CSCI 241

Scott Wehrwein

Tree Rotations

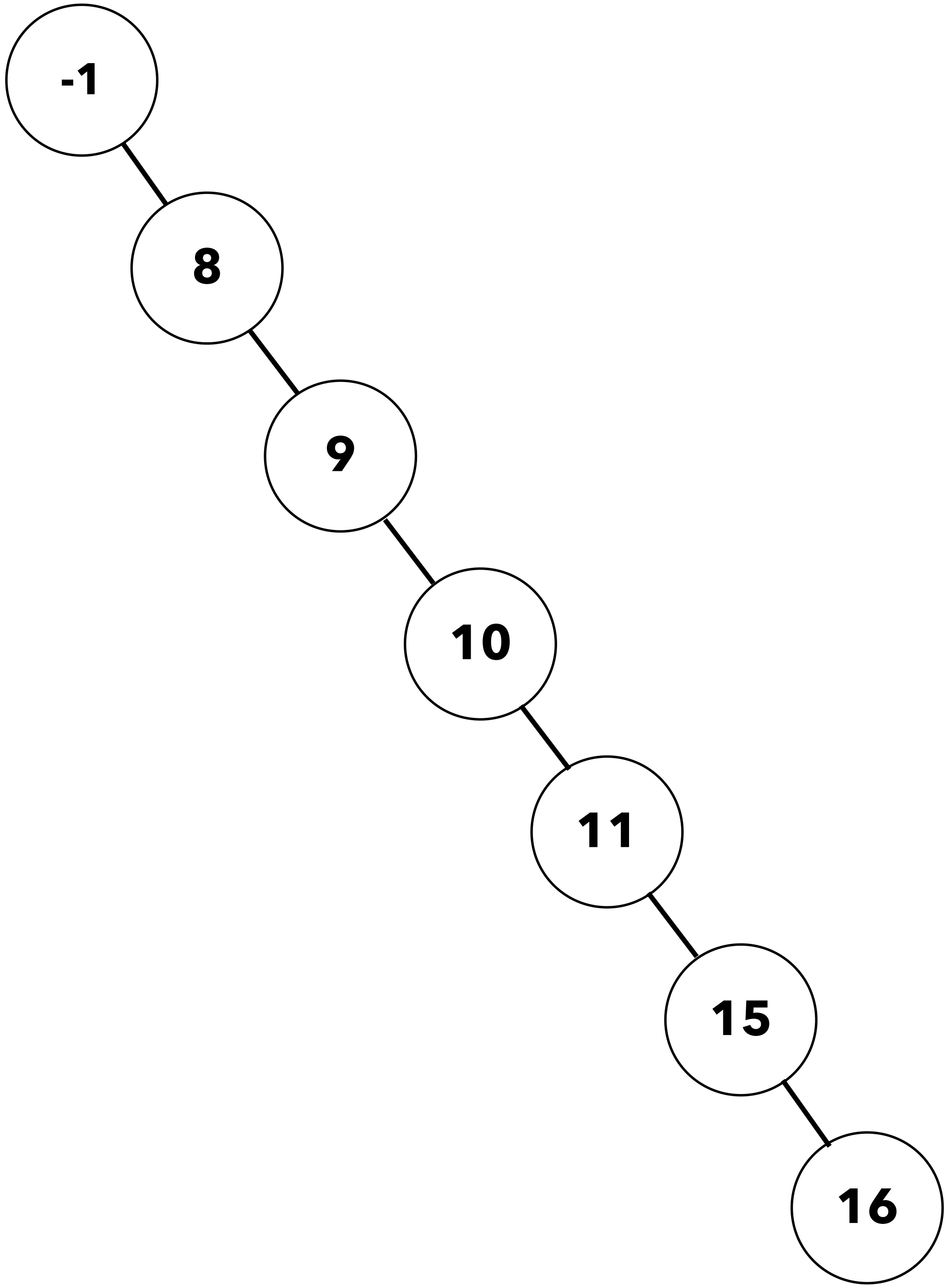
Goals

Be able to execute BST rotations on paper.

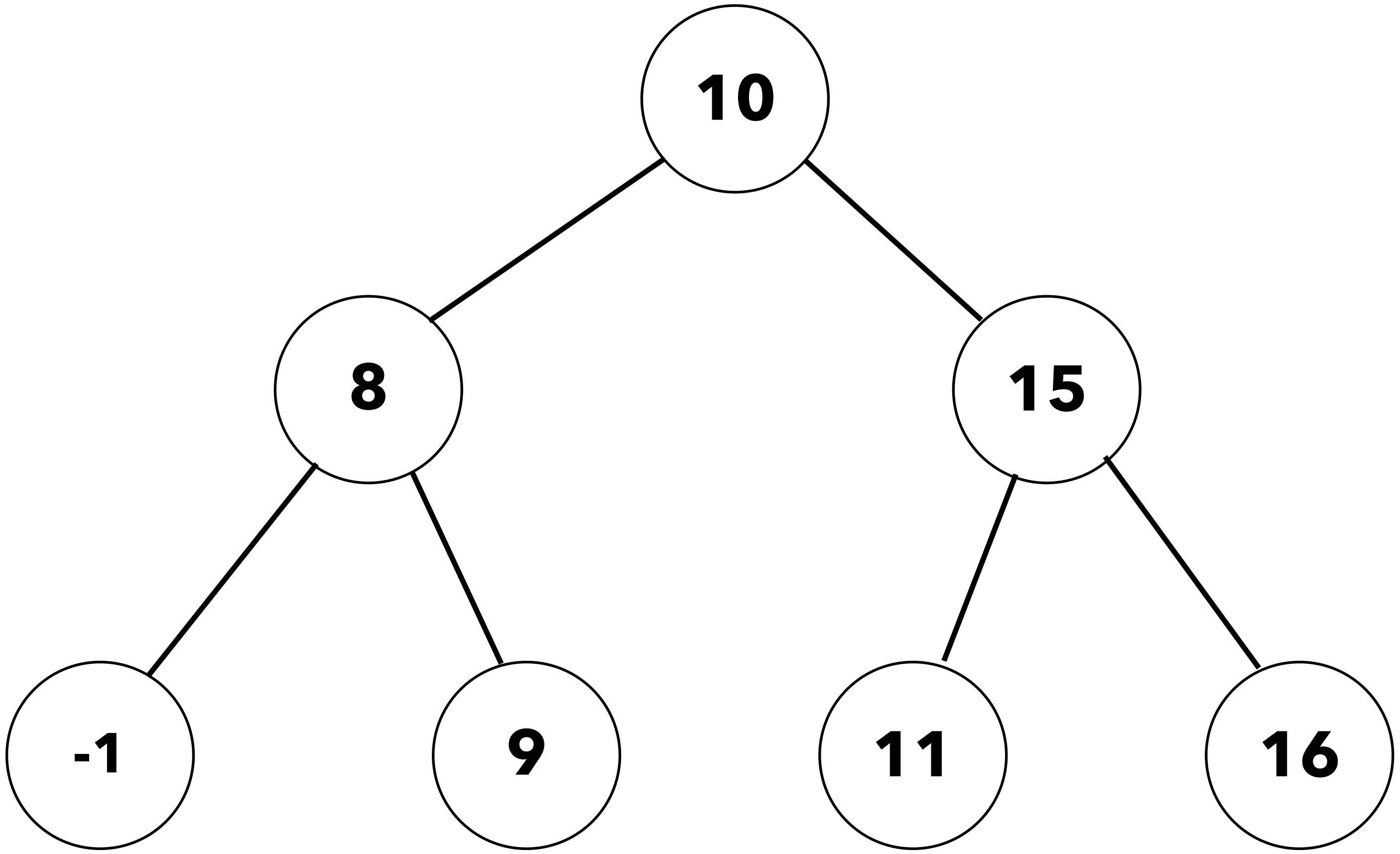
Be prepared to implement BST rotations.

Measuring Badness

Bad tree =(



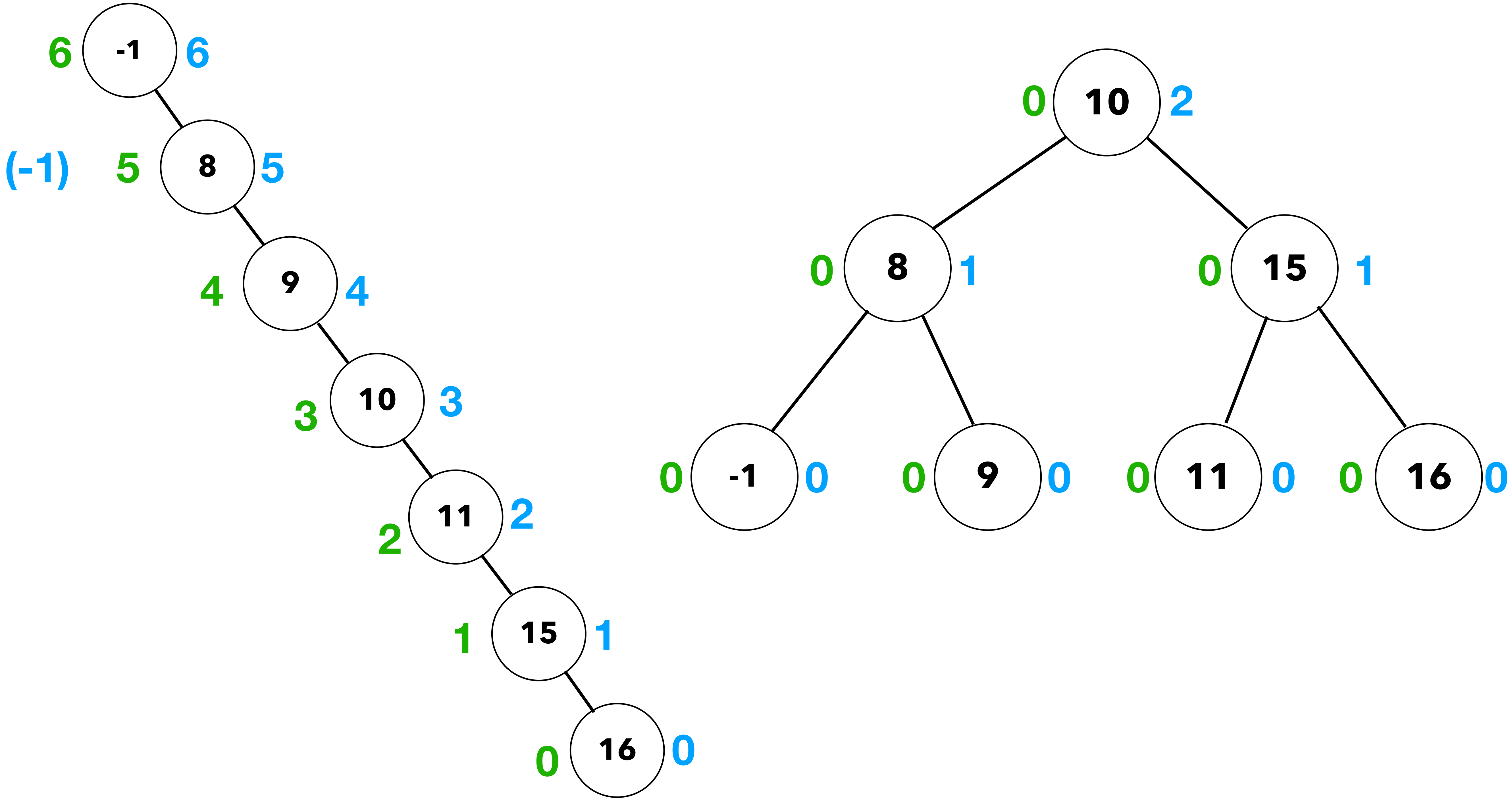
Good tree =)



how bad? how good?

