

CSCI 241

Lecture N-1
 Graph Planarity
 Topological Sort

Announcements

- Quiz 6 grades are out.
 - video coming soon
- A4 is due Wednesday
- No lab deliverable this week.
 - TAs will be available in lab sections for questions.
- Material from today onward will not be on the exam
 - this week: a mix of fun bonus topics and review

Tentative Goals - This week

- Analyze the runtime of Dijkstra's algorithm.
- Know the definition of graph planarity
- Know how to use Topological Sort to determine whether a graph is acyclic.
- Know the definition of a spanning tree.
- Know how to build spanning trees using:
 - Prim's algorithm
 - Kruskal's algorithm
- Coding trees?
- Tries?

Exercise: Analyze the runtime of Dijkstra's Algorithm.

```
S = { }; F = {v}; v.d = 0;
while (F ≠ { }) {
  f = node in F with min d value;
  Remove f from F, add it to S;
  for each neighbor w of f {
    if (w not in S or F) {
      w.d = f.d + weight(f, w);
      add w to F;
    } else if (f.d + weight(f, w) < w.d) {
      w.d = f.d + weight(f, w);
    }
  }
}
```

Let $e = |E|$, $v = |V|$

Assume hash table lookups are $O(1)$.

For all else, assume worst-case.

One group member: submit your group's answer via Socrative.

Pseudocode is available at:

https://facultyweb.cs.wwu.edu/~wehrwes/courses/csci241_20s/lectures/L21/algorithm.pdf

Course webpage > Schedule > 5/27 > algorithm

Exercise: Analyze the runtime of Dijkstra's Algorithm.

```
S = { }; F = {v}; v.d = 0;
while (F ≠ { }) {
  f = node in F with min d value;
  Remove f from F, add it to S;
  for each neighbor w of f {
    if (w not in S or F) {
      w.d = f.d + weight(f, w);
      add w to F;
    } else if (f.d + weight(f, w) < w.d) {
      w.d = f.d + weight(f, w);
    }
  }
}
```

Let $e = |E|$, $v = |V|$

Assume hash table lookups are $O(1)$.

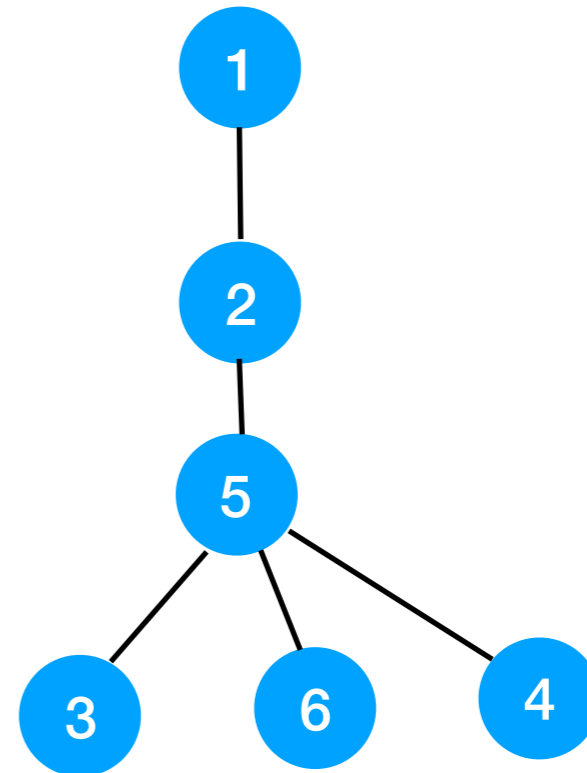
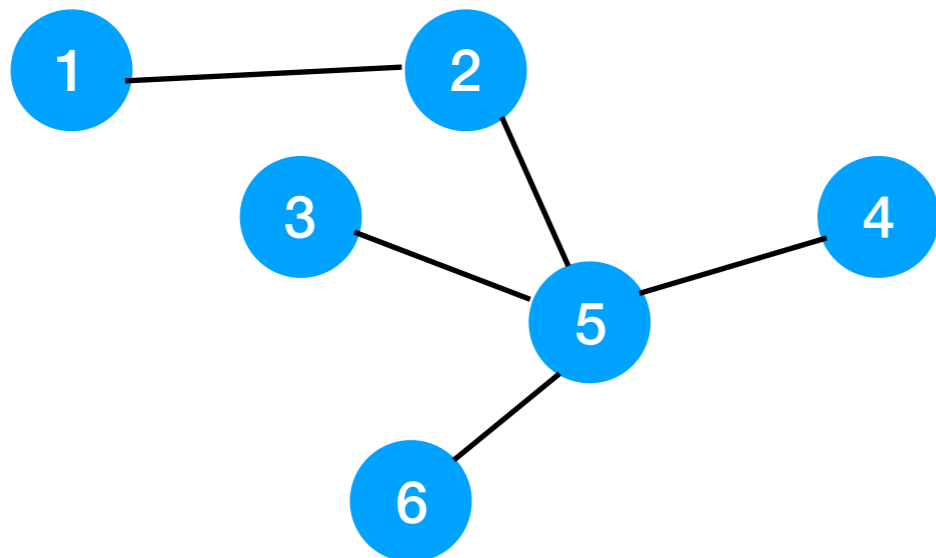
For all else, assume worst-case.

Drawing Graphs

- The same graph can be drawn (infinitely!) many different ways.

$$V = \{1,2,3,4,5,6\}$$

$$E = \{(1,2), (2,5), (3,5), (4,5), (5,6)\}$$

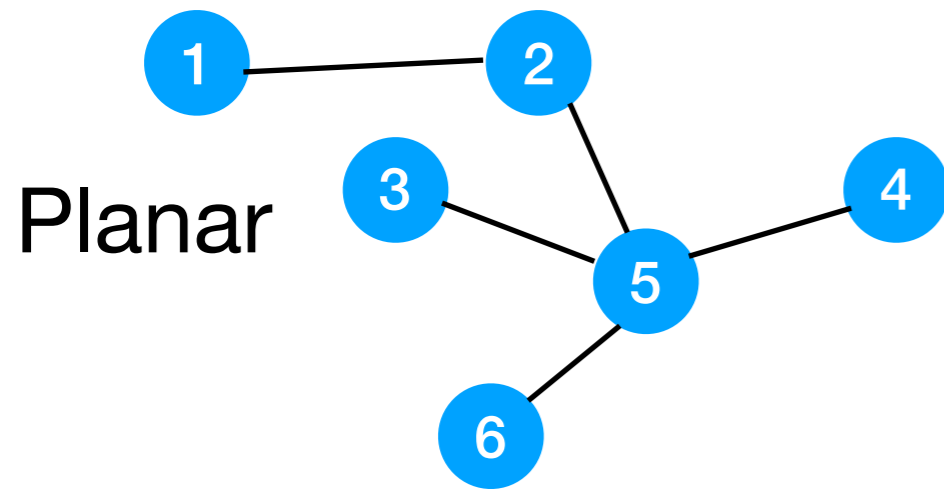


Planarity

- If a graph can be drawn without crossing edges, it is **planar**.

