Graph Traversals

CSCI 241 Spring 2020

Lecture 20

Announcements

- Quiz today – same as usual

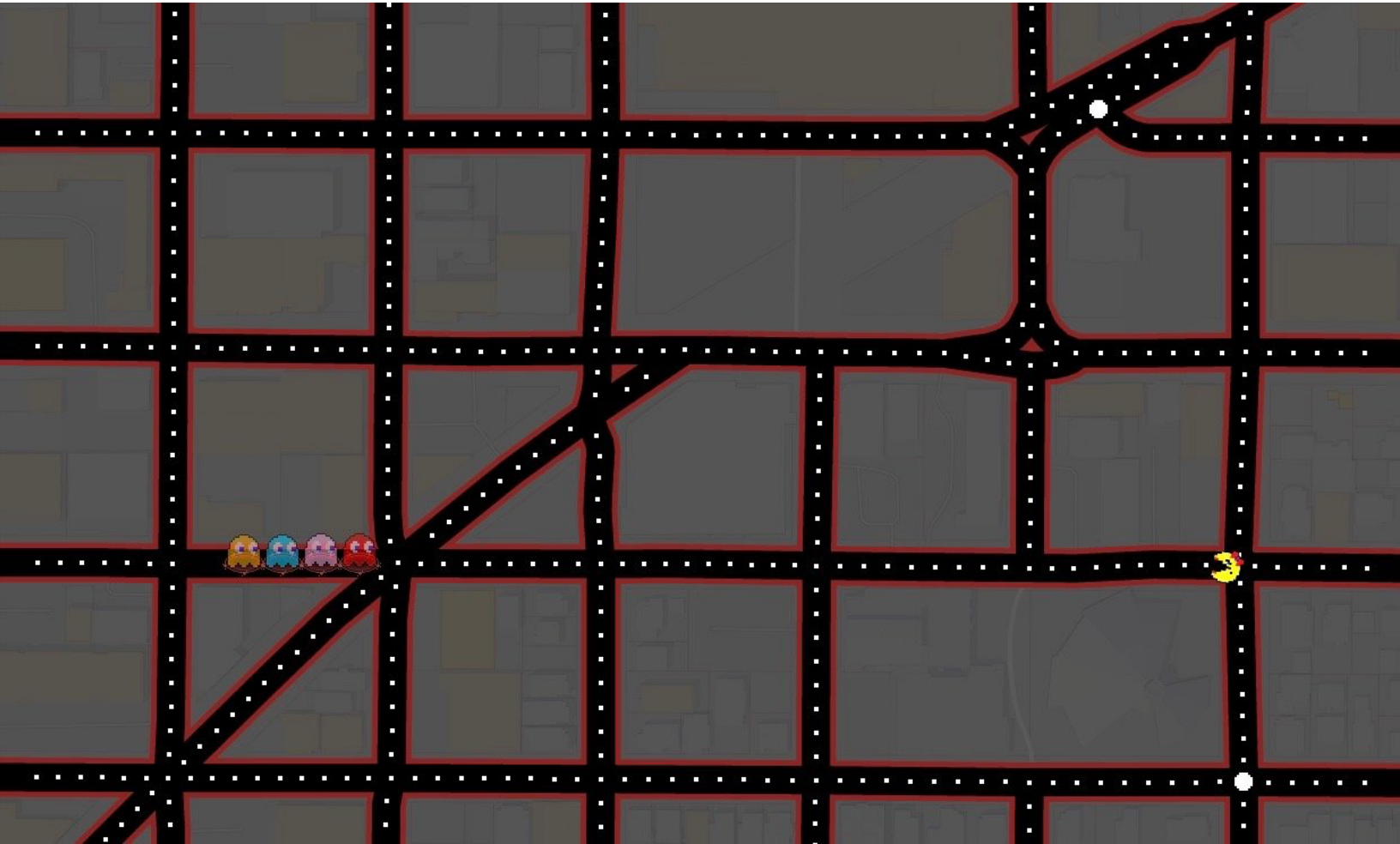
- A4: Dijkstra's Single Source Shortest Paths
  - We'll cover Dijkstra's algorithm Wednesday
  - Assignment released Wednesday
  - Due a week later, Wednesday 6/3

- Basic graph operation runtime - see end of Wednesday's slides.