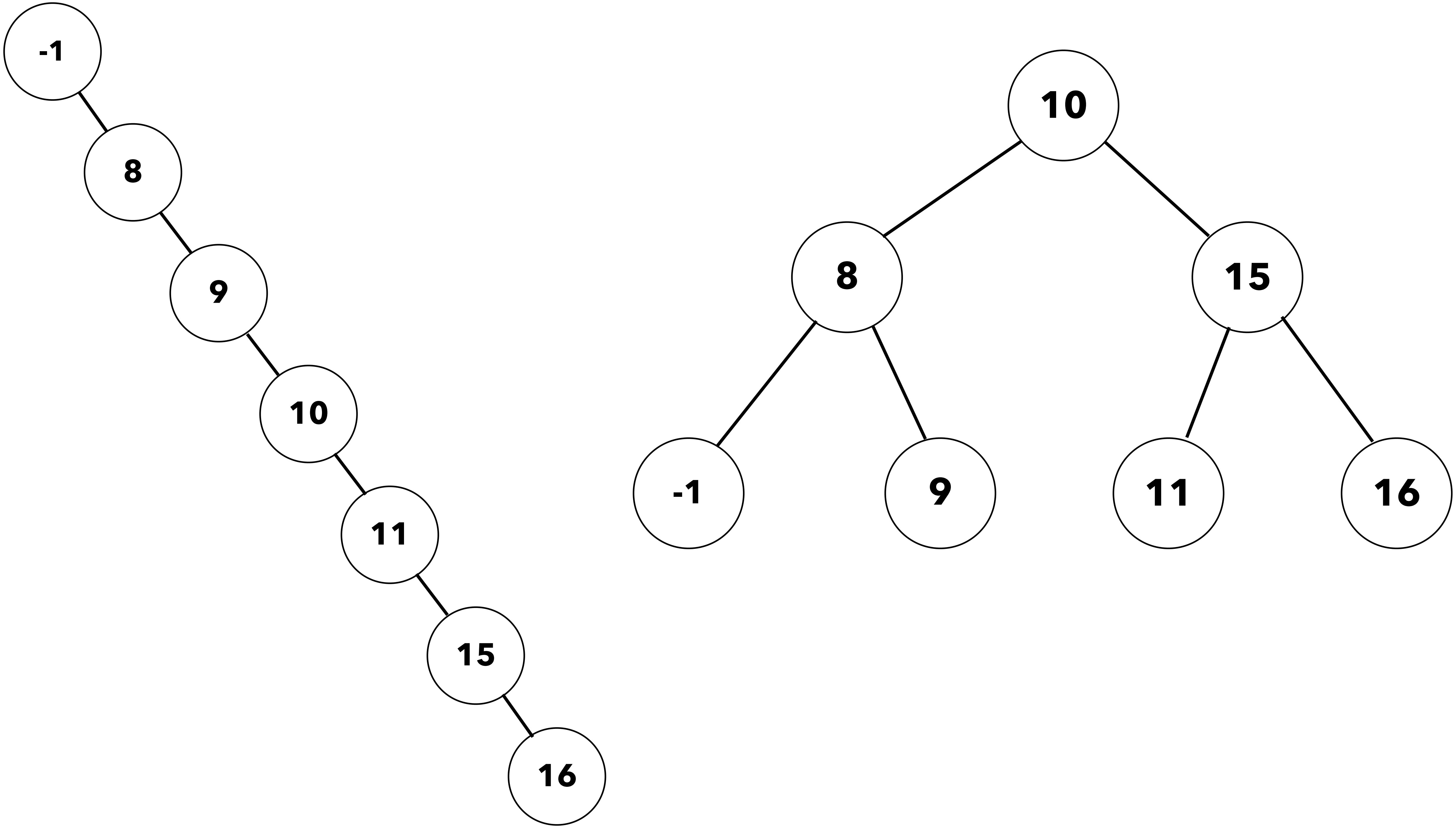
Measuring Badness

Balance(n): height(n.right) - height(n.left)



Tree Badness

Hey Jude: can we take a bad tree and make it better?



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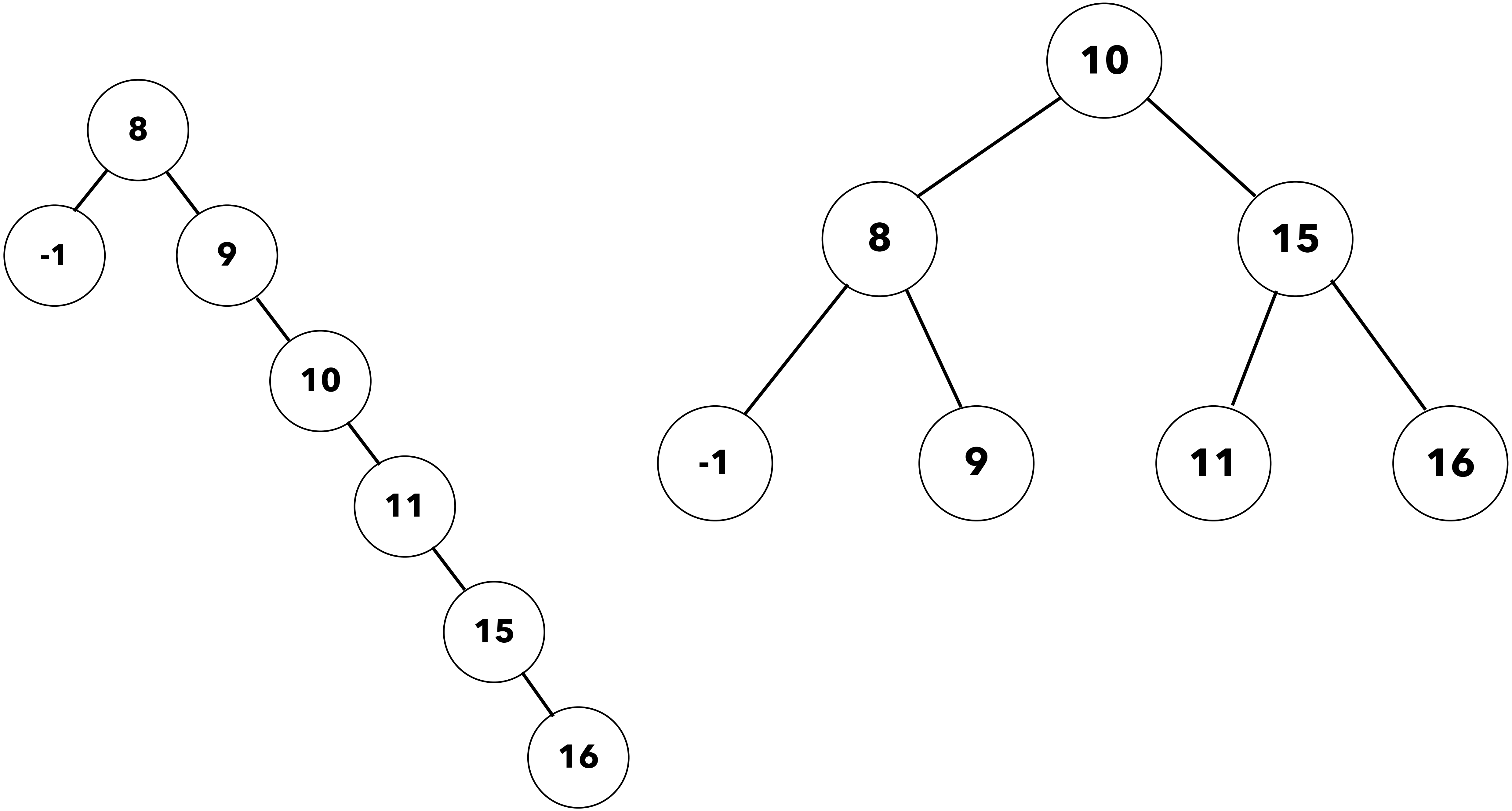

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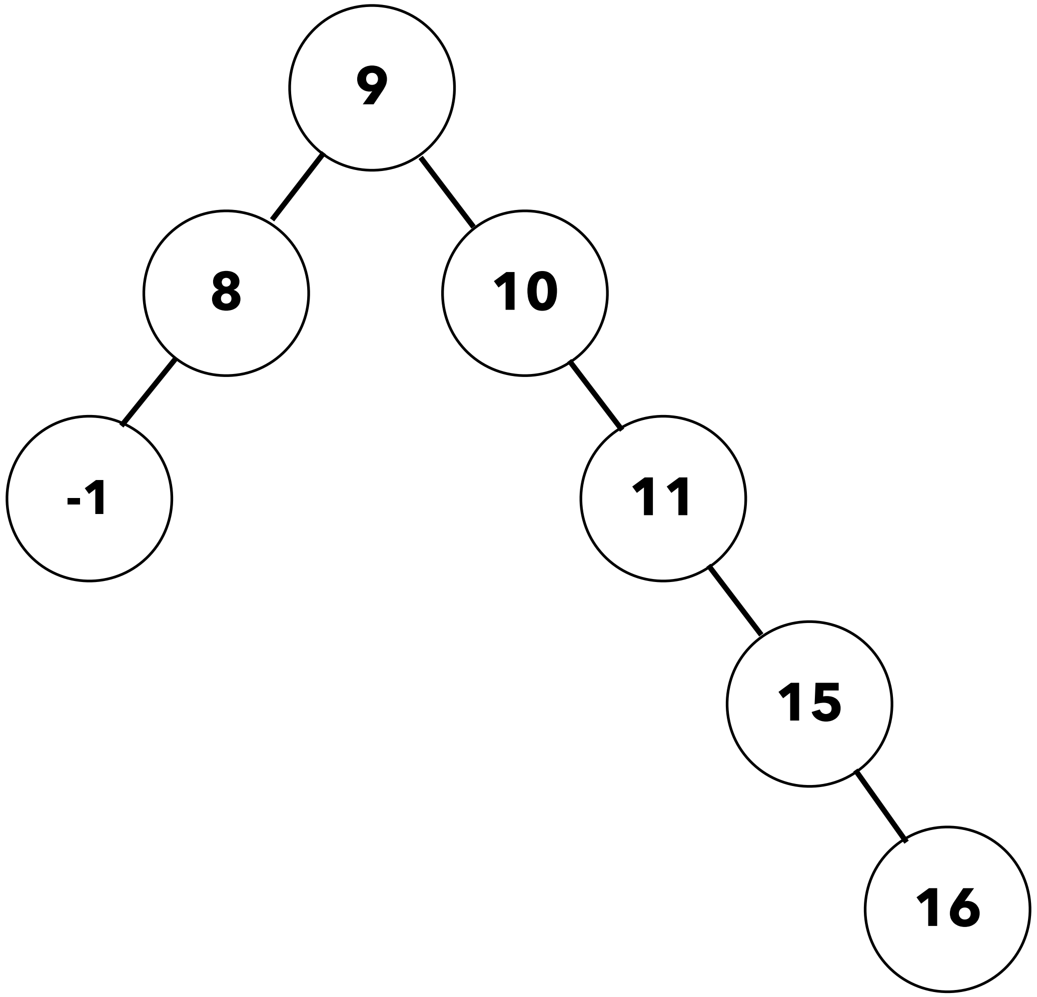