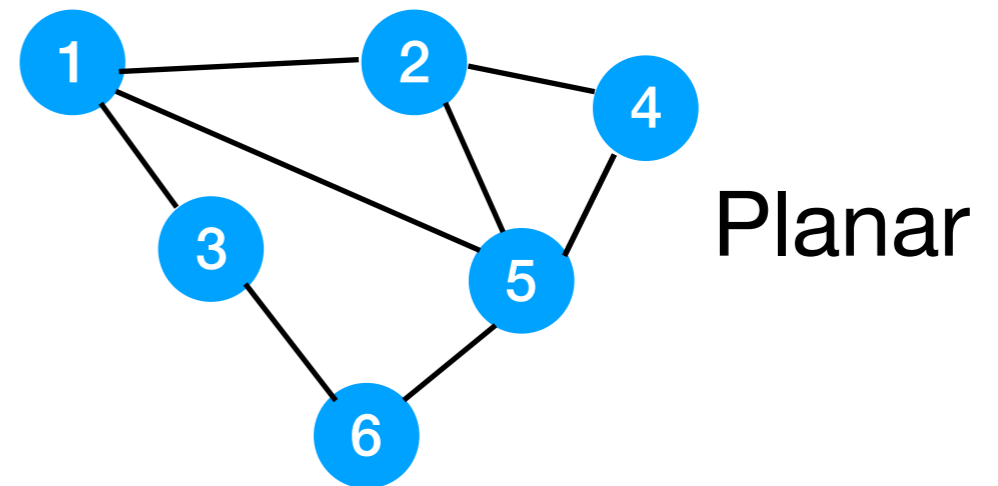
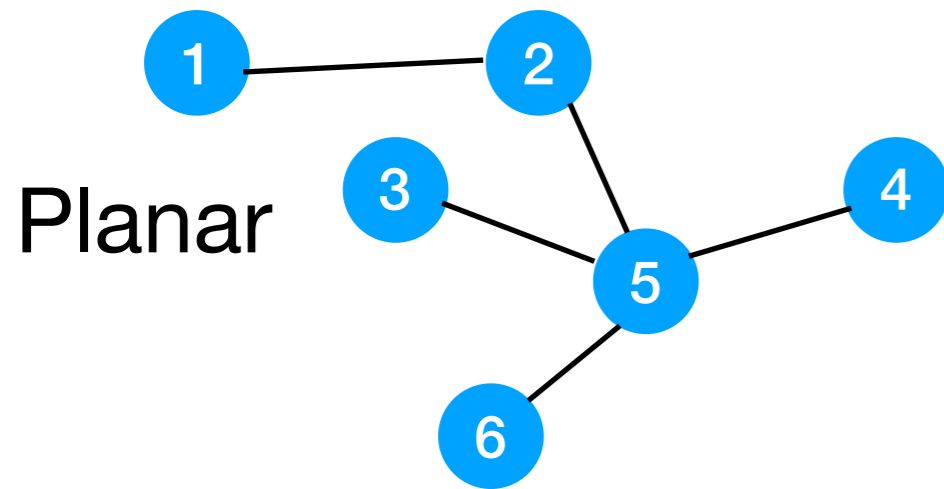
Planarity

- If a graph can be drawn without crossing edges, it is **planar**.



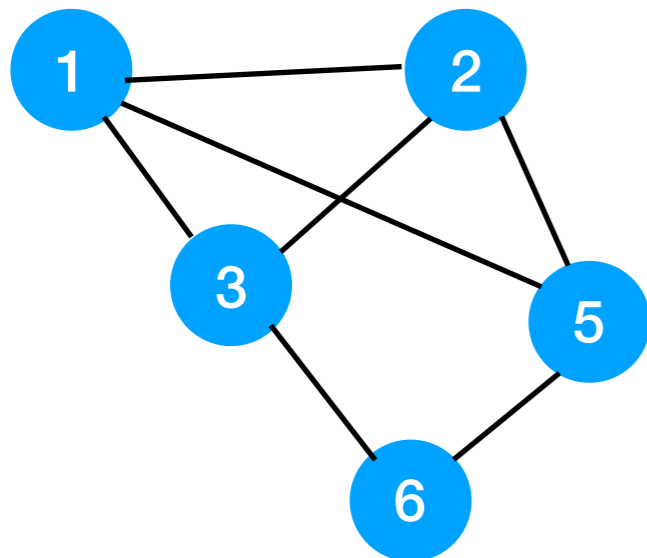
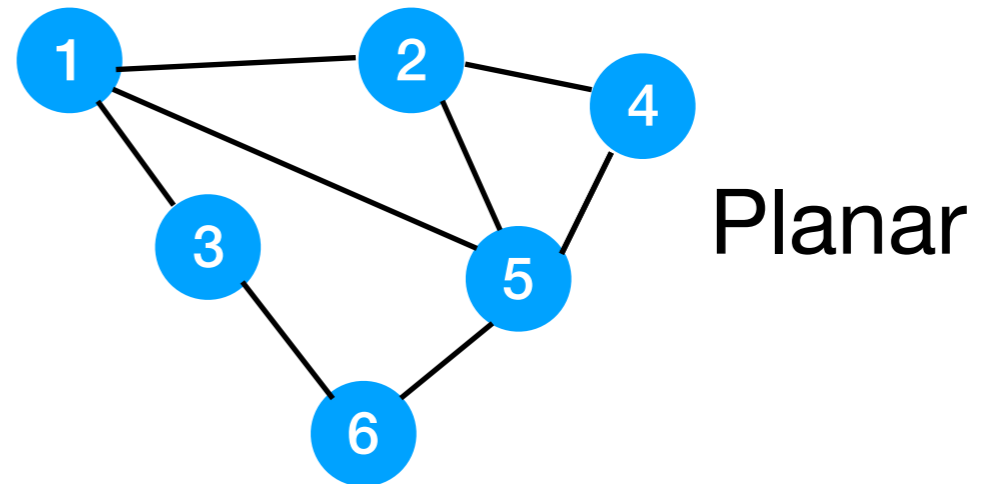
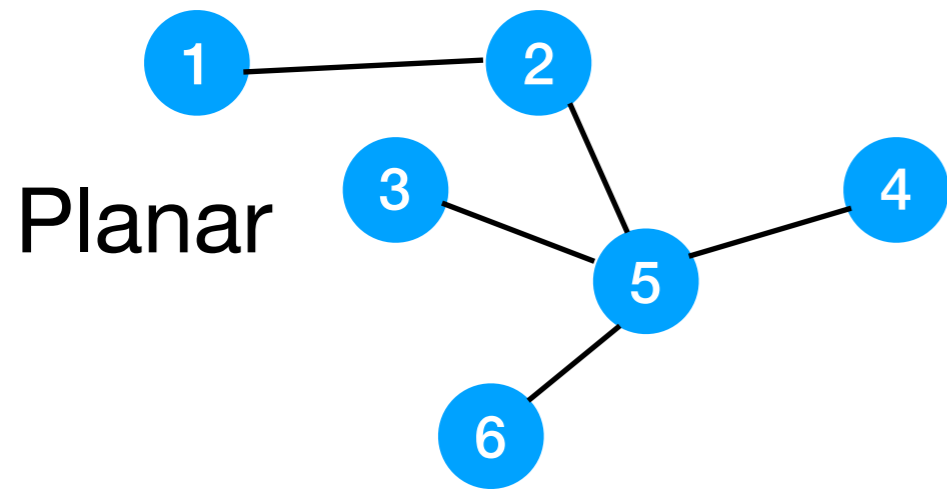
Planarity

- If a graph can be drawn without crossing edges, it is **planar**.



