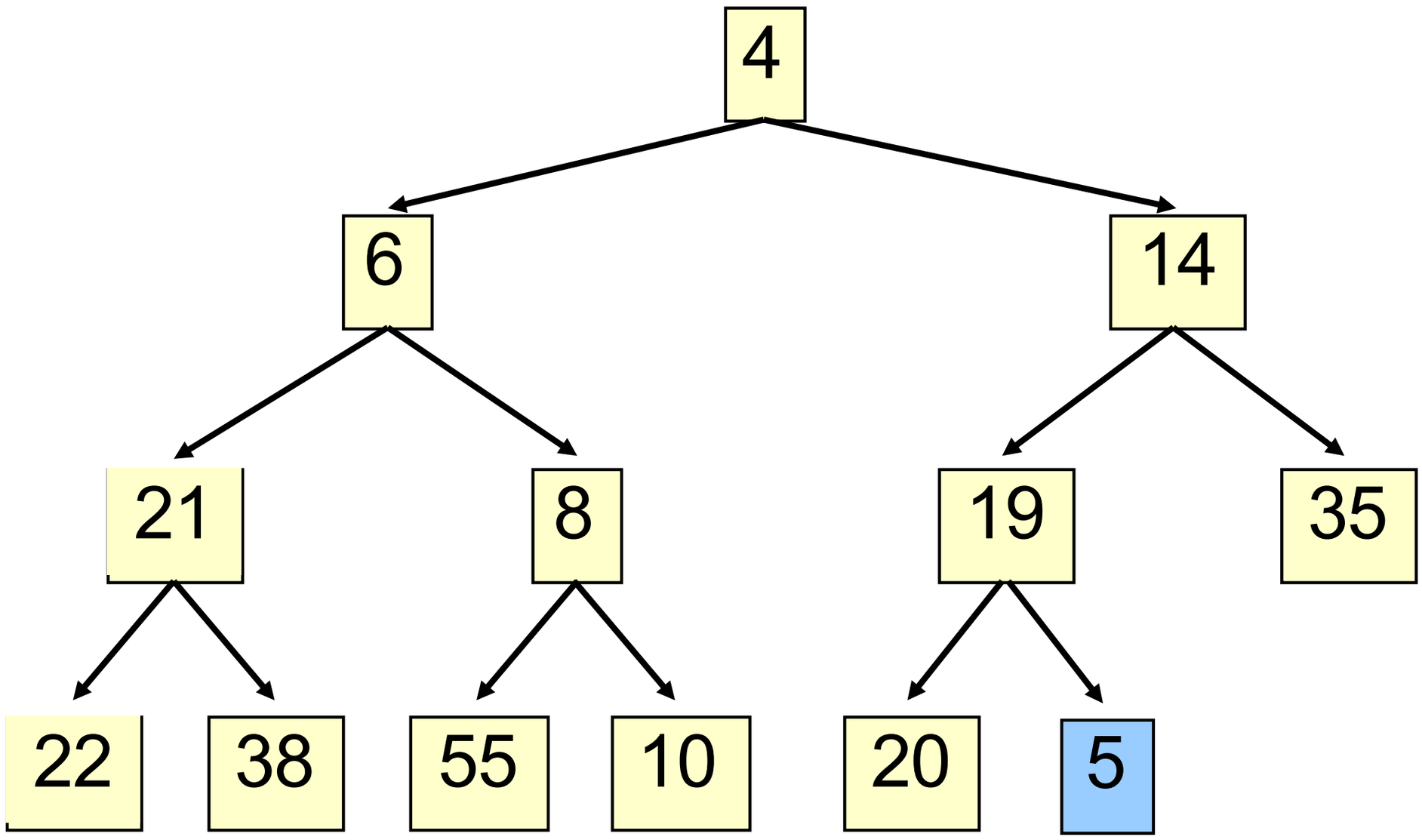

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void add(V v, P p);
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