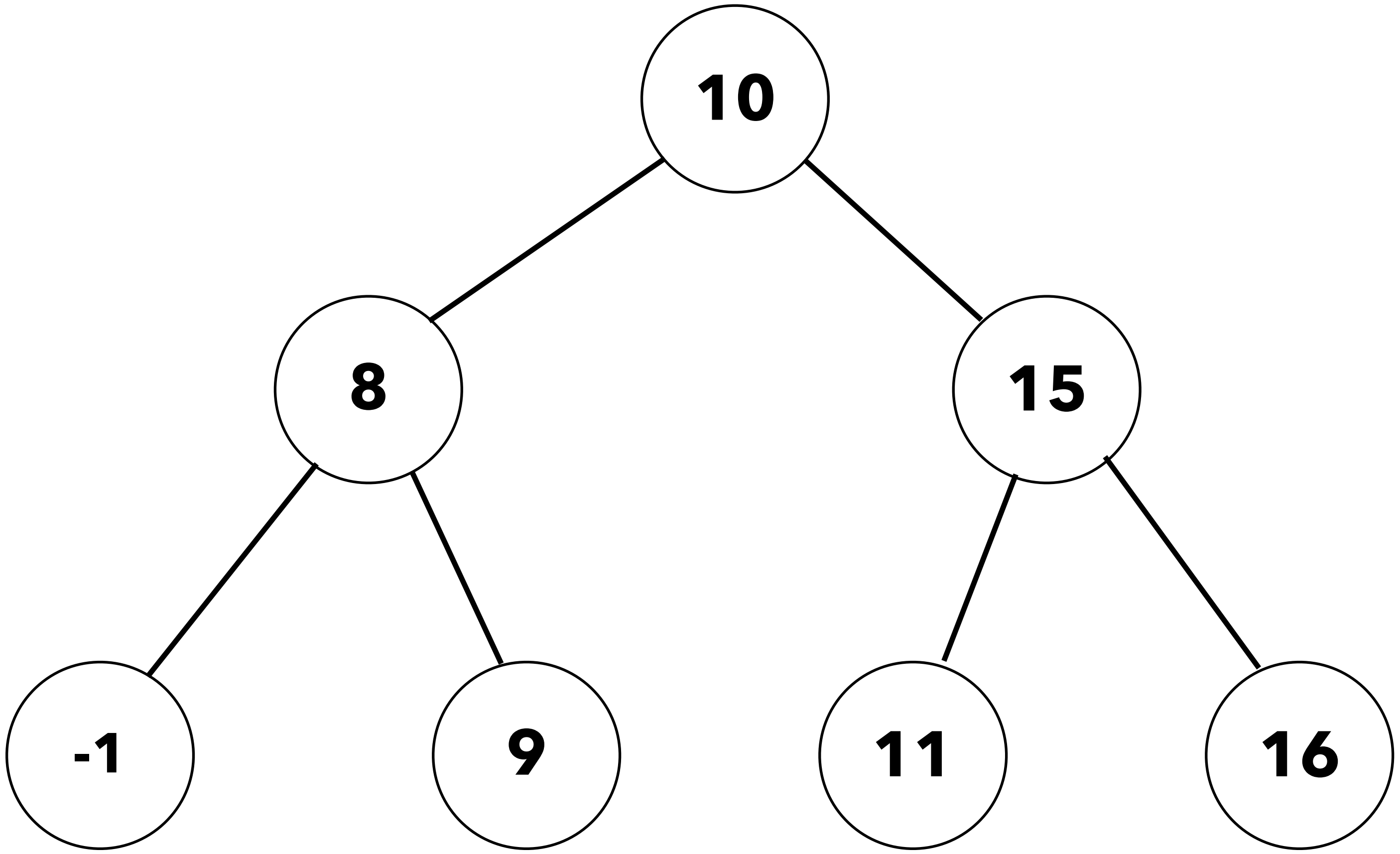

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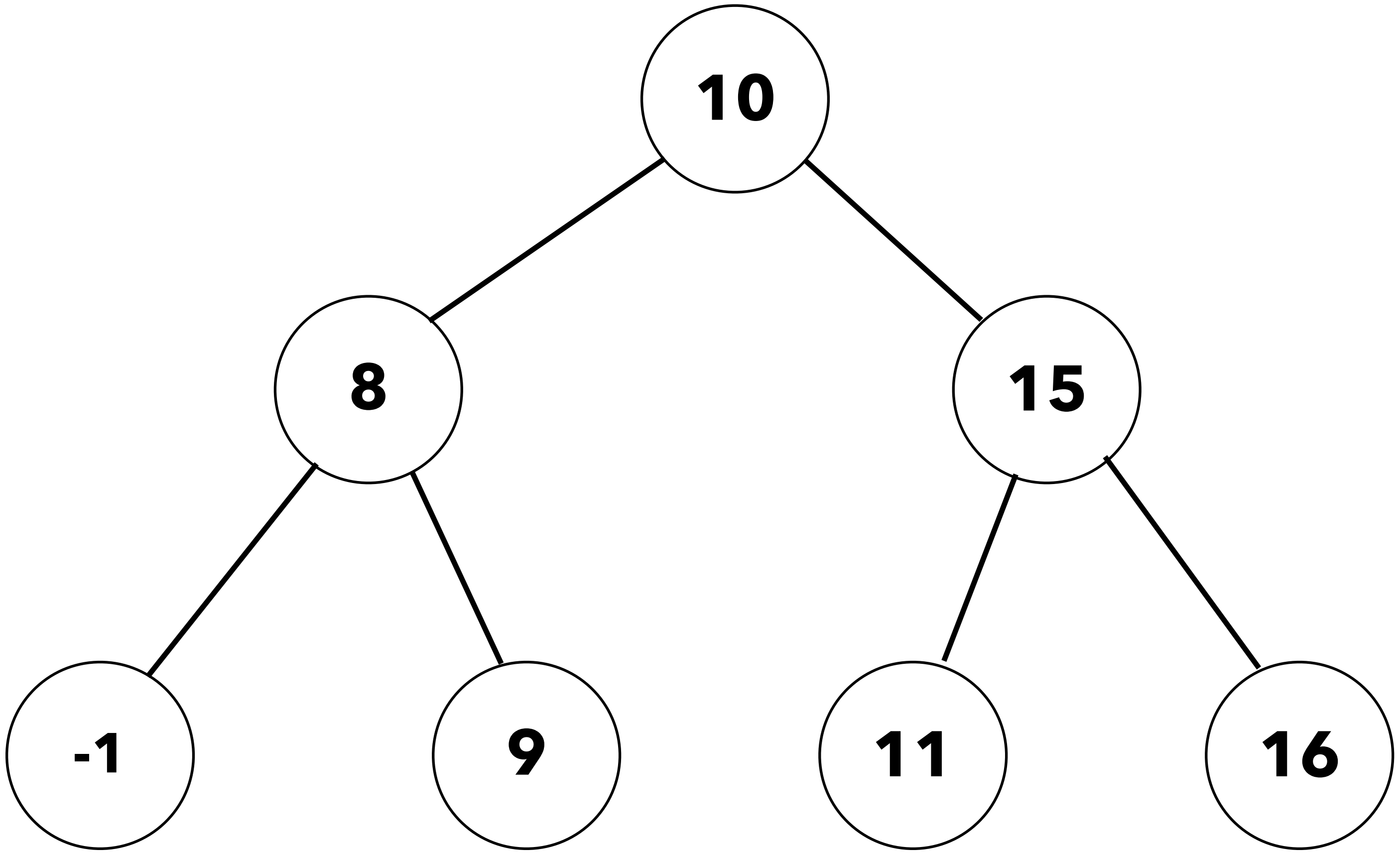

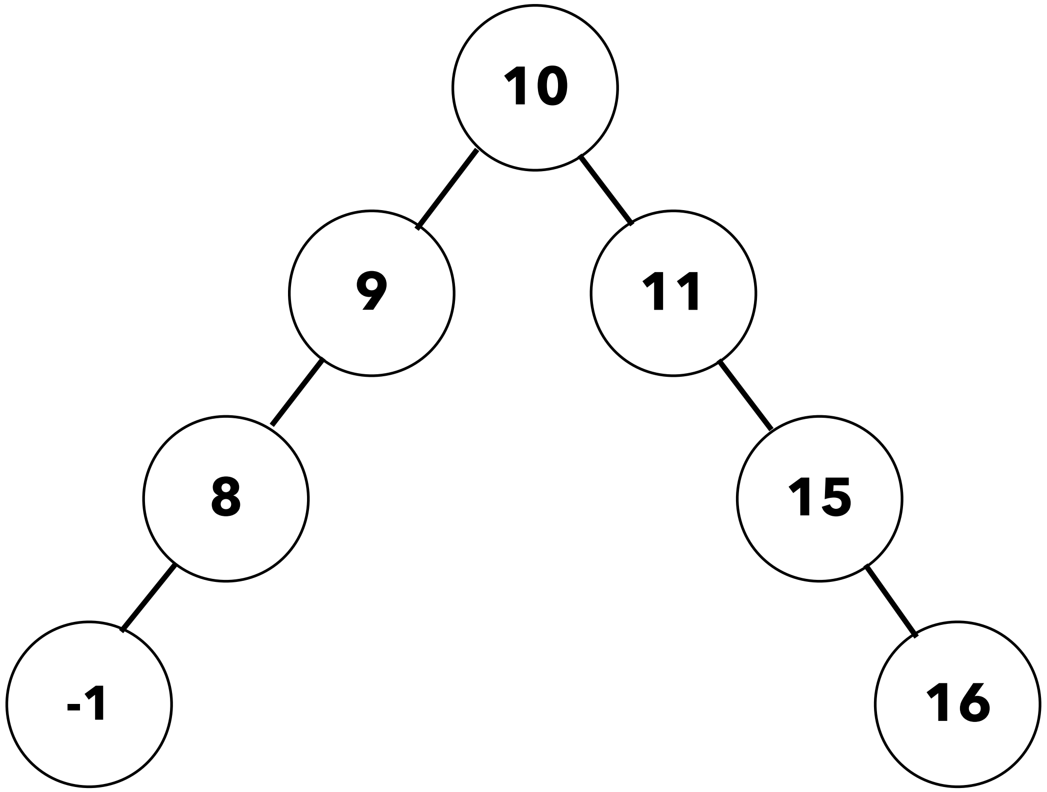

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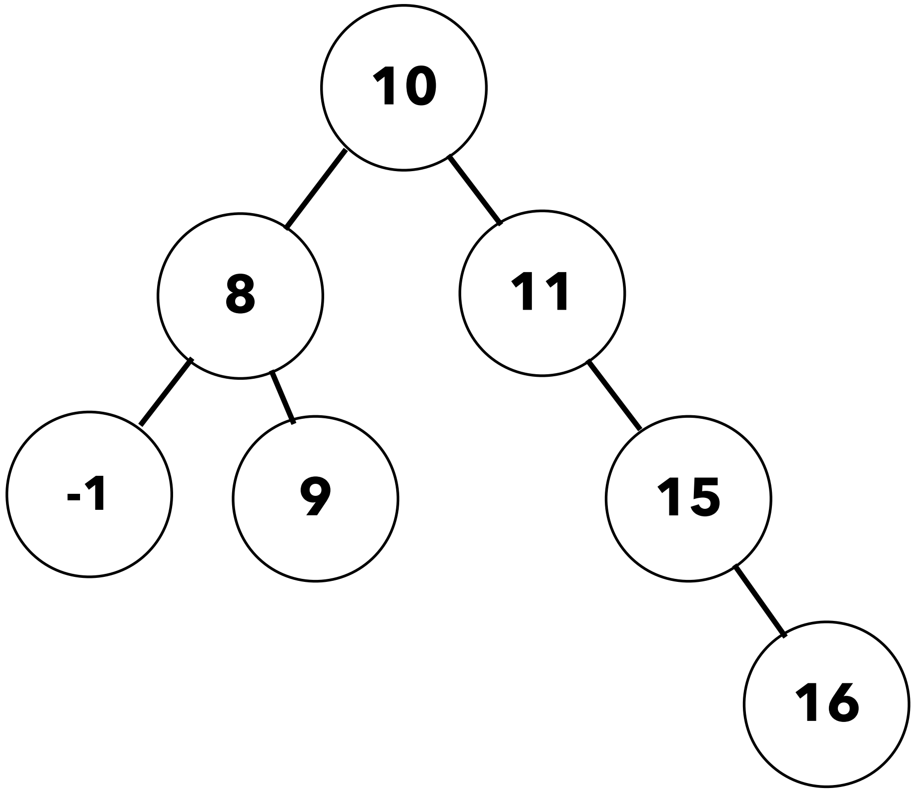