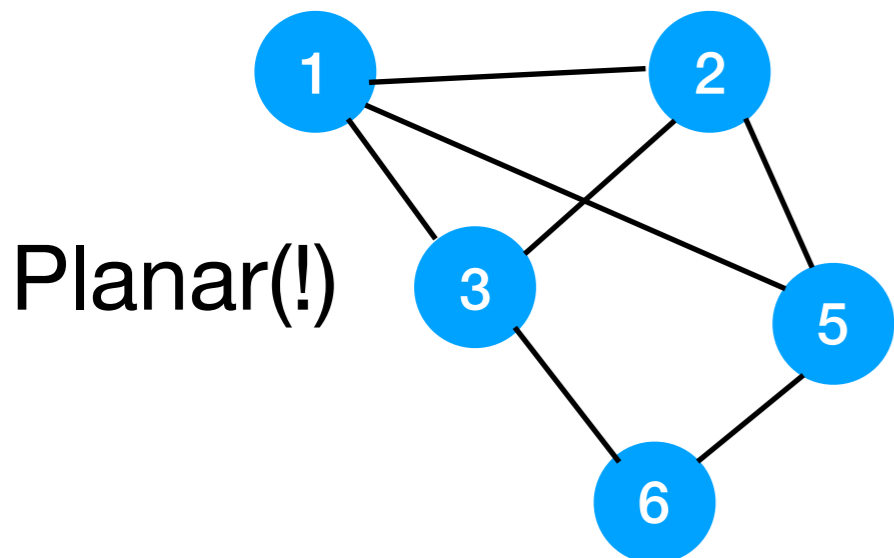
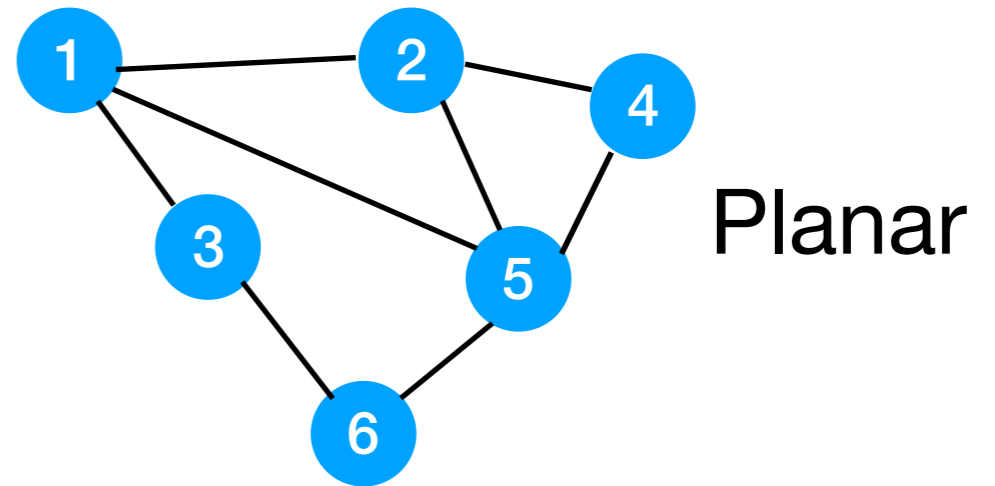
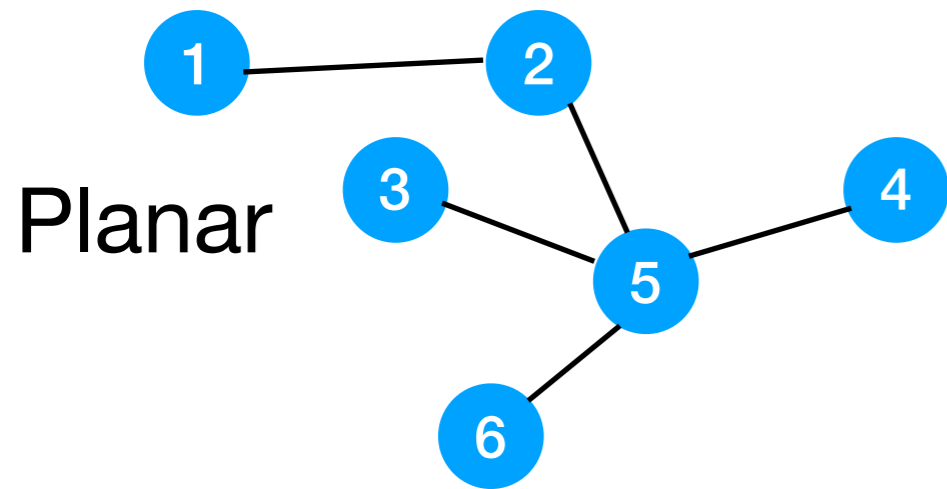
Planarity

- If a graph can be drawn without crossing edges, it is **planar**.