Goals

- Understand and be able to implement graph traversal/search algorithms:
  - Depth-first search
  - Breadth-first search

Look, a graph!



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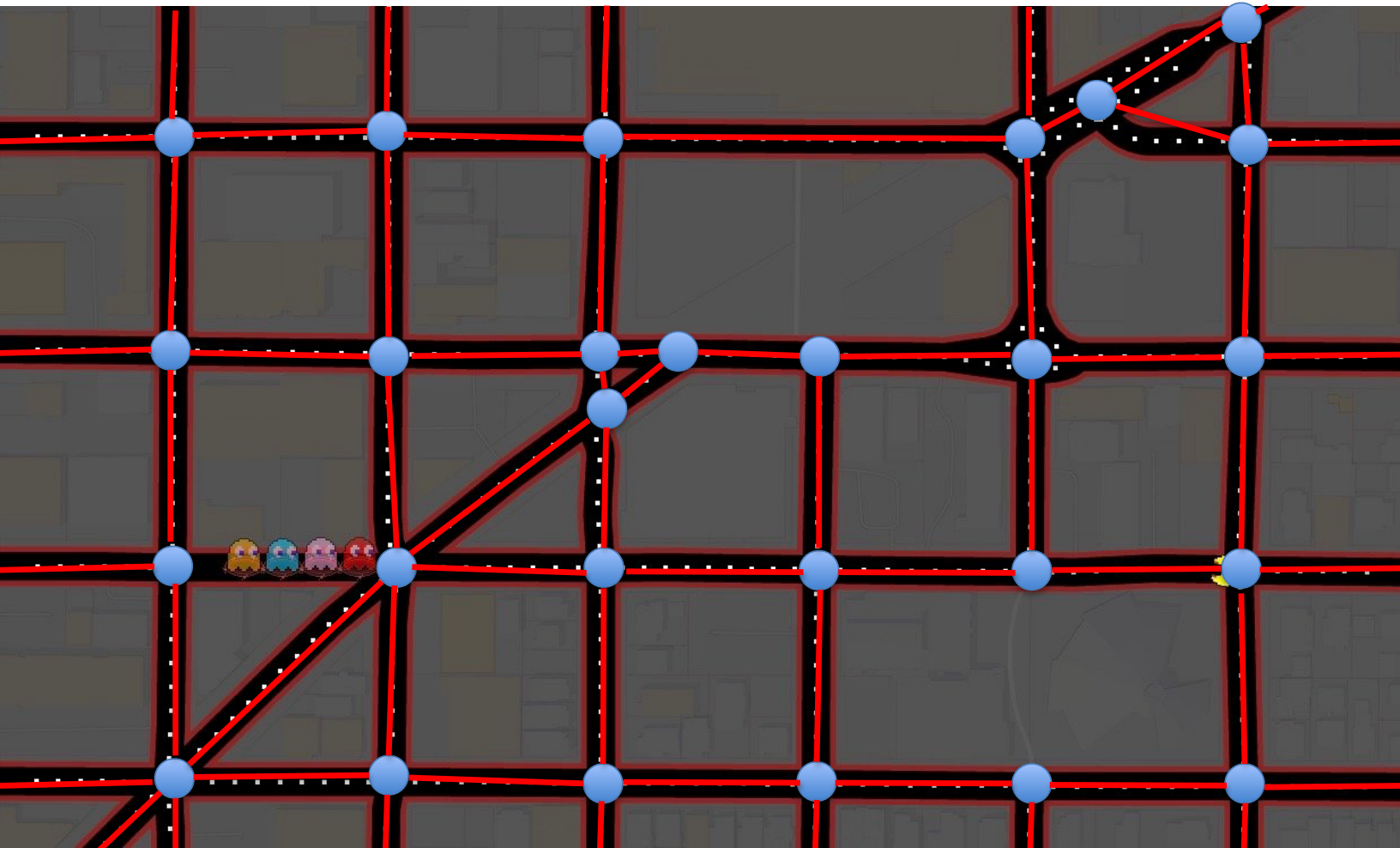