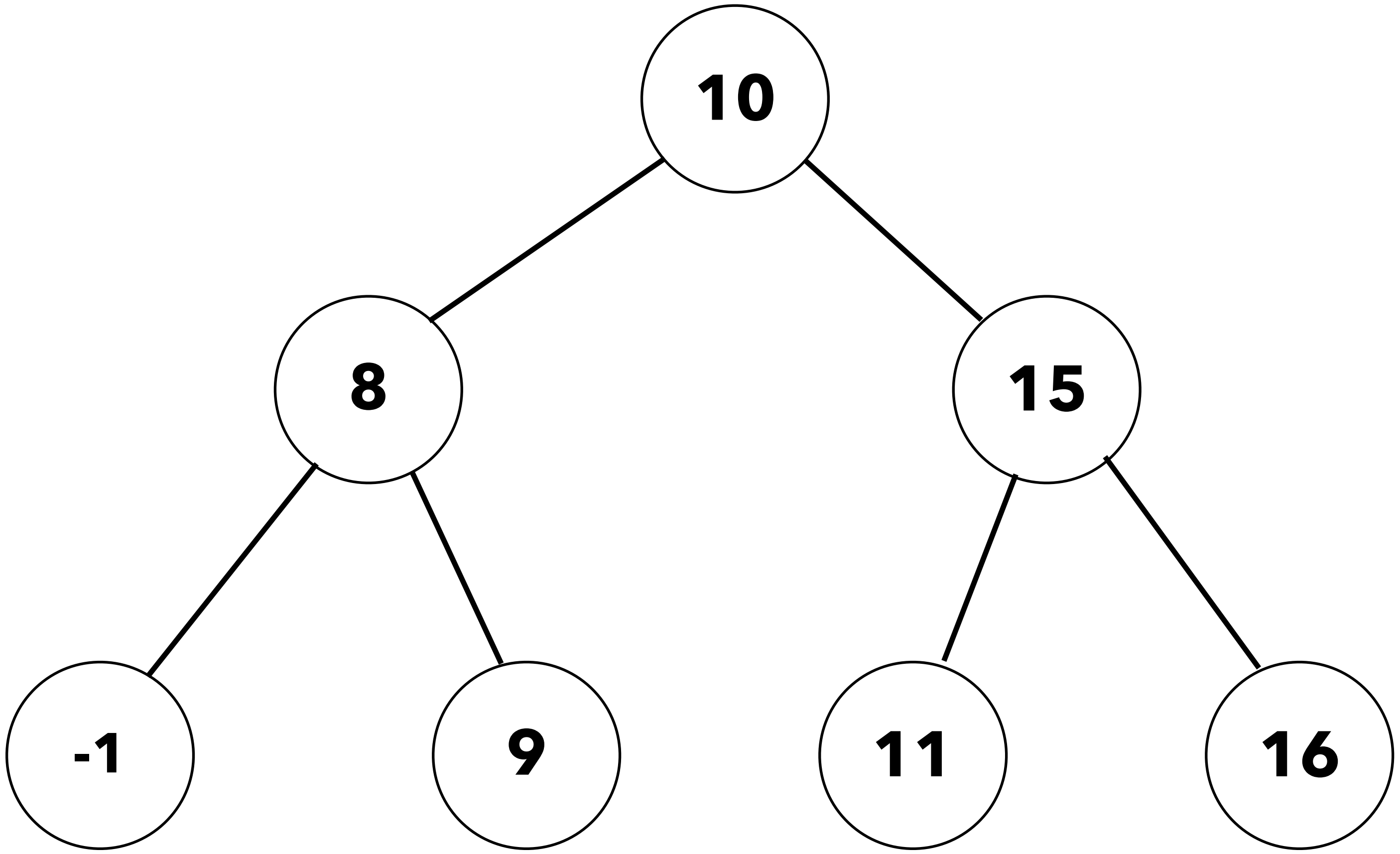

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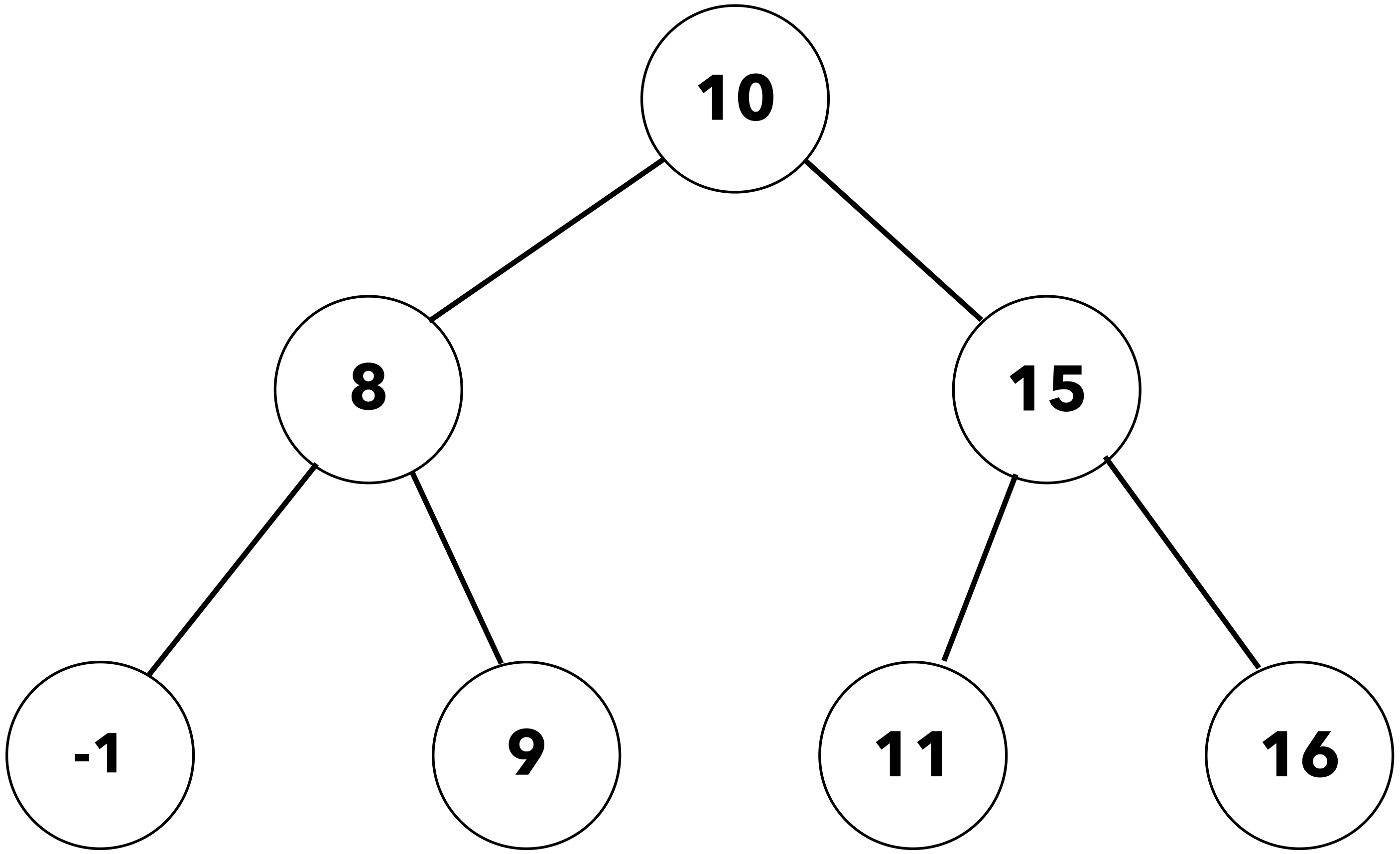
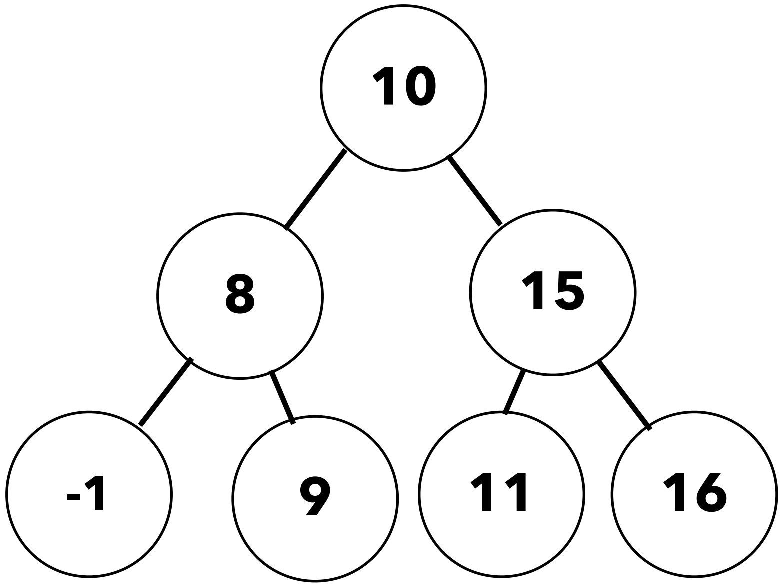
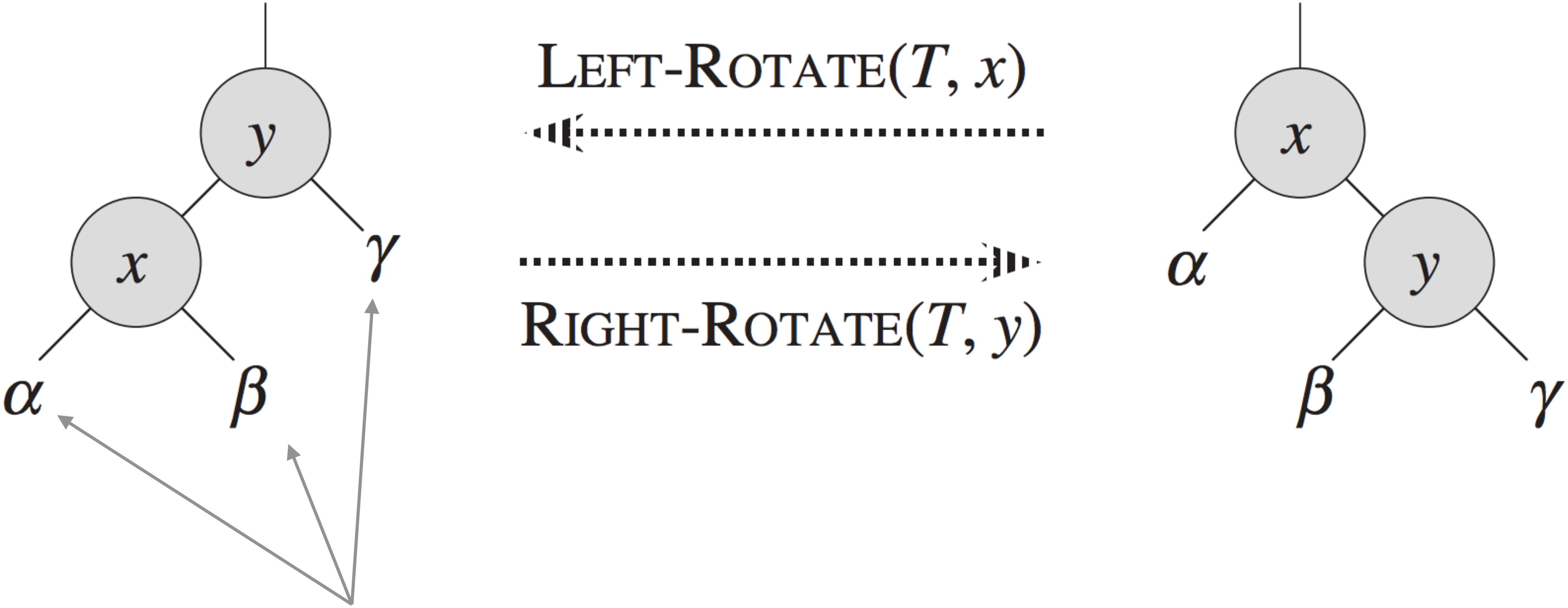




Tree Rotations

modify tree structure without violating the BST property.



subtrees (could be null, leaf, or tree with many nodes)

CLRS Fig 13.2, pg 313

Tree Rotations

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Steps in left rotation (move y up to its parent's position):

1. Transfer β: x's right subtree becomes y's old left subtree (β)
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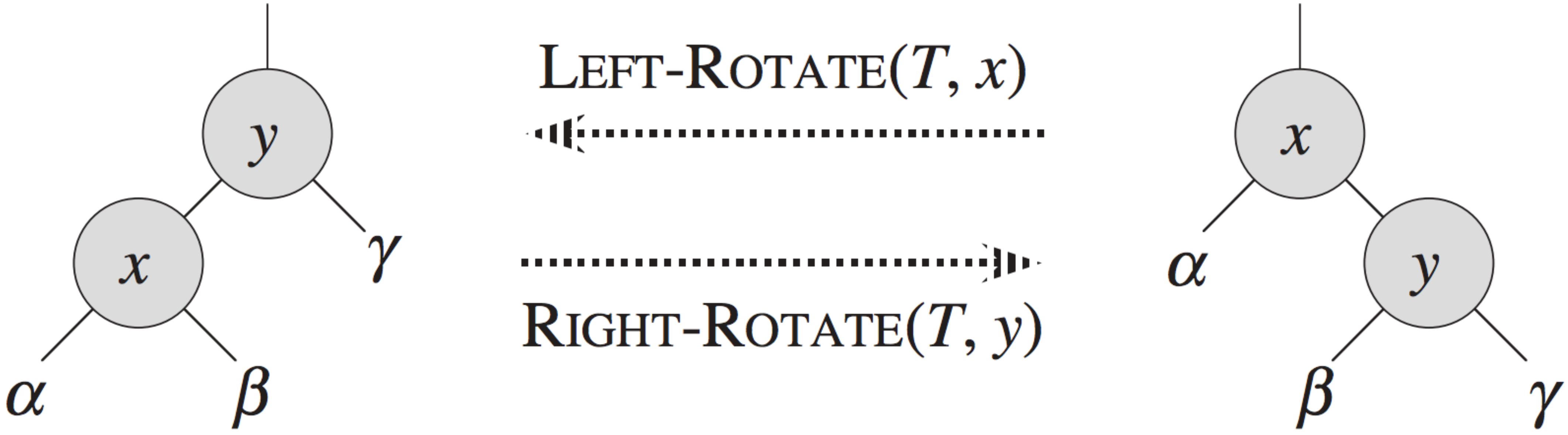