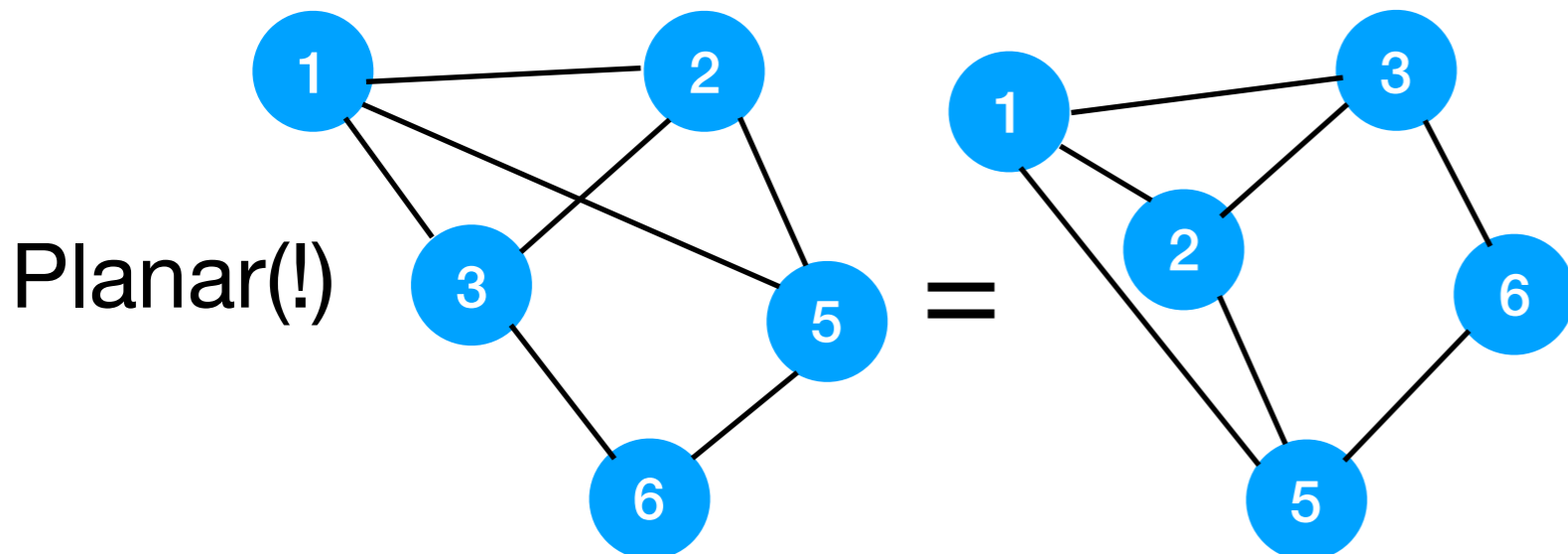
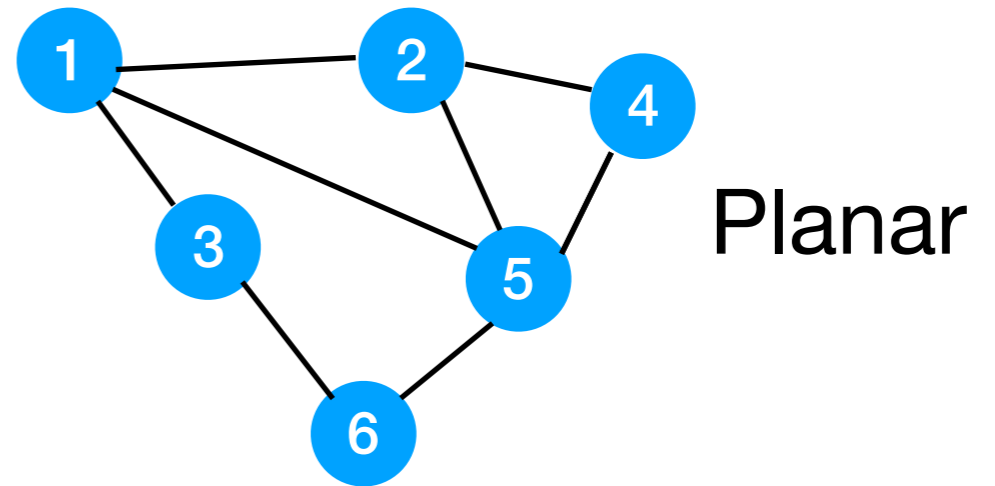
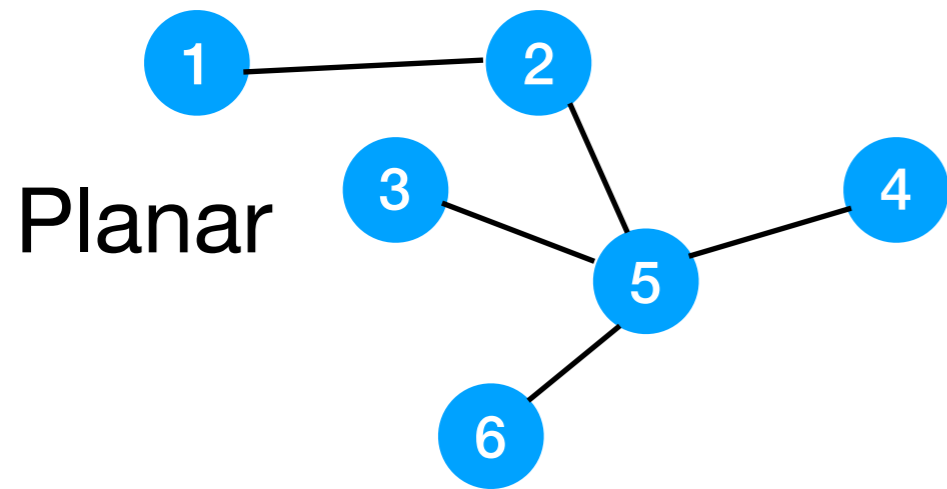
Planarity

- If a graph can be drawn without crossing edges, it is **planar**.