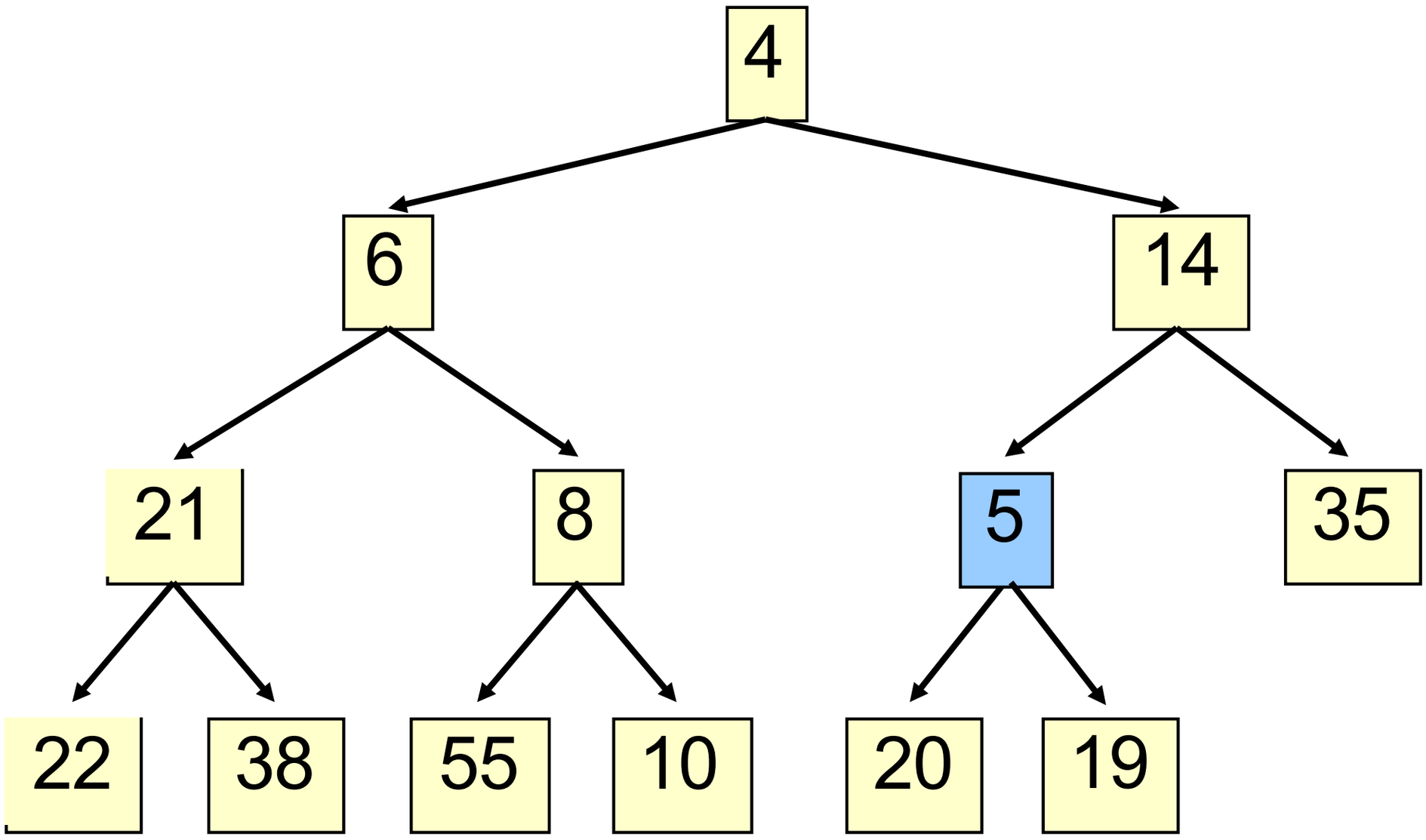

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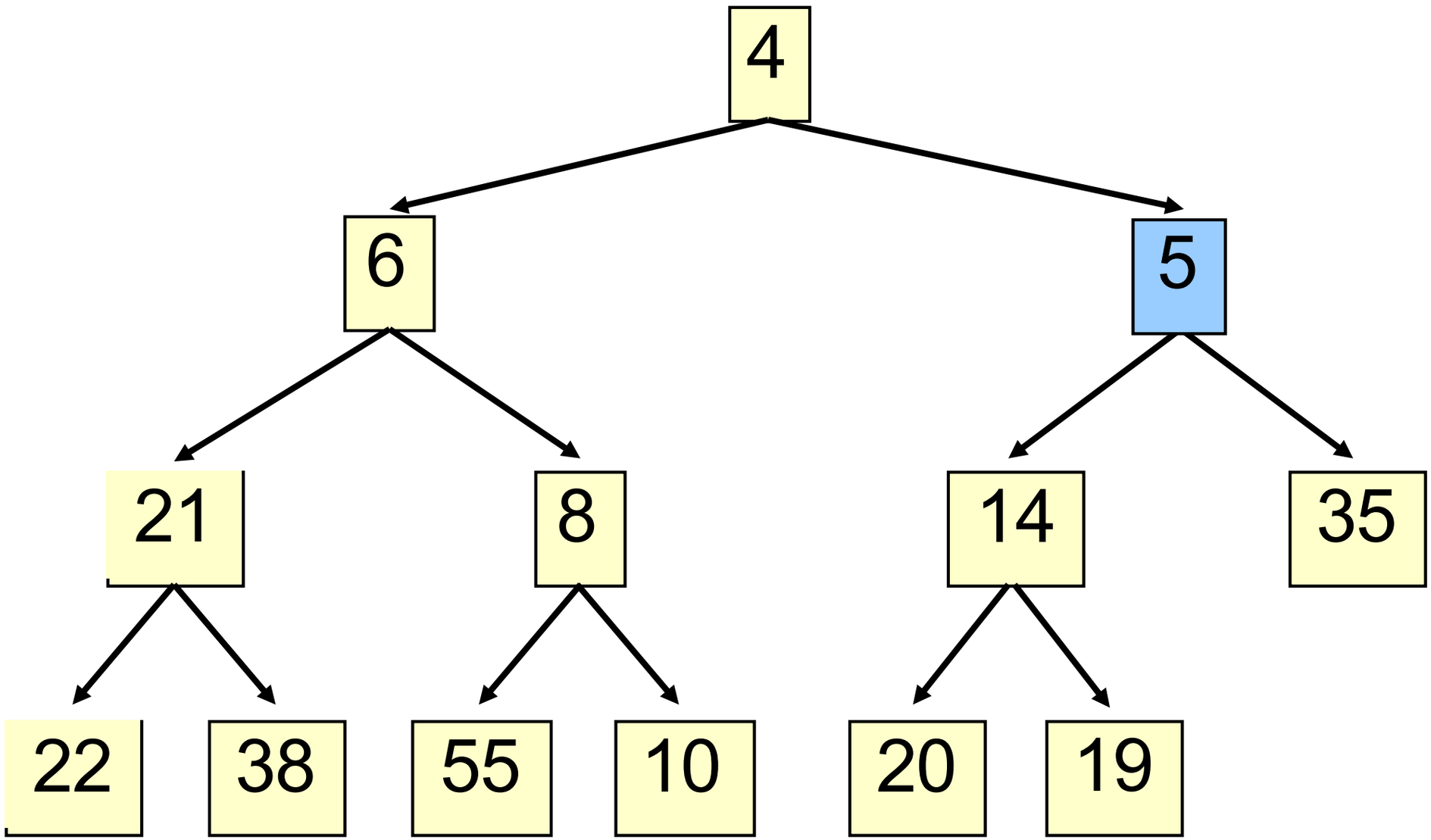