CLRS Fig 13.2, pg 313

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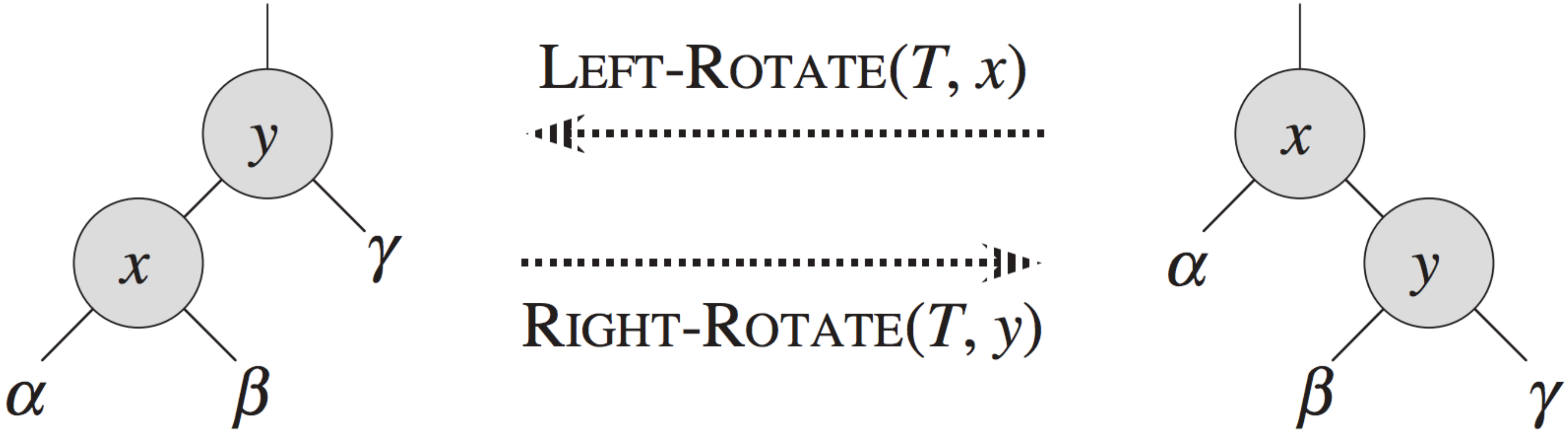
Details: need to update child, parent, and (possibly) root pointers.



CLRS Fig 13.2, pg 313

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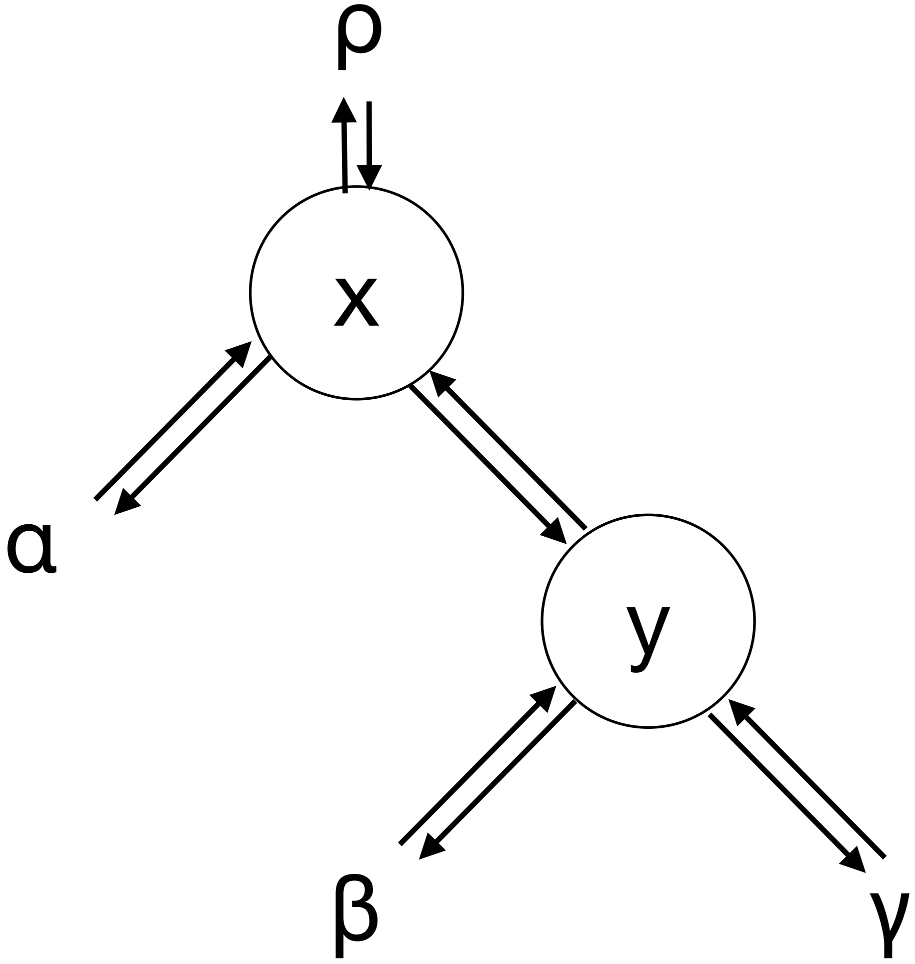

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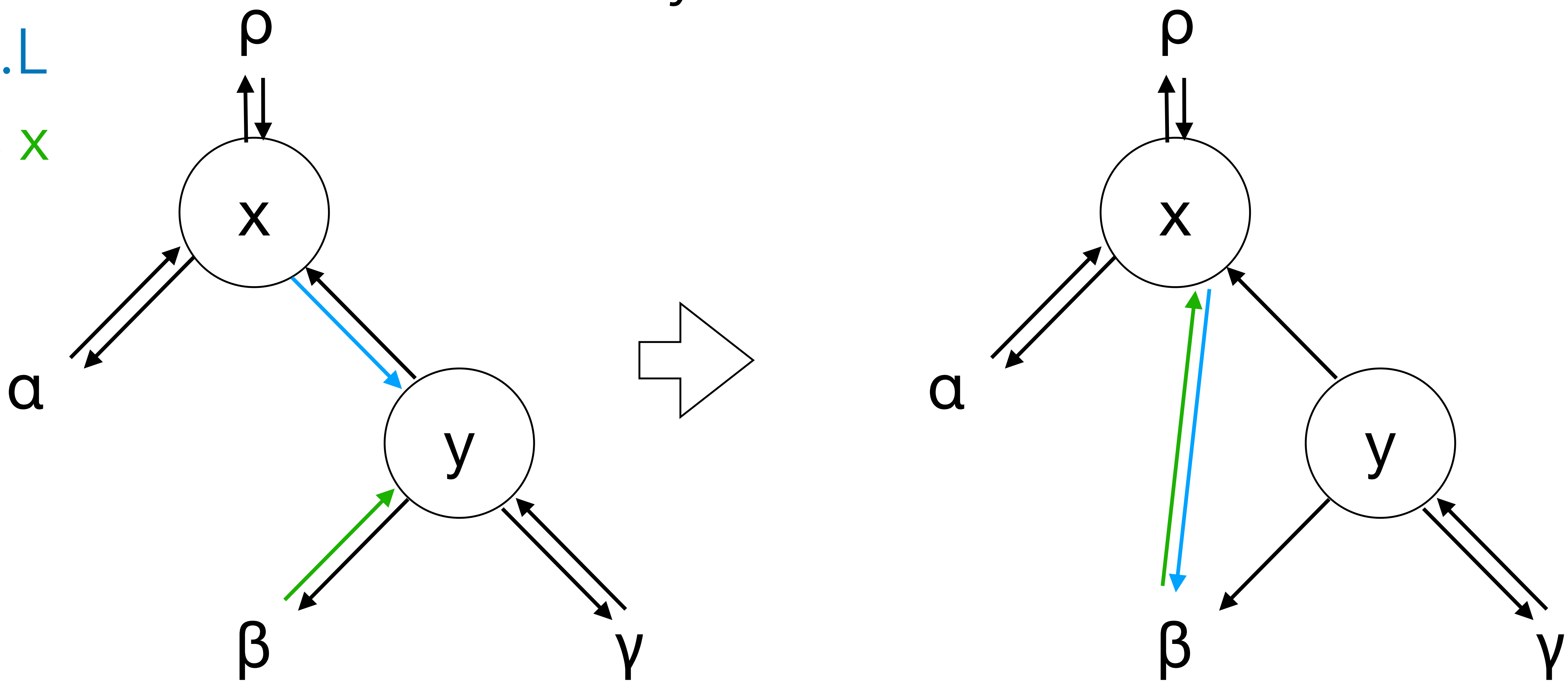
x.R gets y.L

y.L.p gets x



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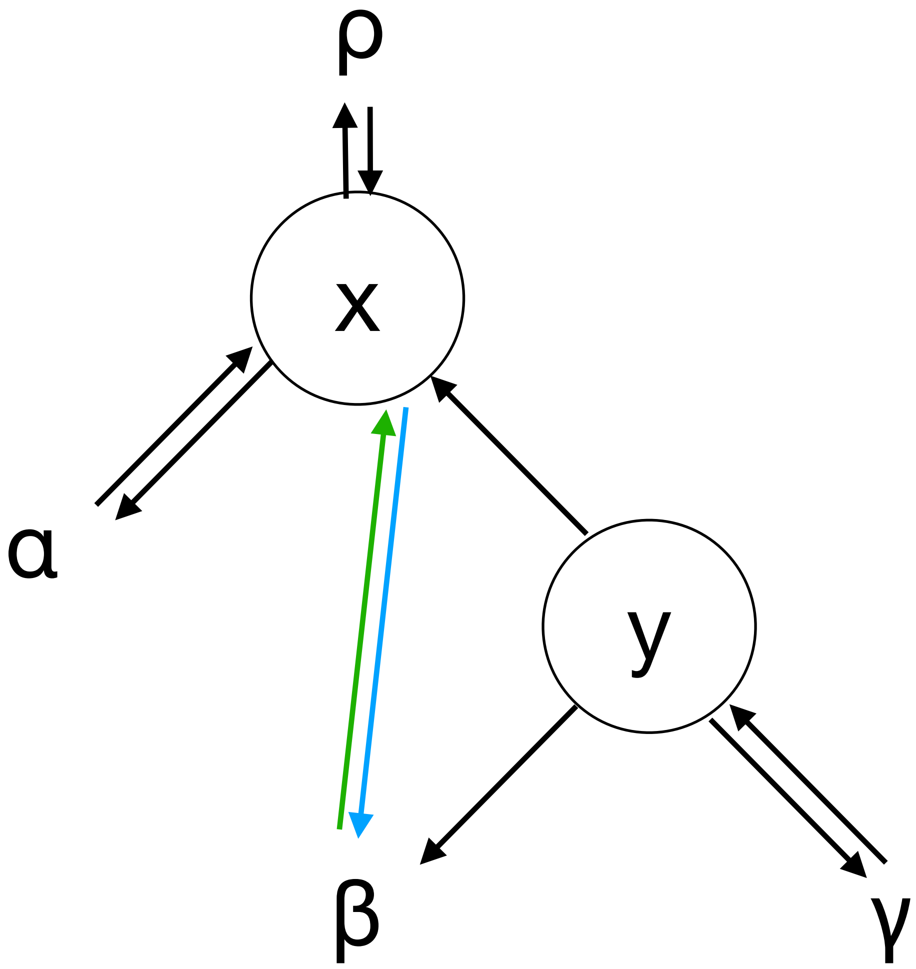