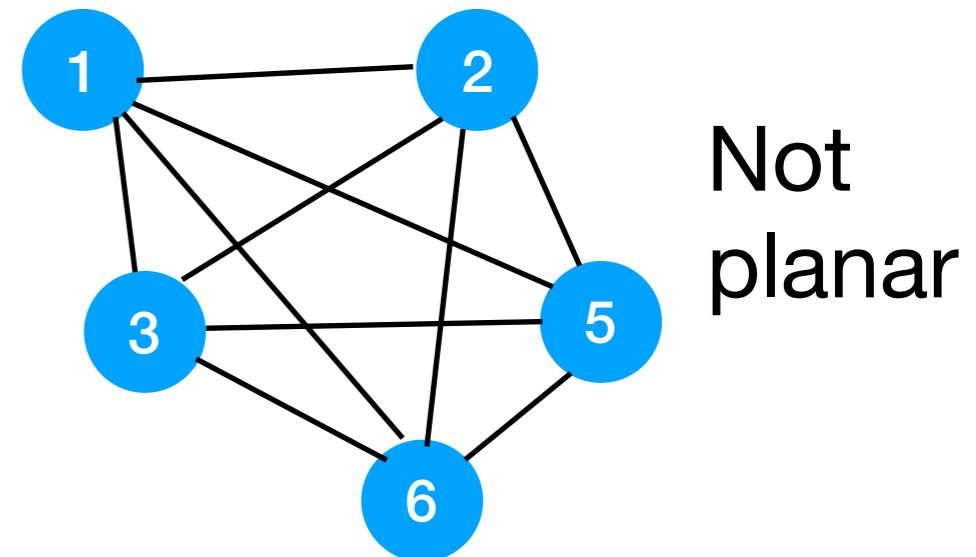
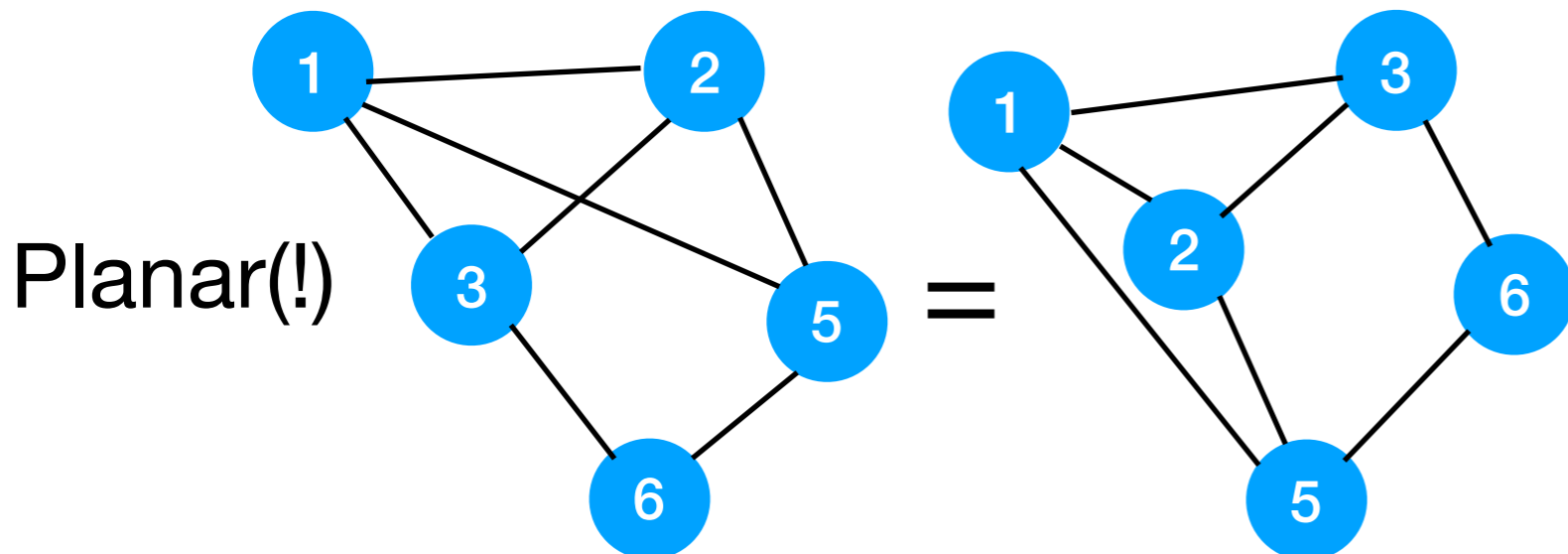
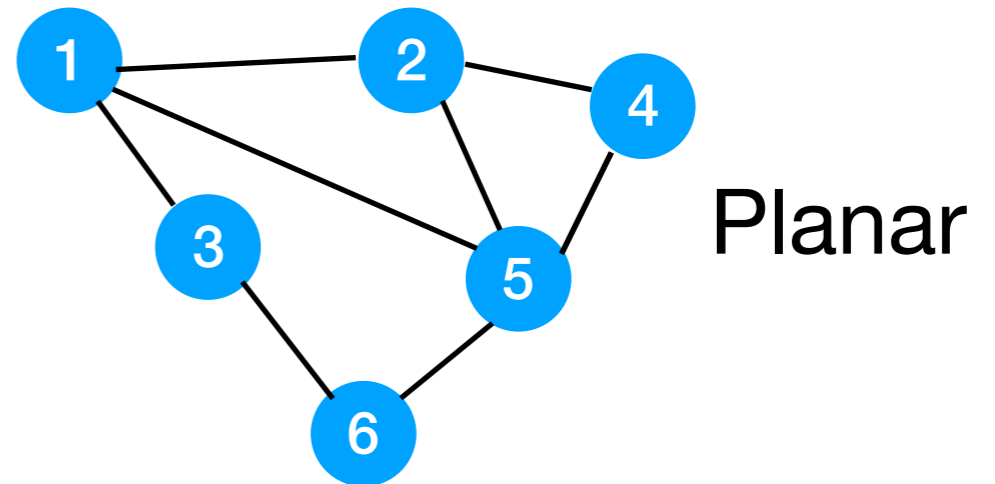
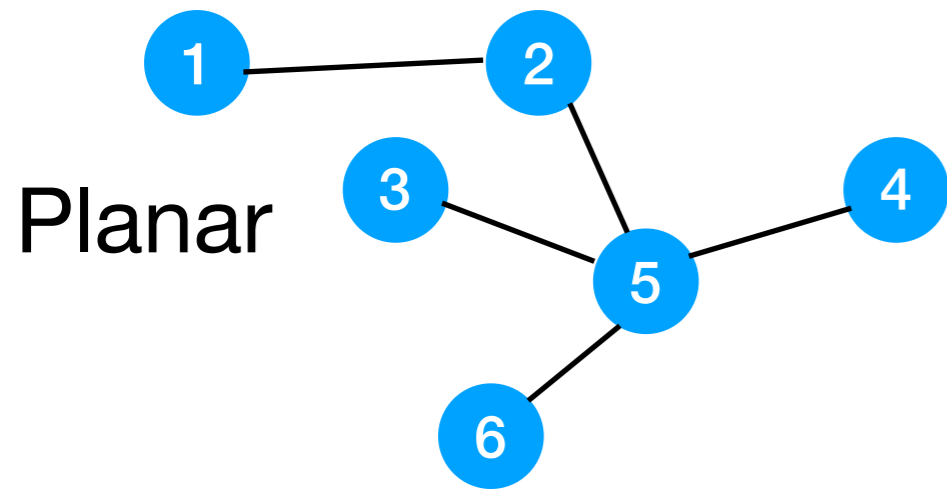
Planarity

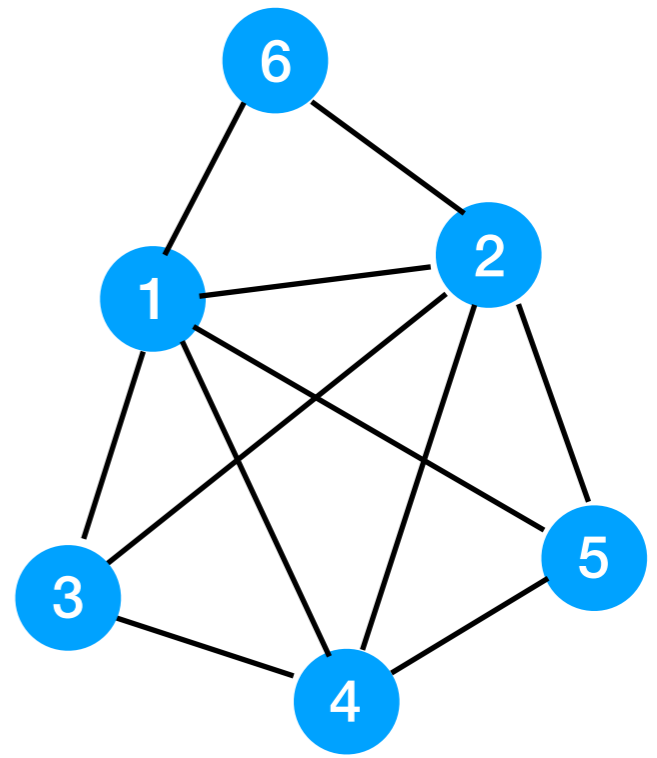
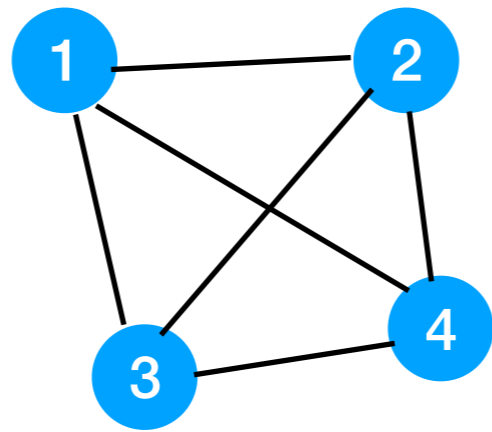
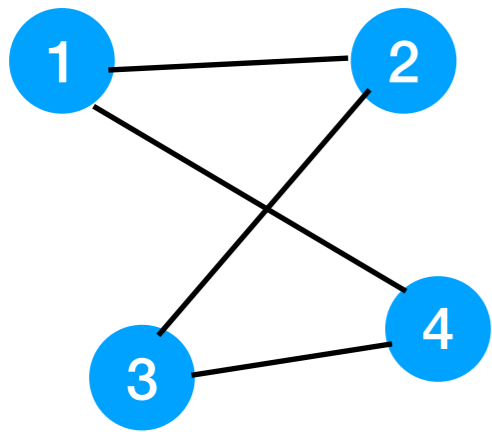
- If a graph can be drawn without crossing edges, it is **planar**.