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void add(V v, P p);
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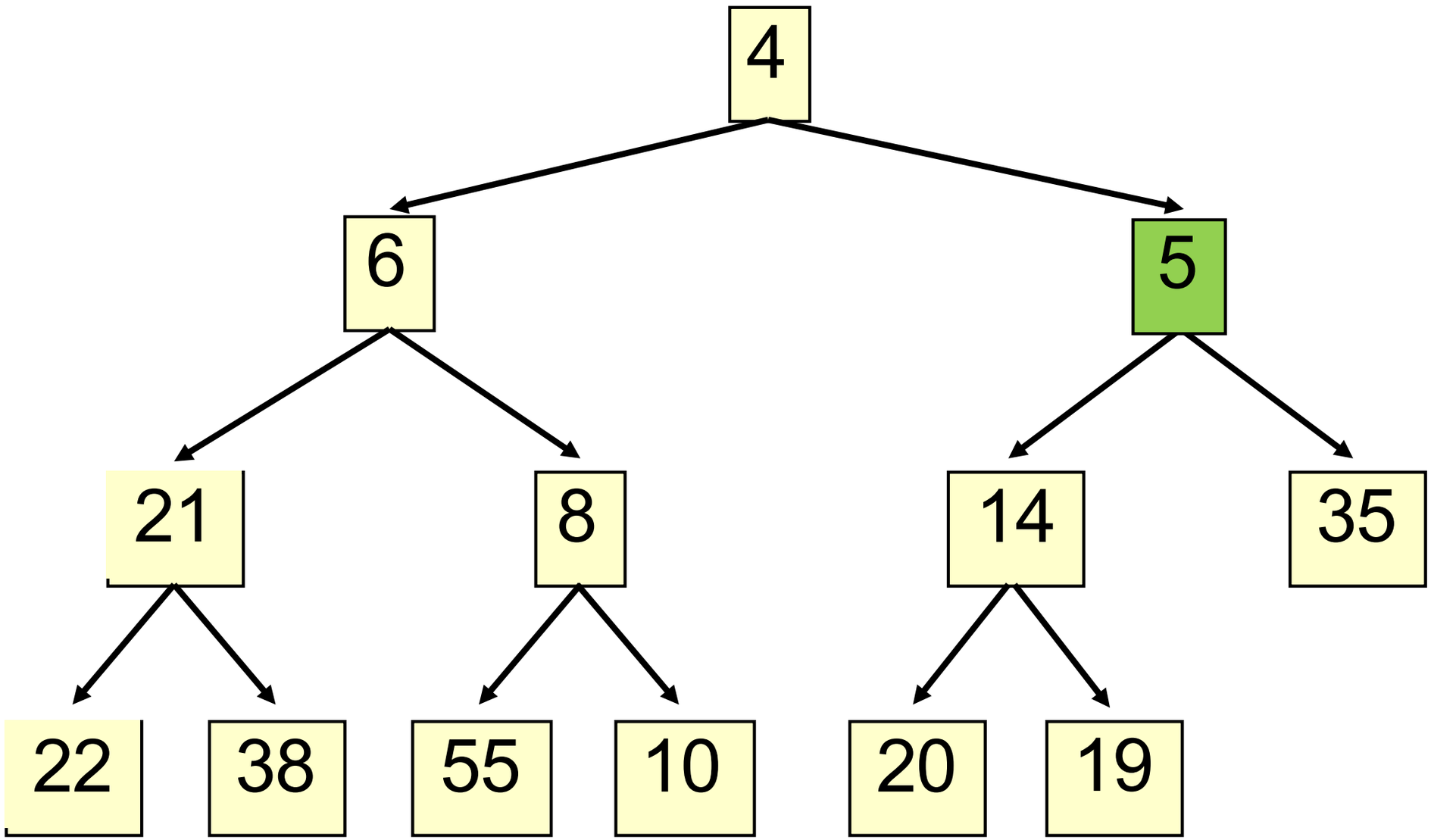


```
void add(V v, P p);
```