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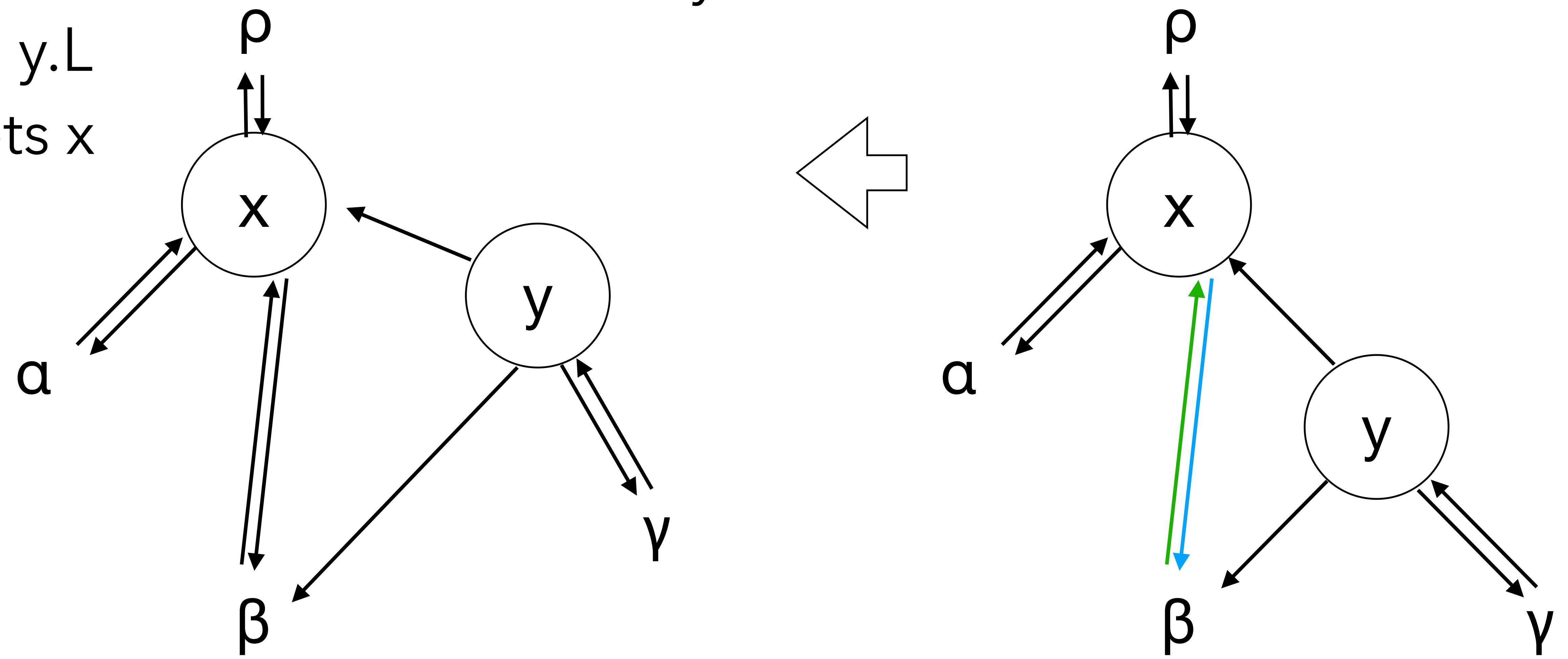

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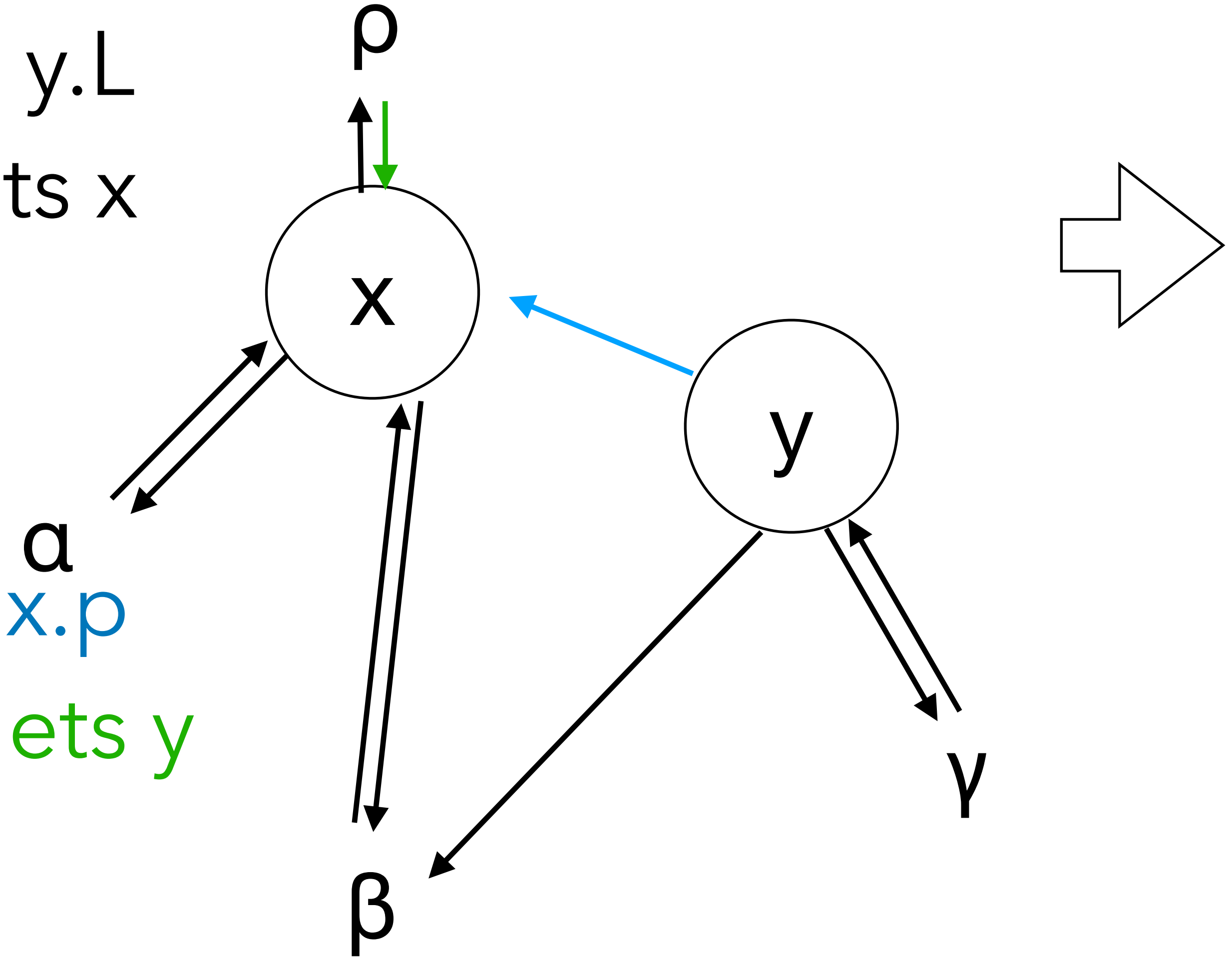

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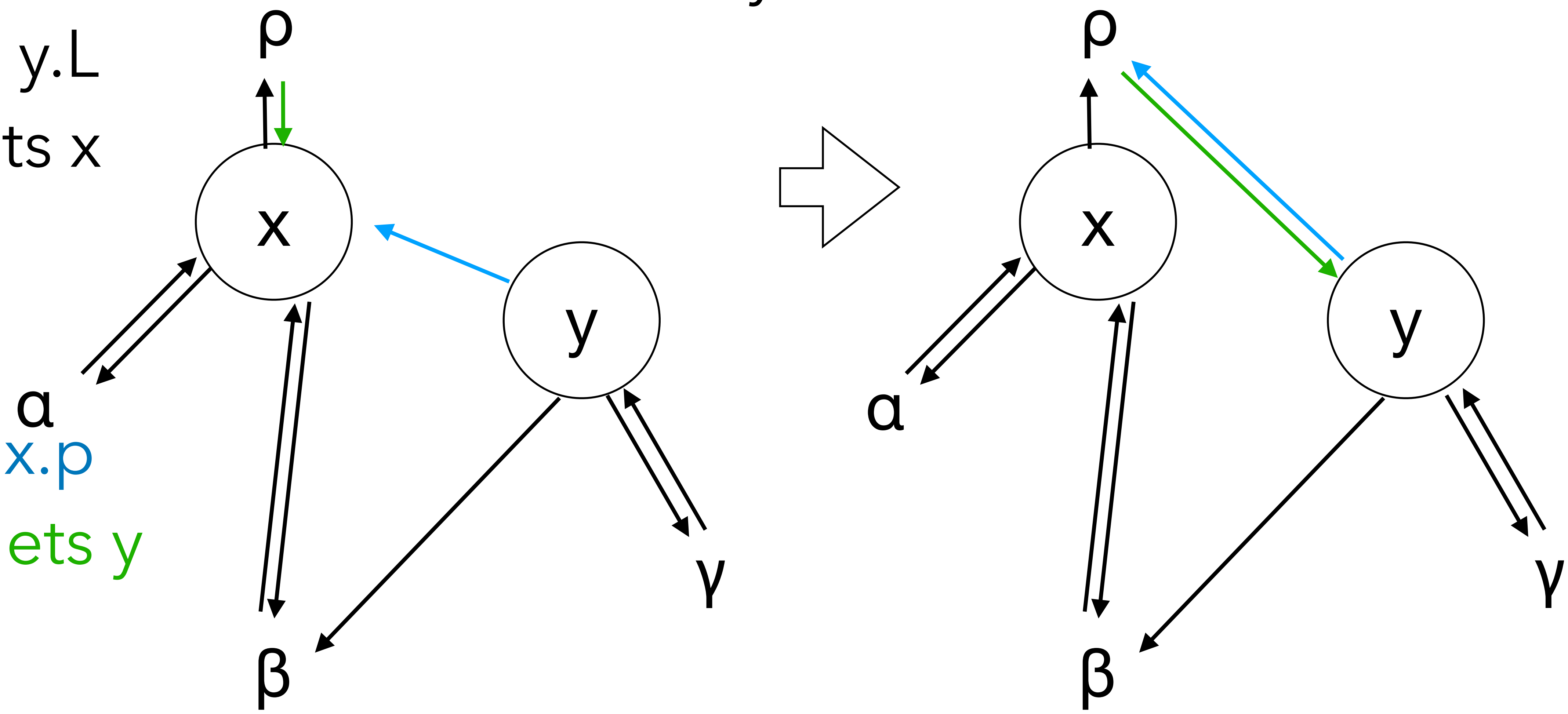

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(what if ρ is null / x was root?)

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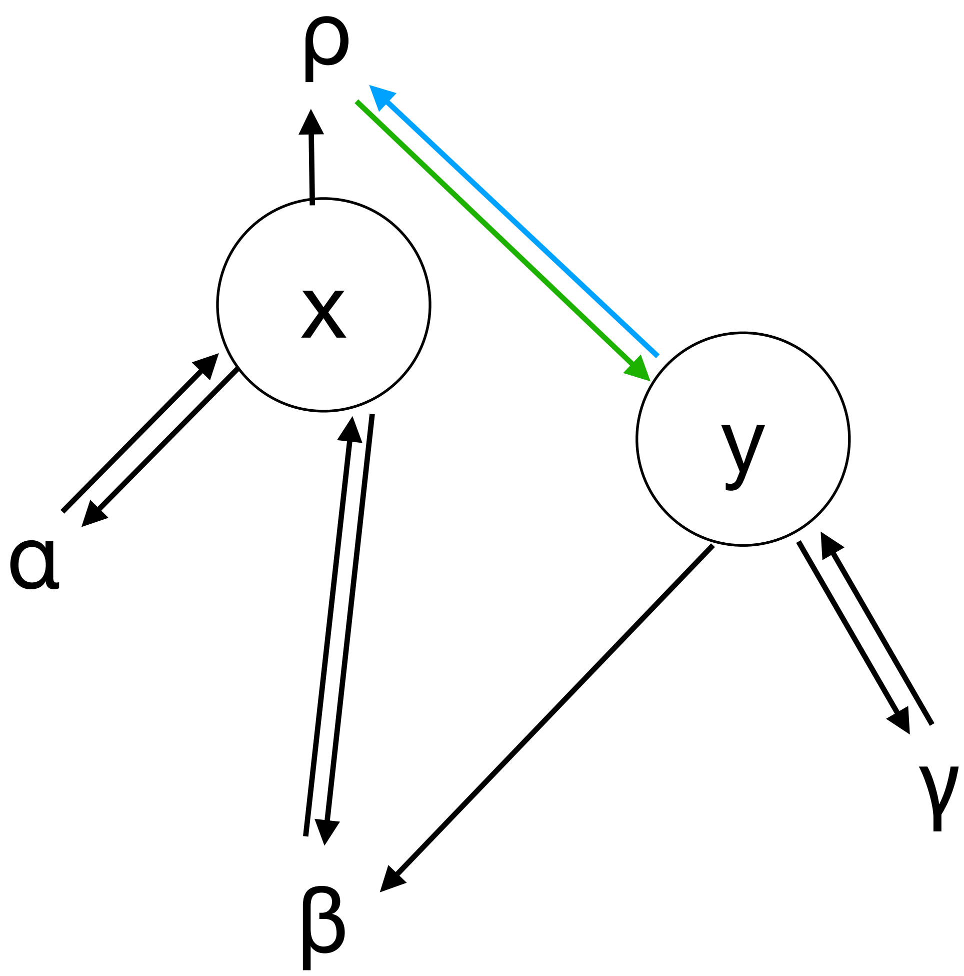