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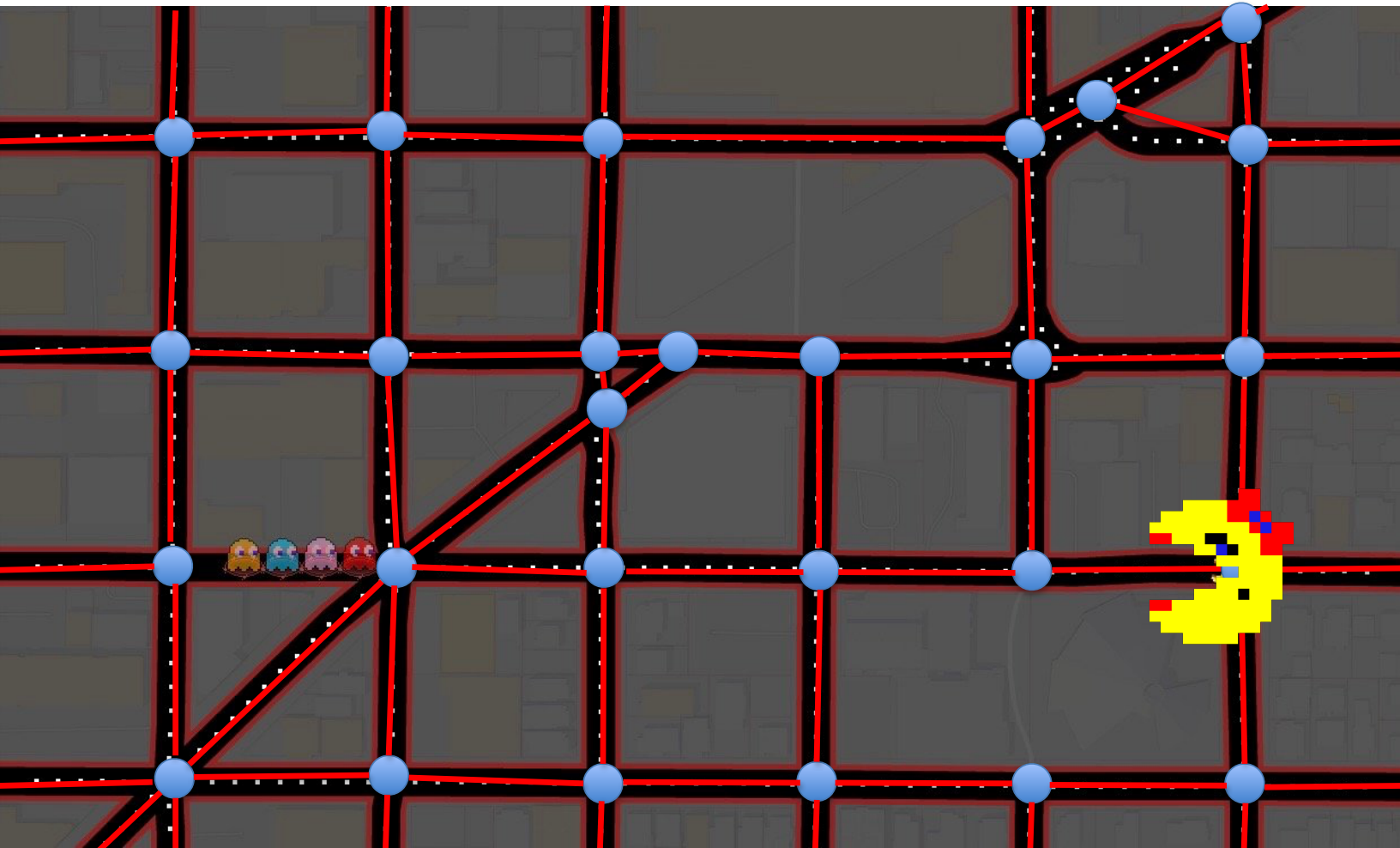