Algorithm:

- Add v in the wrong place (the leftmost empty leaf)
- While v is in the wrong place (its p is less than its parent's)
  - move v towards the right place (swap with parent)

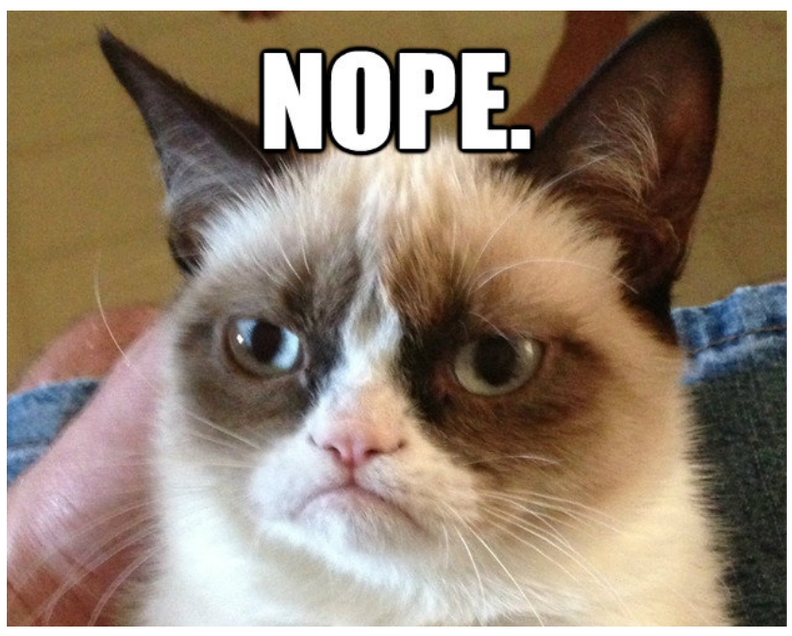
The heap invariant is maintained!

Implementing Heaps

```
public class HeapNode {
   private int value;
   private HeapNode left;
   private HeapNode right;
   ...
}
public class Heap {
   HeapNode root;
   ...
```

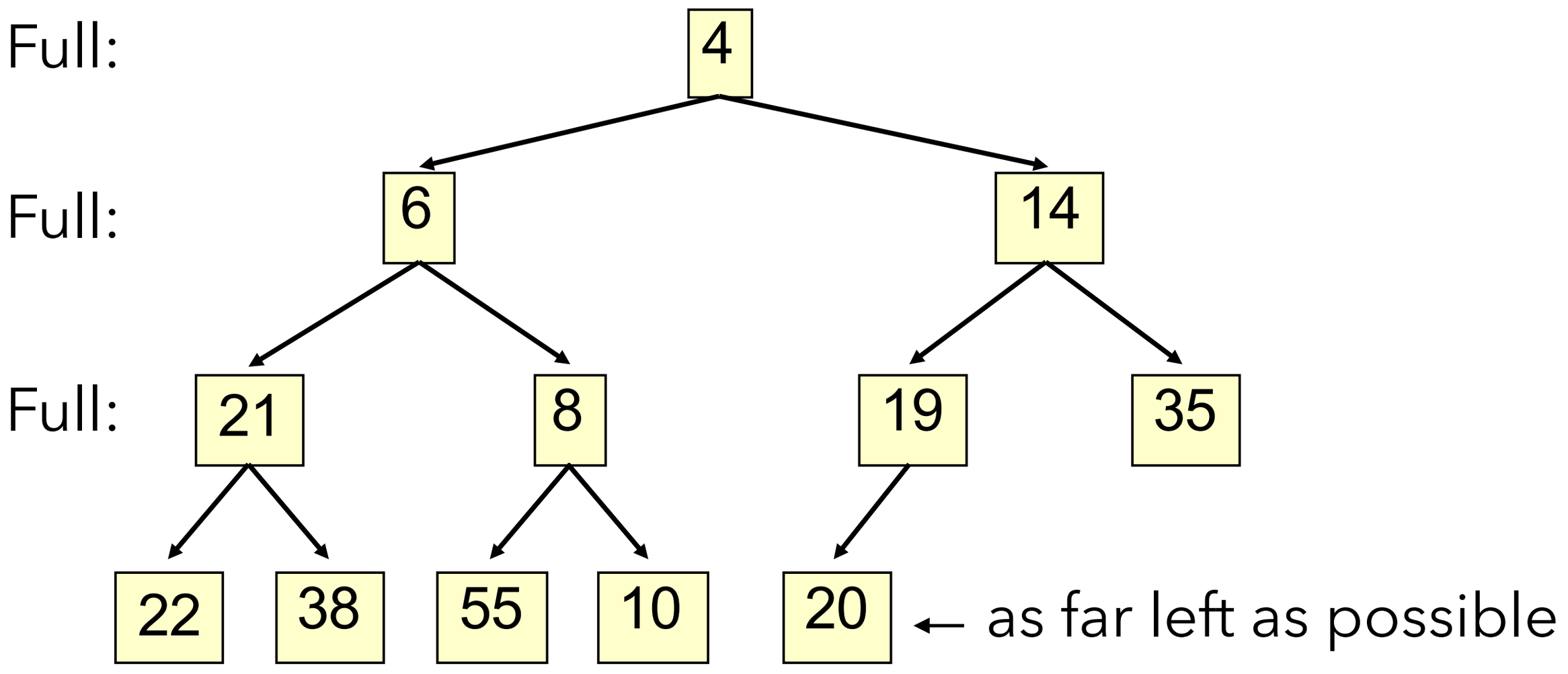
Implementing Heaps

```
public class HeapNope {
   private int value;
   private HeapNope left;
   private HeapNope right;
   ...
```