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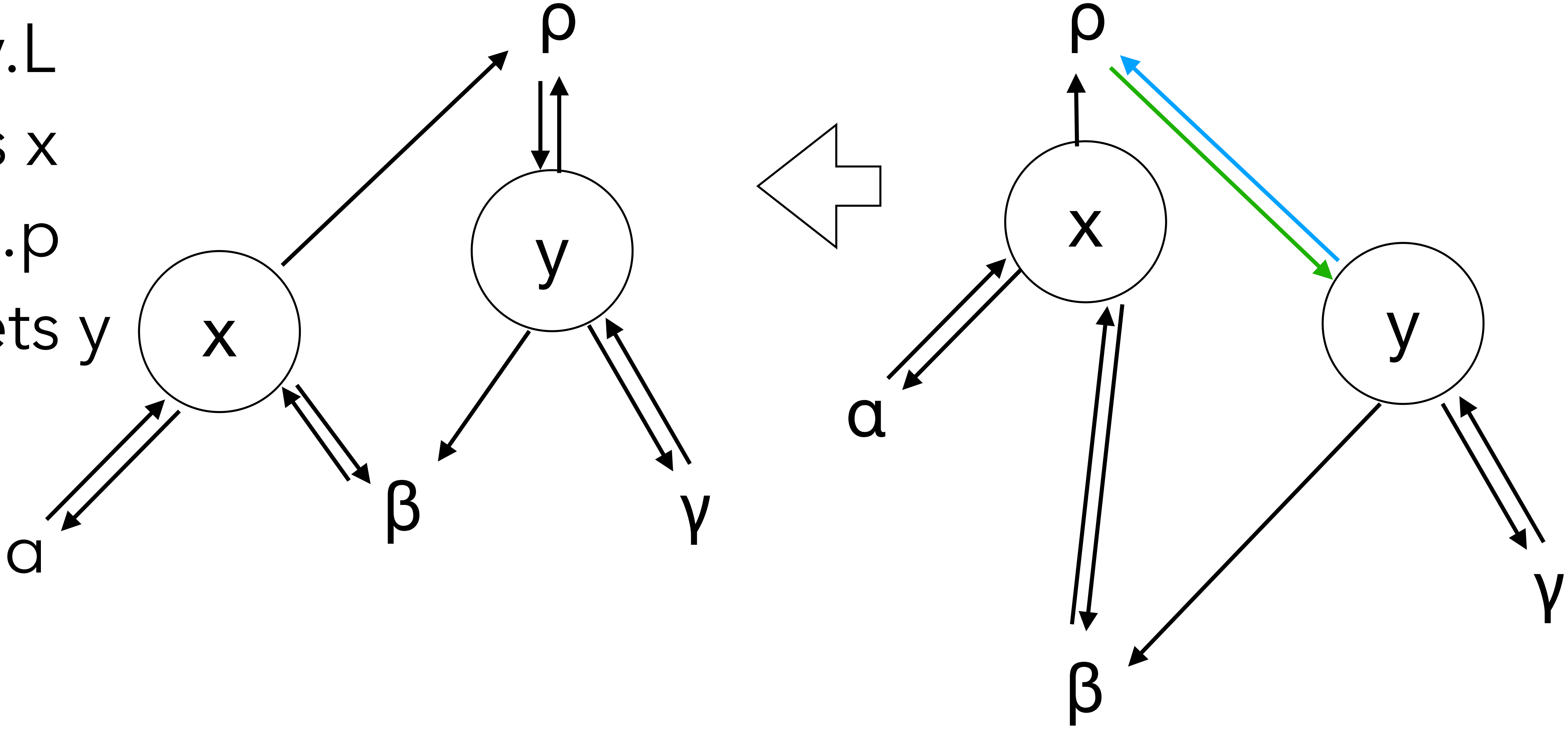

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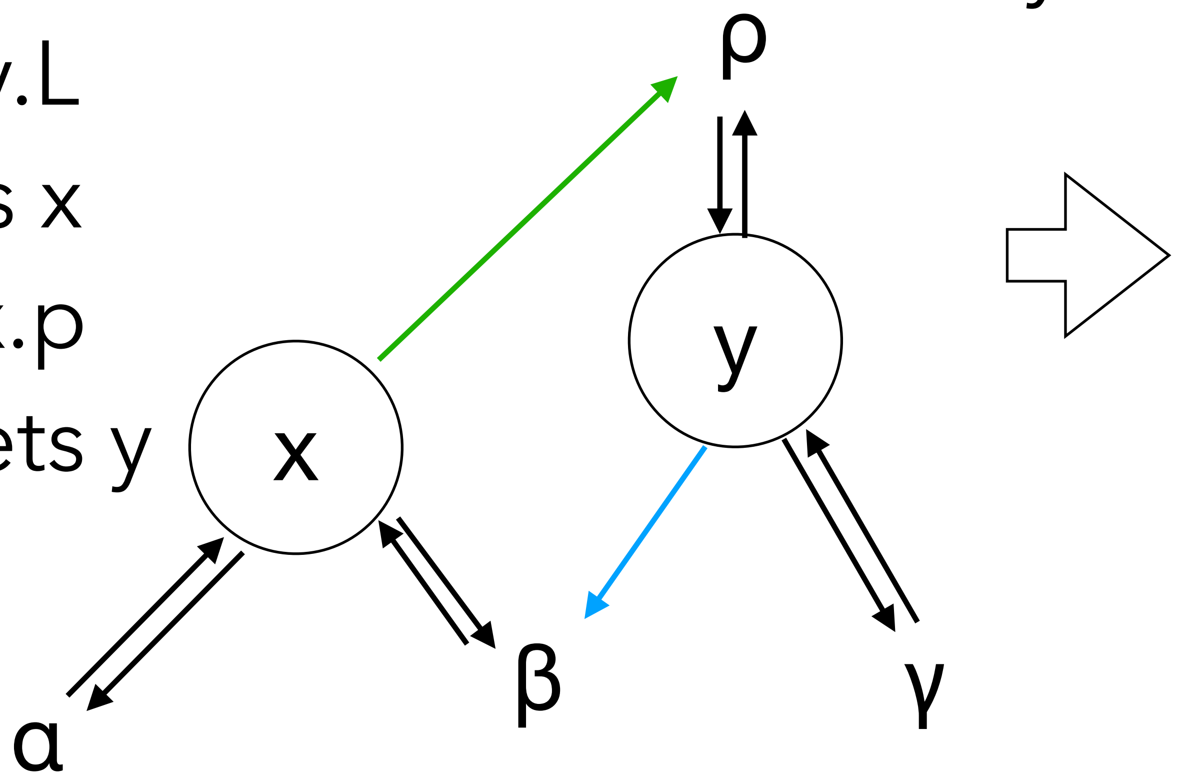

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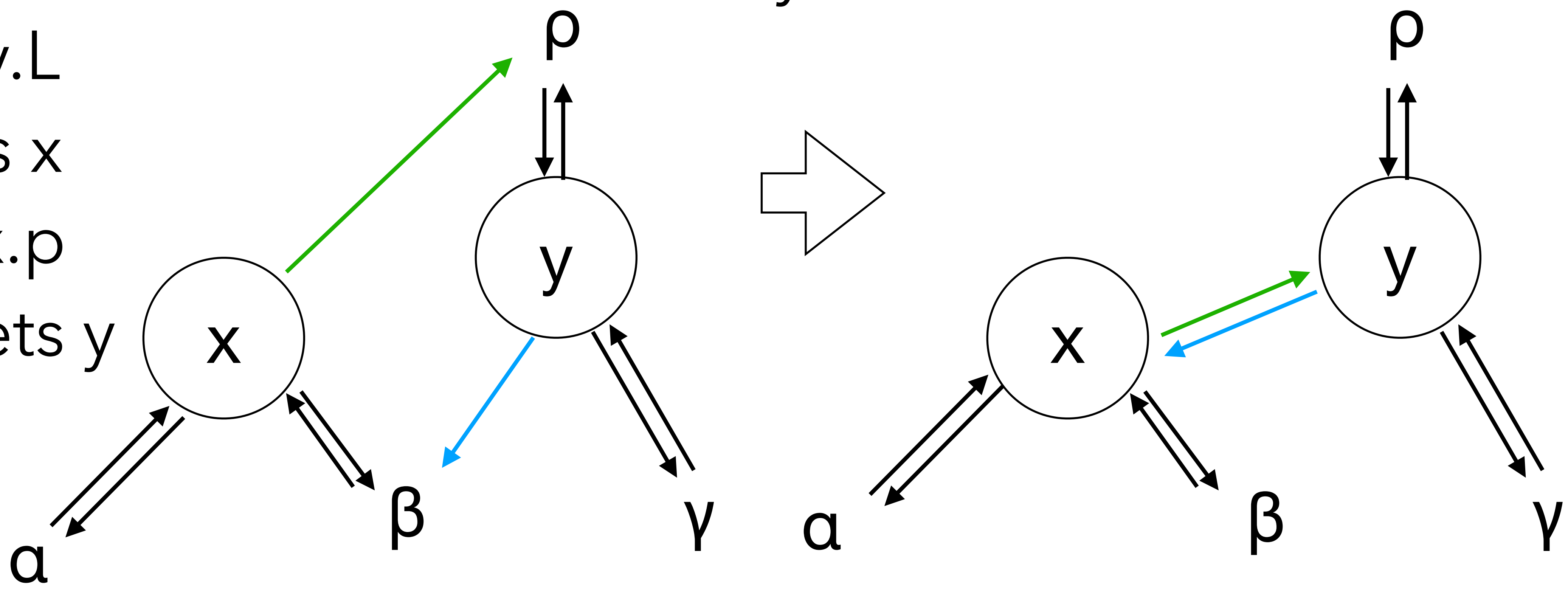

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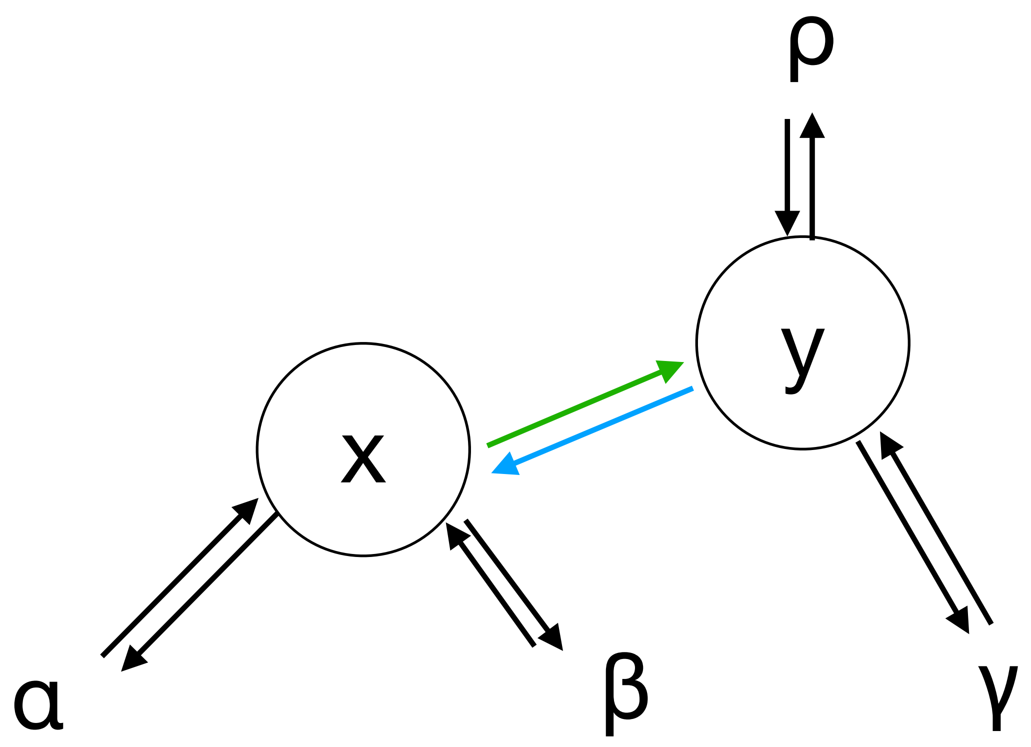