Planarity

- If a graph can be drawn without crossing edges, it is **planar**.

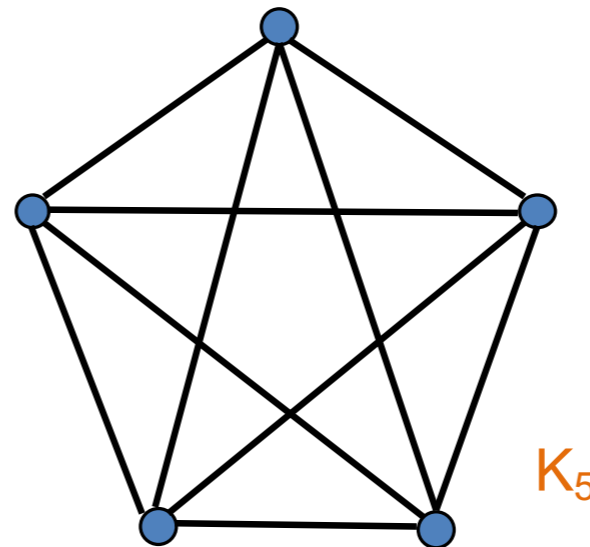




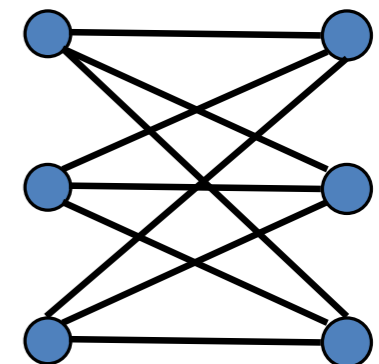
Detecting Planarity

A **subgraph** of a graph is a graph whose vertex and edge sets are subsets of the larger graph's.

- Elements of the edge subset can only contain nodes in the vertex subset.
- There's a (non-obvious) theorem that says a graph is **planar** if and only if it does not contain* one of these as a **subgraph**:



K_5

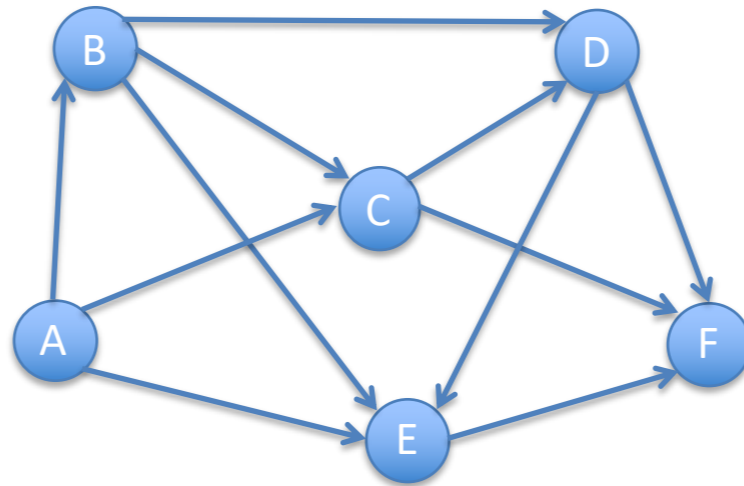


$K_{3,3}$

*The definition of “contain” is slightly more general than having one of these directly as a subgraph.

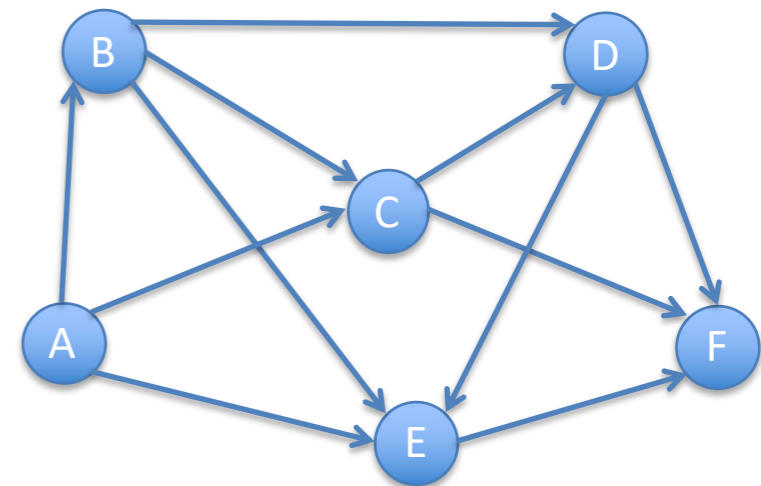
Detecting DAGs

- A **DAG**, or **Directed Acyclic Graph** is a...
graph that is **directed** and **acyclic**.



Is this a DAG?

- How do we tell if a directed graph is acyclic?
 - If a node has indegree 0, it can't be part of a cycle.
 - Edges coming from that node also can't be part of a cycle.



Is this a DAG?

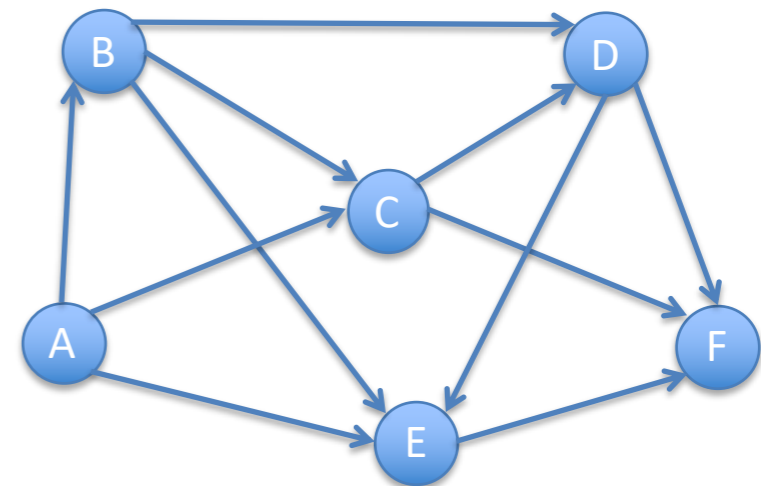
- How do we tell if a directed graph is acyclic?
 - If a node has indegree 0, it can't be part of a cycle.
 - Edges coming from that node also can't be part of a cycle.

Algorithm:

while there is a node with indegree 0:

delete the node and all edges coming from it

if the graph is empty, the original graph was a DAG



Is this a DAG?

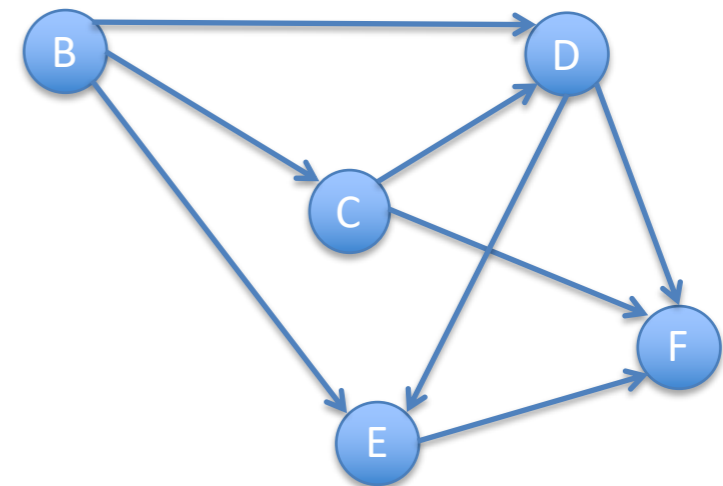
- How do we tell if a directed graph is acyclic?
 - If a node has indegree 0, it can't be part of a cycle.
 - Edges coming from that node also can't be part of a cycle.