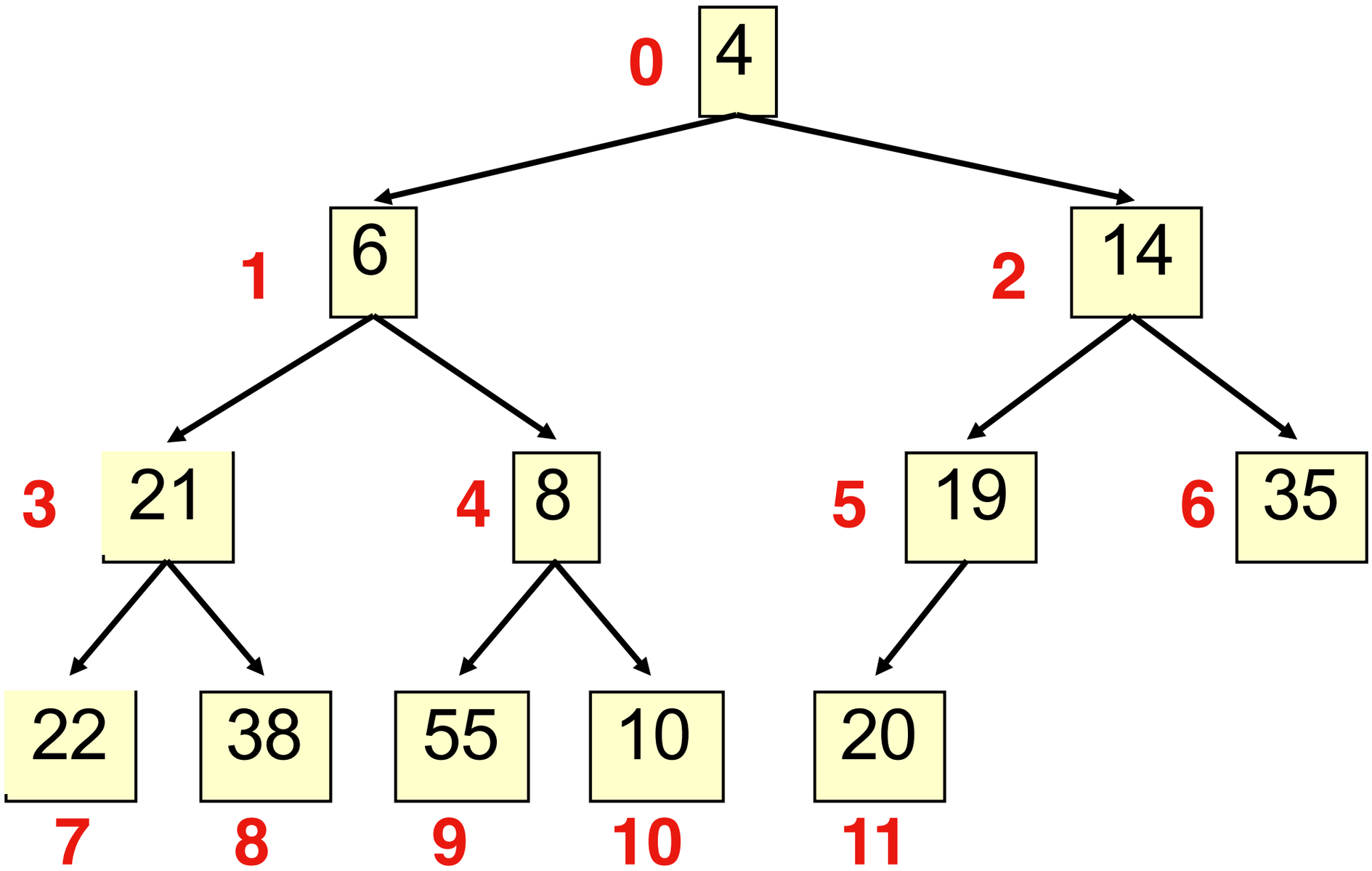
A heap is a special binary tree.

2. **Complete:** no holes!