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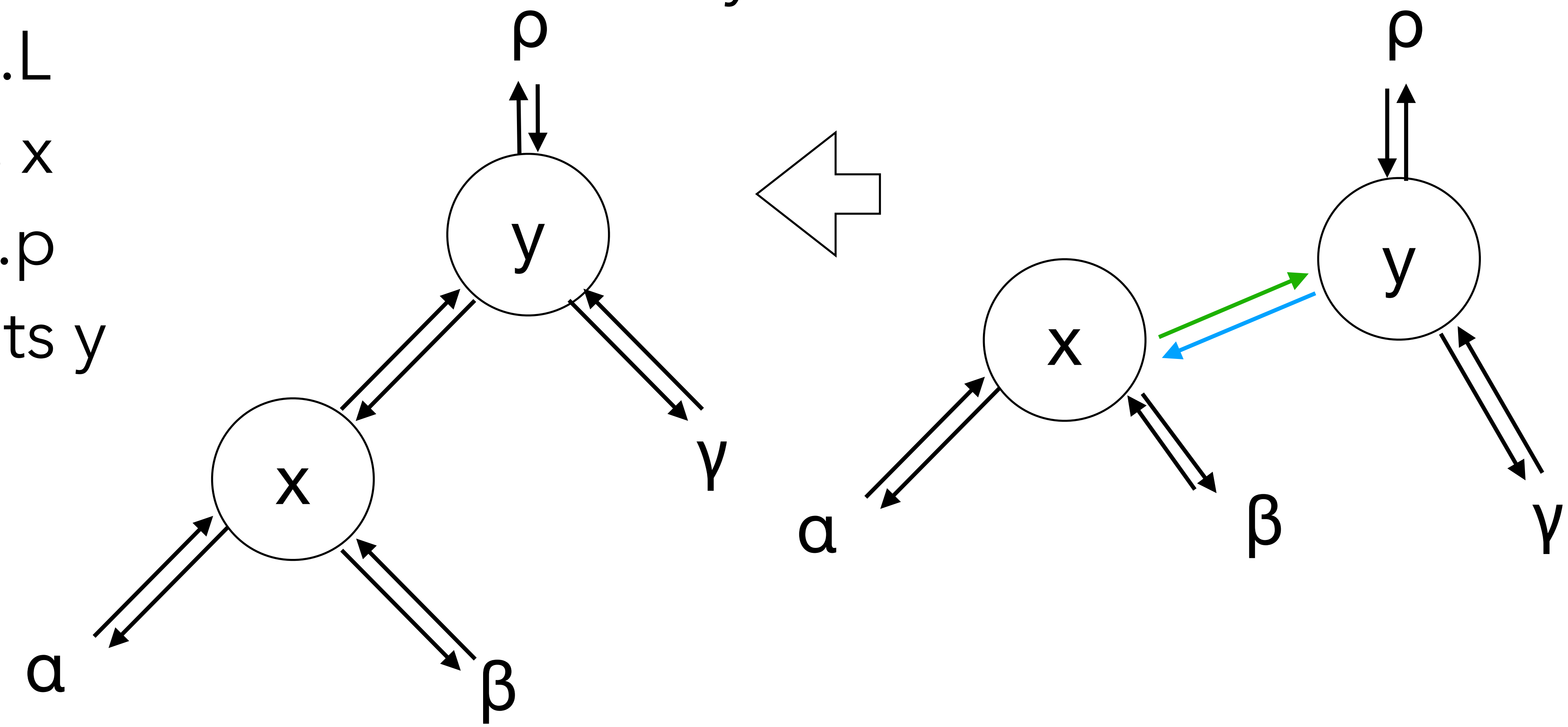

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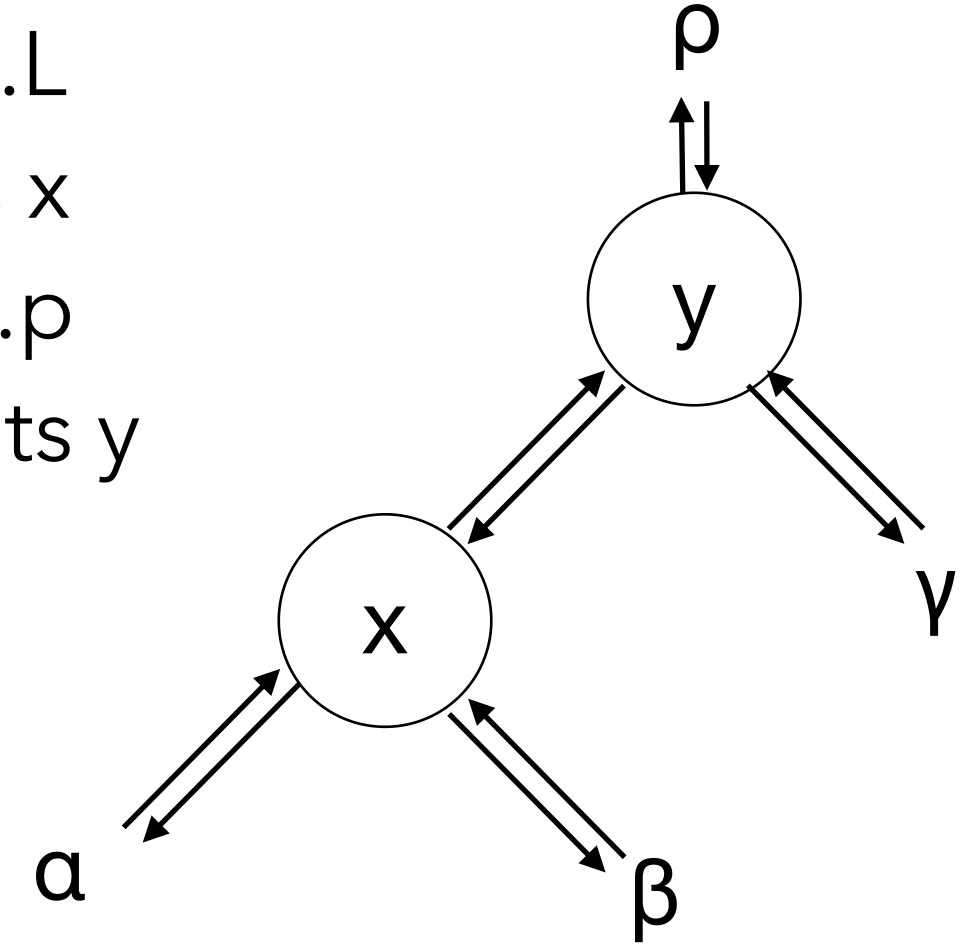

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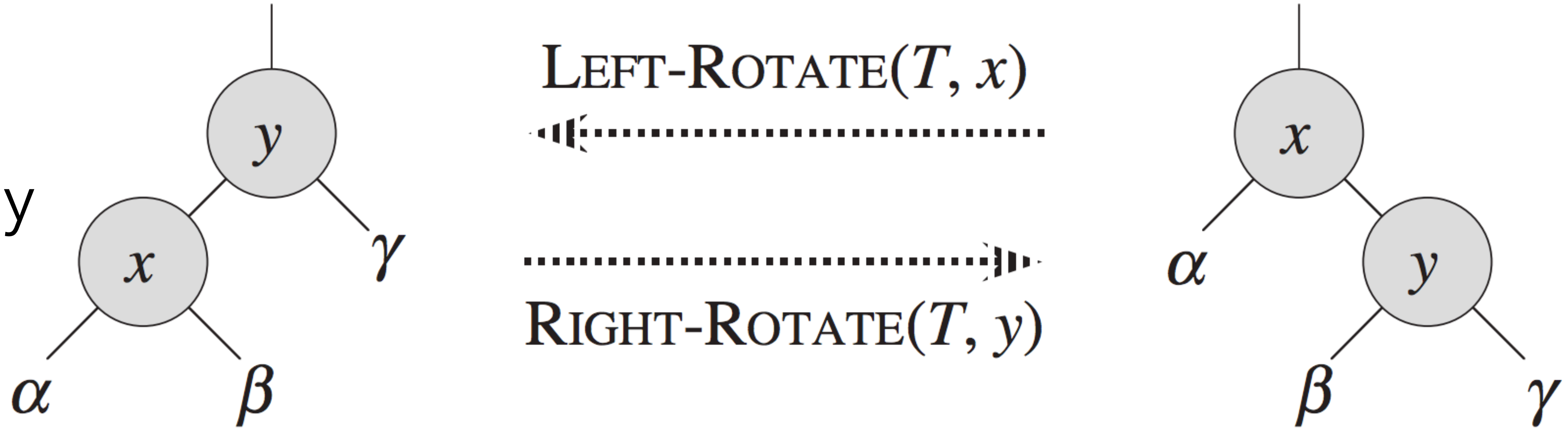
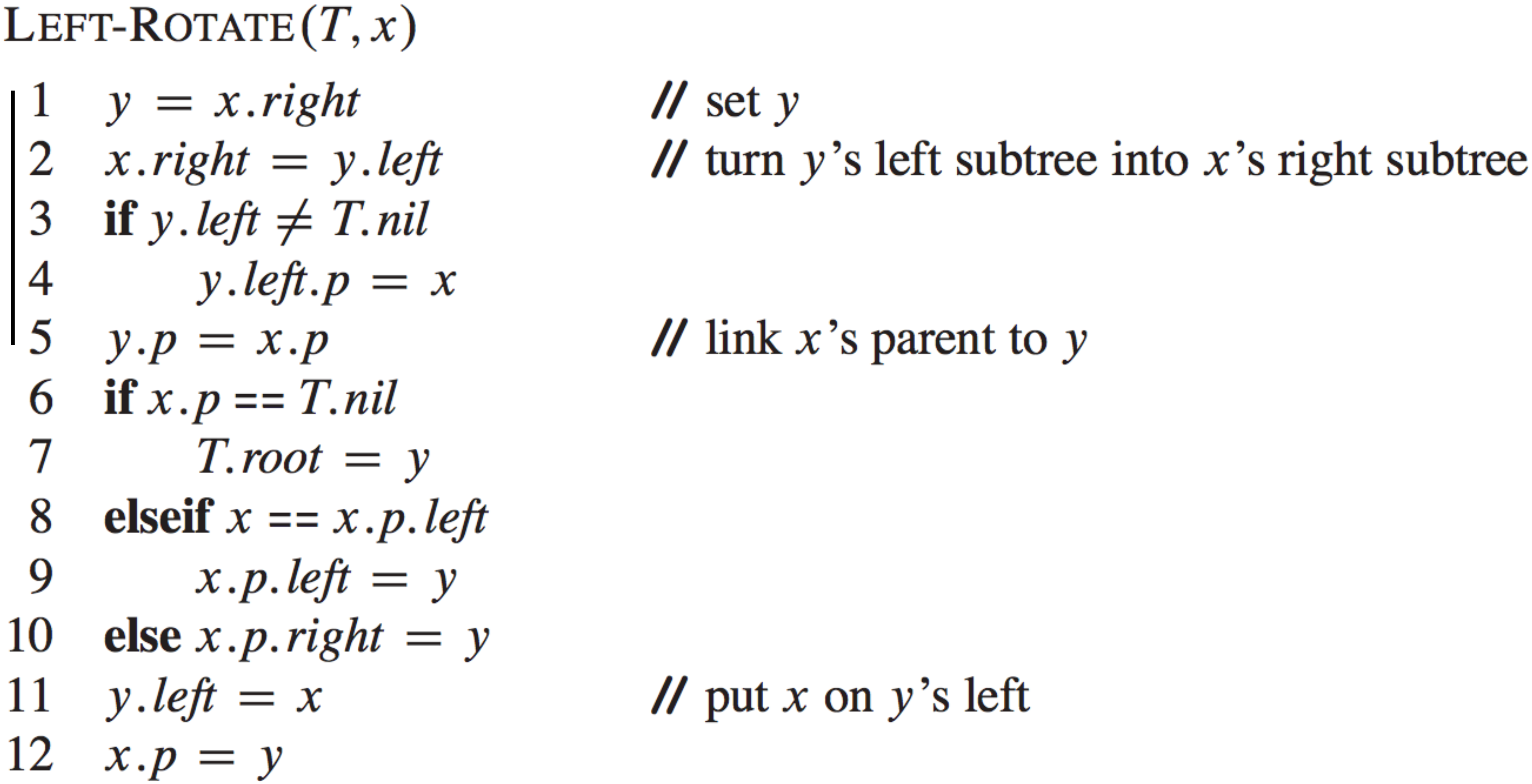


Overall Transformation

Pseudocode from CLRS



LEFT-ROTATE(T, x)

1. xfer β

```
y = x.right                      // set y
x.right = y.left                 // turn y's left subtree into x's right subtree
if y.left ≠ T.nil
    y.left.p = x
y.p = x.p                        // link x's parent to y
```

2. xfer parent

```
if x.p == T.nil
    T.root = y
elseif x == x.p.left
    x.p.left = y
else x.p.right = y
```

3. xfer x

```
y.left = x                       // put x on y's left
x.p = y
```

Notational quirk: assume T.nil means "null"