Algorithm:

while there is a node with indegree 0:

delete the node and all edges coming from it

if the graph is empty, the original graph was a DAG



Is this a DAG?

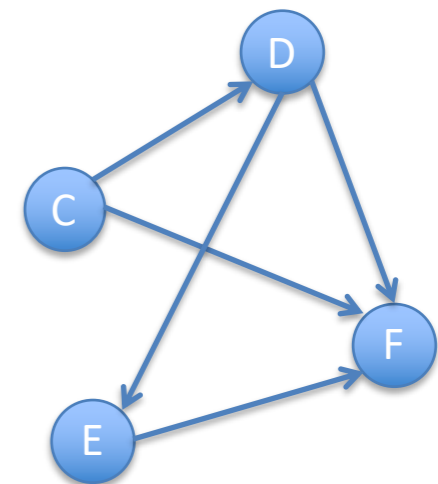
- How do we tell if a directed graph is acyclic?
 - If a node has indegree 0, it can't be part of a cycle.
 - Edges coming from that node also can't be part of a cycle.

Algorithm:

while there is a node with indegree 0:

delete the node and all edges coming from it

if the graph is empty, the original graph was a DAG



Is this a DAG?

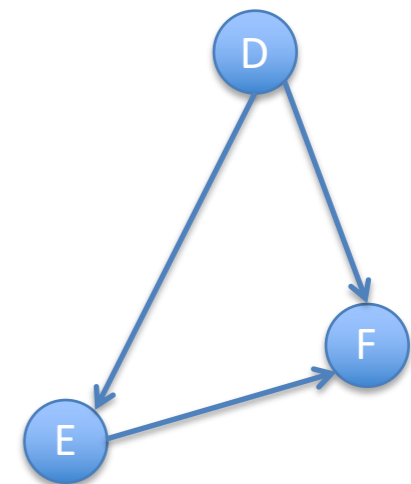
- How do we tell if a directed graph is acyclic?
 - If a node has indegree 0, it can't be part of a cycle.
 - Edges coming from that node also can't be part of a cycle.

Algorithm:

while there is a node with indegree 0:

delete the node and all edges coming from it

if the graph is empty, the original graph was a DAG



Is this a DAG?

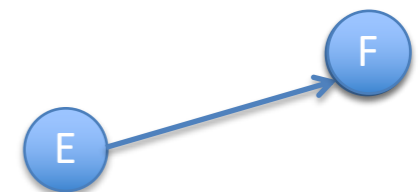
- How do we tell if a directed graph is acyclic?
 - If a node has indegree 0, it can't be part of a cycle.
 - Edges coming from that node also can't be part of a cycle.

Algorithm:

while there is a node with indegree 0:

delete the node and all edges coming from it

if the graph is empty, the original graph was a DAG



Is this a DAG?

- How do we tell if a directed graph is acyclic?
 - If a node has indegree 0, it can't be part of a cycle.
 - Edges coming from that node also can't be part of a cycle.

Algorithm:

while there is a node with indegree 0:

 delete the node and all edges coming from it

if the graph is empty, the original graph was a DAG



Topological Sort

Topological sort (or toposort):

$i = 0$

while there is a node with indegree 0:

 delete* the node and all edges coming from it

 label* the deleted node i

 increment i

if the graph is empty, the original graph was a DAG

Topological Sort

Topological sort (or toposort):

$i = 0$

while there is a node with indegree 0:

delete* the node and all edges coming from it

label* the deleted node i

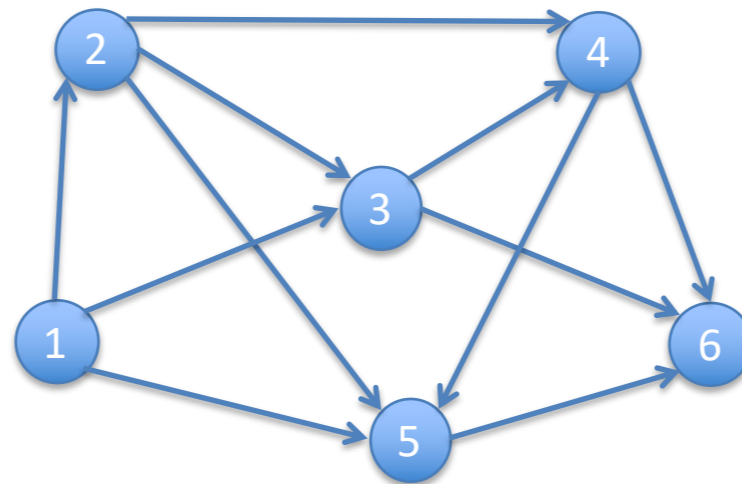
increment i

if the graph is empty, the original graph was a DAG

***This is pseudocode: we probably don't want to actually modify the graph. We'll need to store extra data with nodes and edges, and possibly overlay additional data structures to make it efficient.**