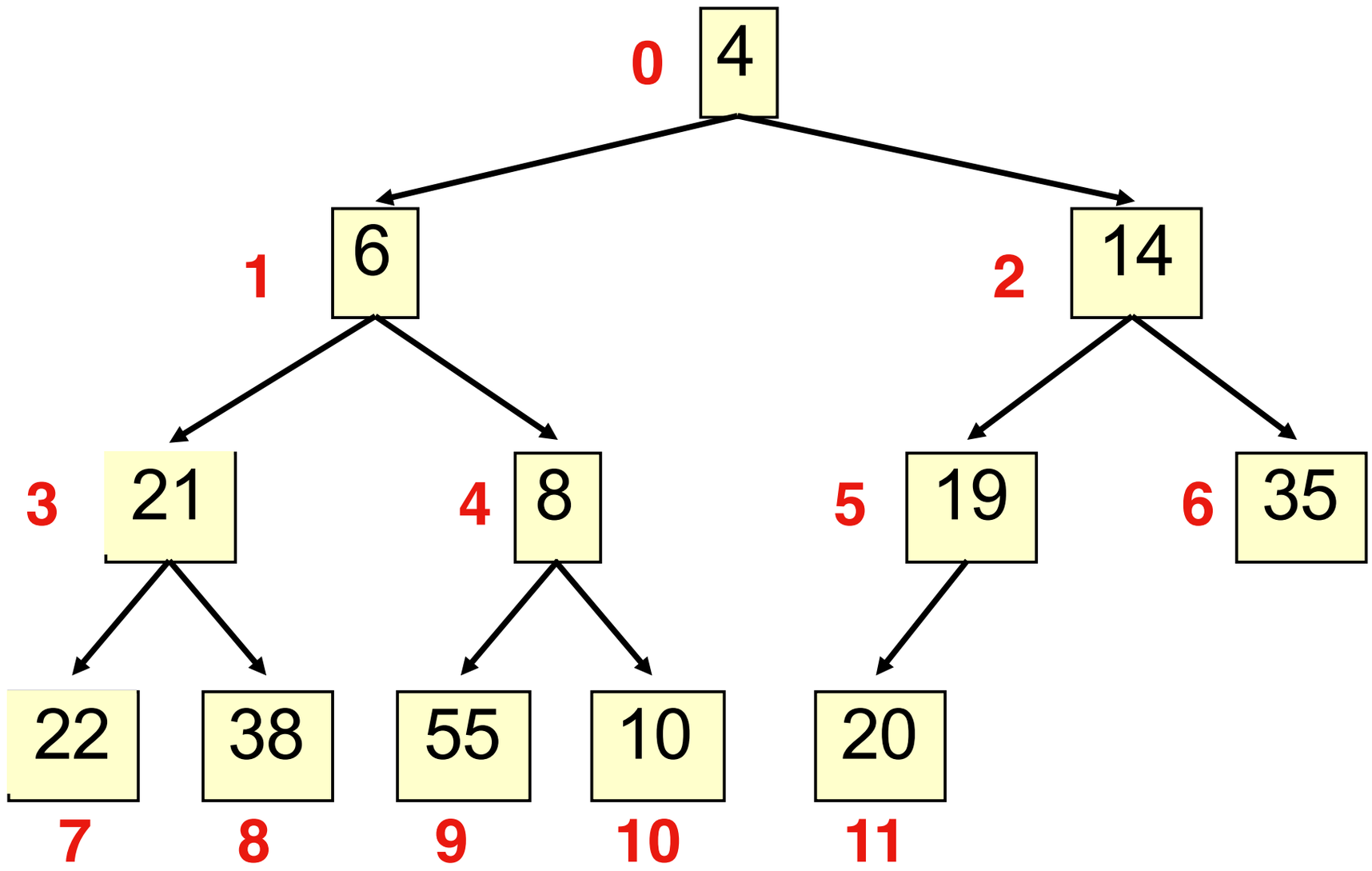
Numbering Nodes

Level-order traversal:



2. Complete: **no holes!**

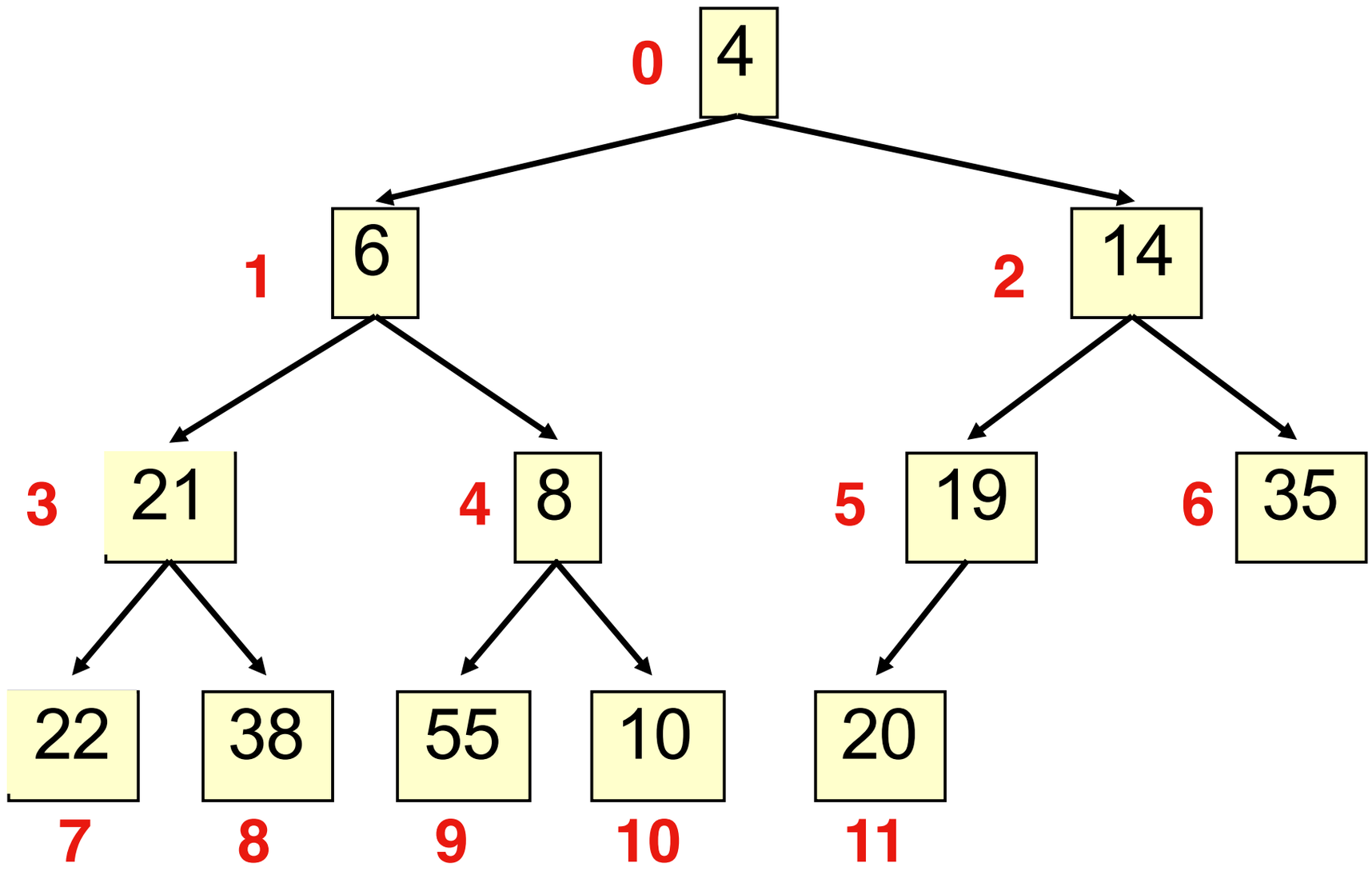
Numbering Nodes



node **k**'s parent is

node **k**'s children are nodes and

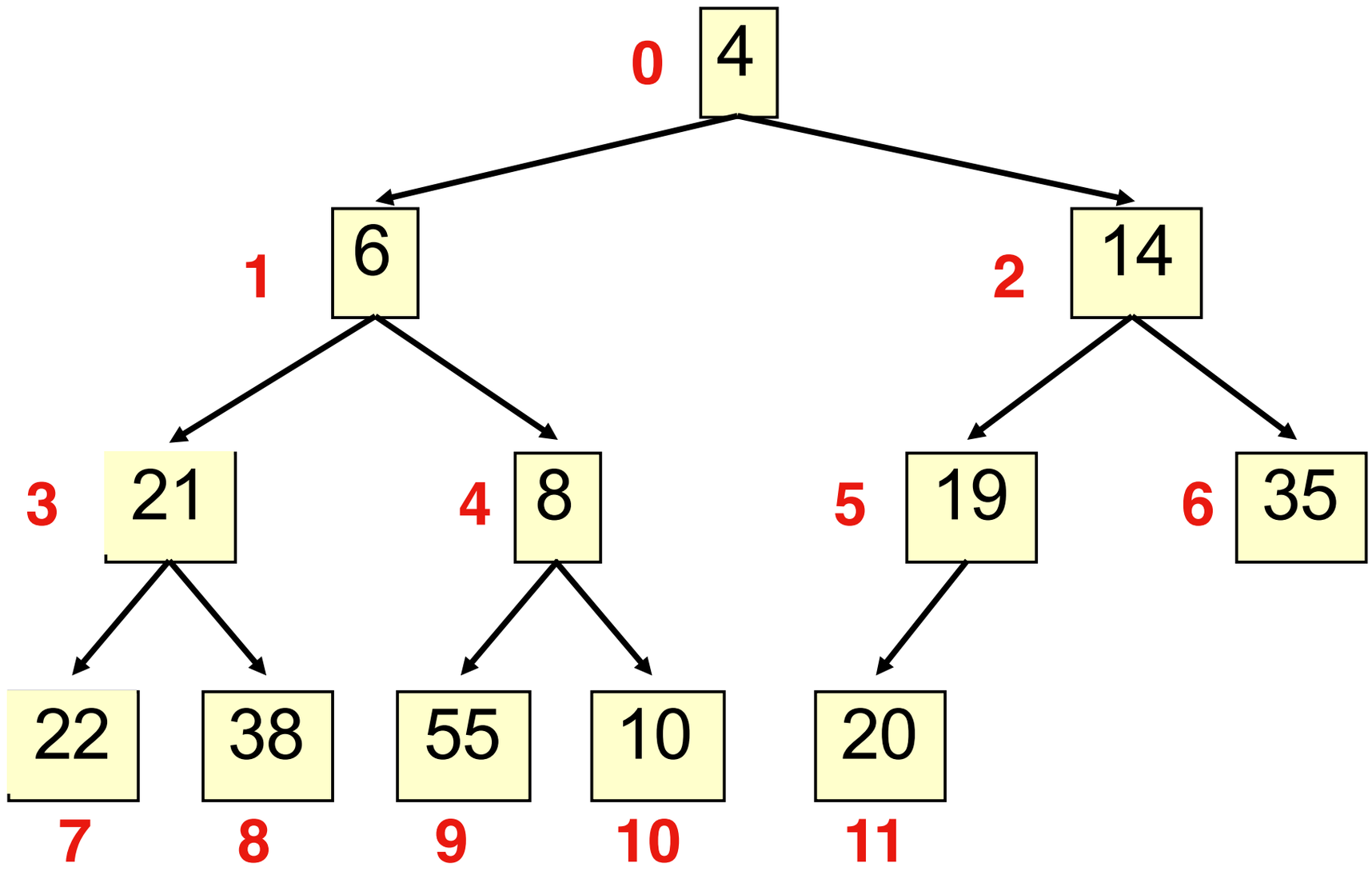
Numbering Nodes



node **k**'s parent is **(k - 1)/2**

node **k**'s children are nodes and

Numbering Nodes



node **k**'s parent is **(k - 1)/2**

node **k**'s children are nodes **2k + 1** and **2k + 2**