Topological Sort

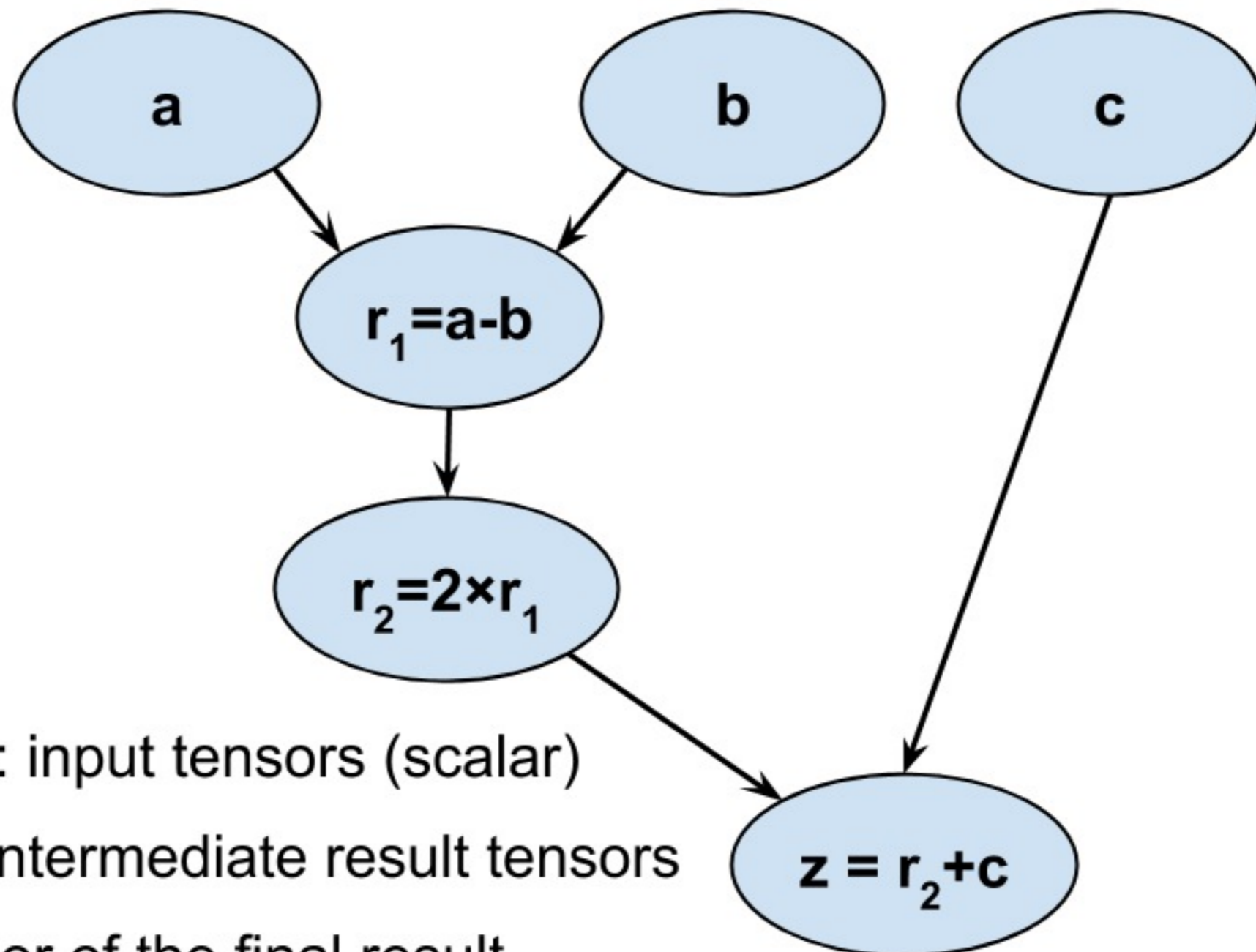
- Here are the labels we applied to the example graph:



- Property: all edges go from a lower-numbered node to a higher-numbered node.
- Useful for dependency resolution, job scheduling,
- Ordering is not necessarily unique: could have chosen from among multiple nodes with indegree 0.

Tensorflow Computation Graphs

Computation graph implementing
the equation $z = 2 \times (a - b) + c$

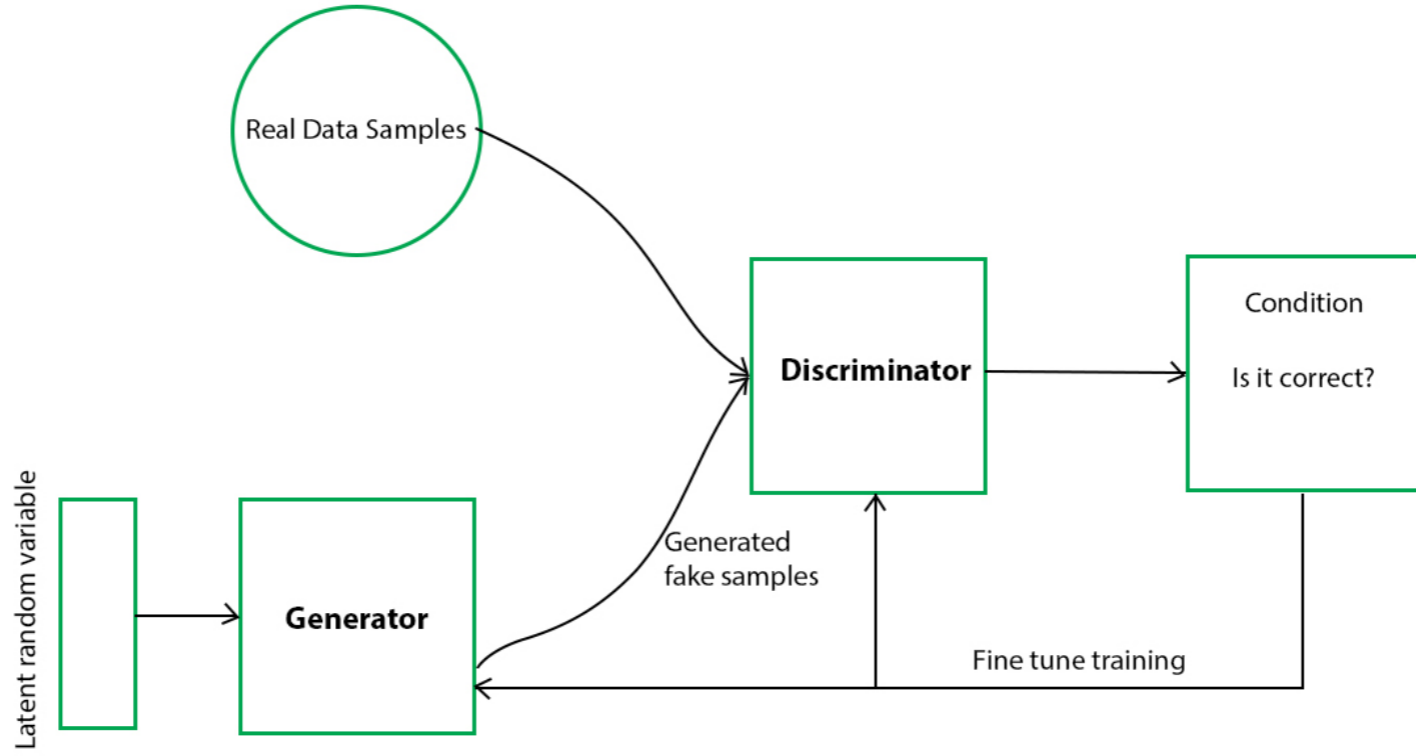
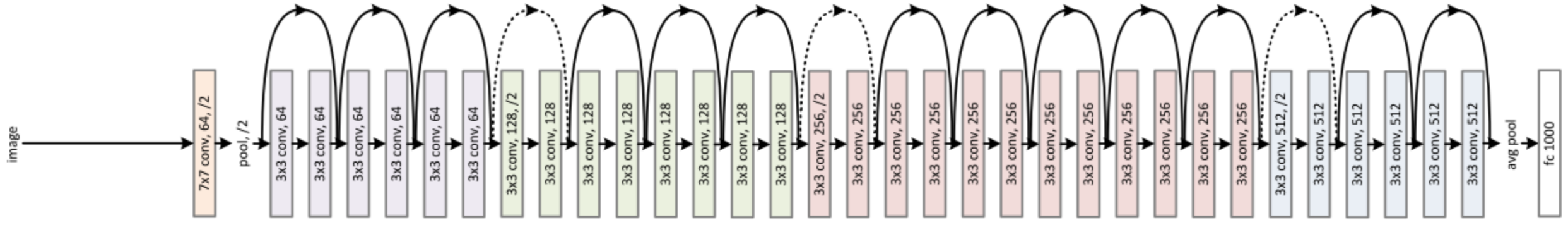


a, b, c: input tensors (scalar)

r₁, r₂: intermediate result tensors

z: tensor of the final result

34-layer residual



Word Problems!