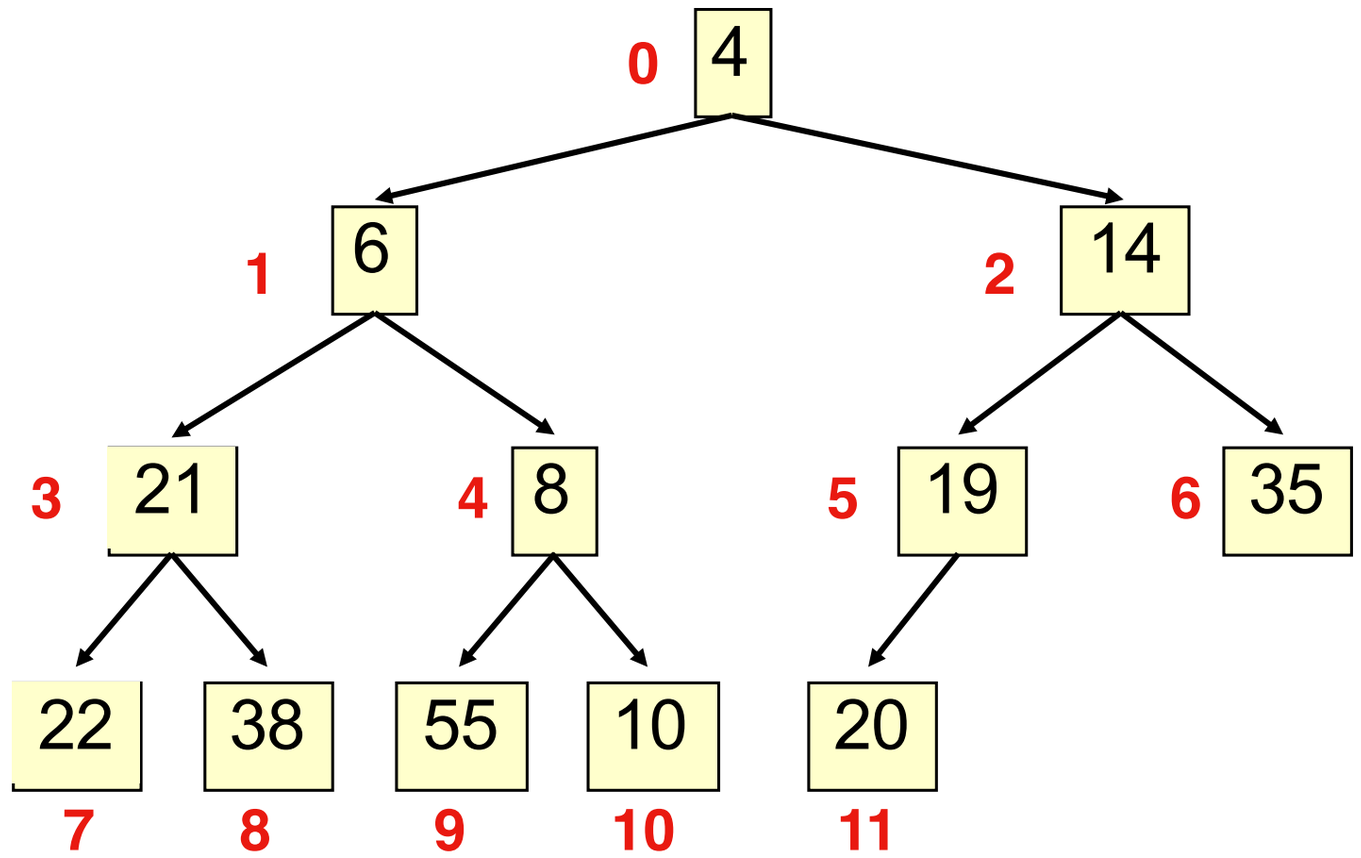
Implementing Heaps

```
public class Heap {
   private Entry[] heap;
   private int size;
   . . .
}
```

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
4	6	14	21	8	19	35	22	38	55	10	20				

Implicit Tree Structure

2. Complete: **no holes!**



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