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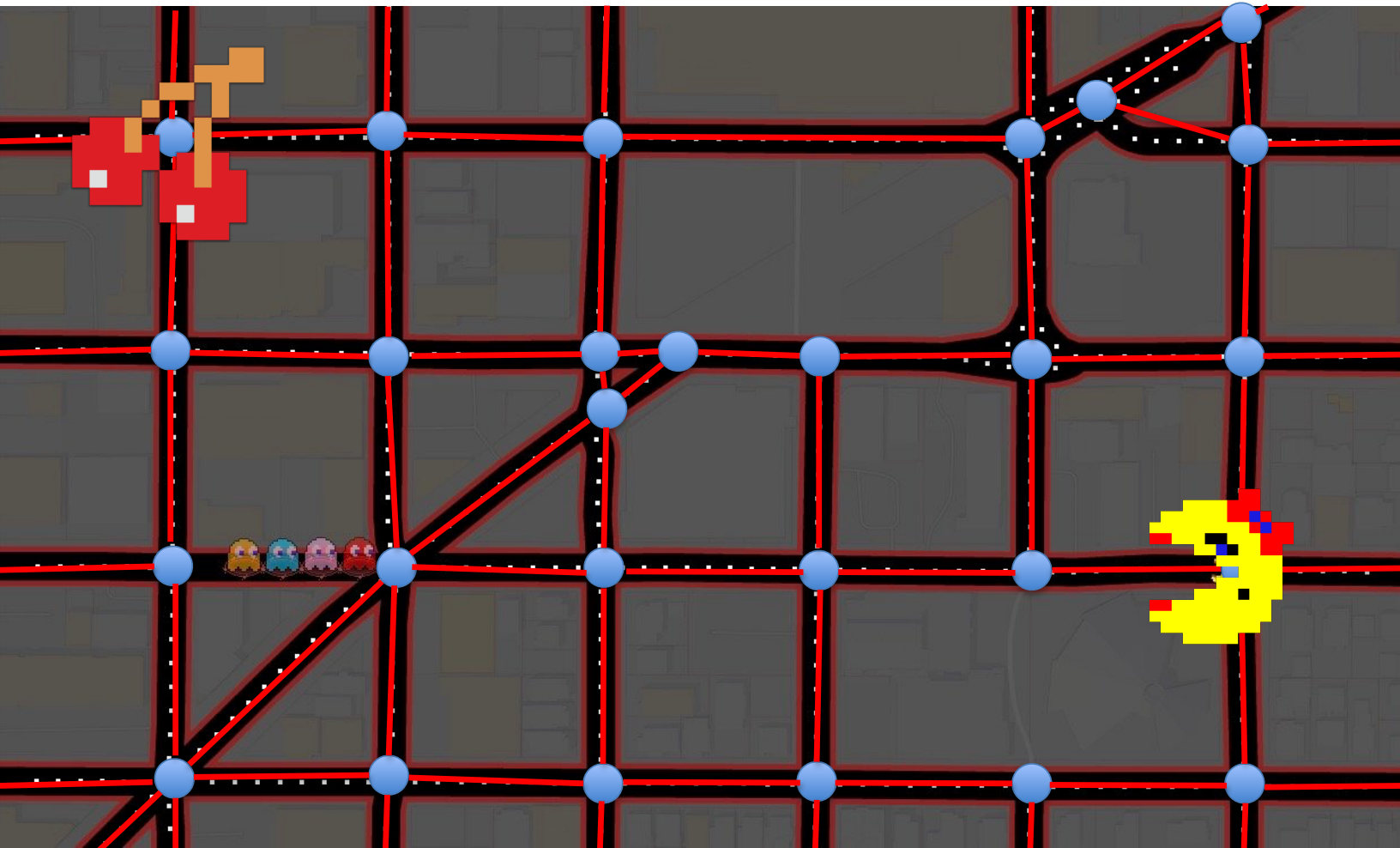


DAG

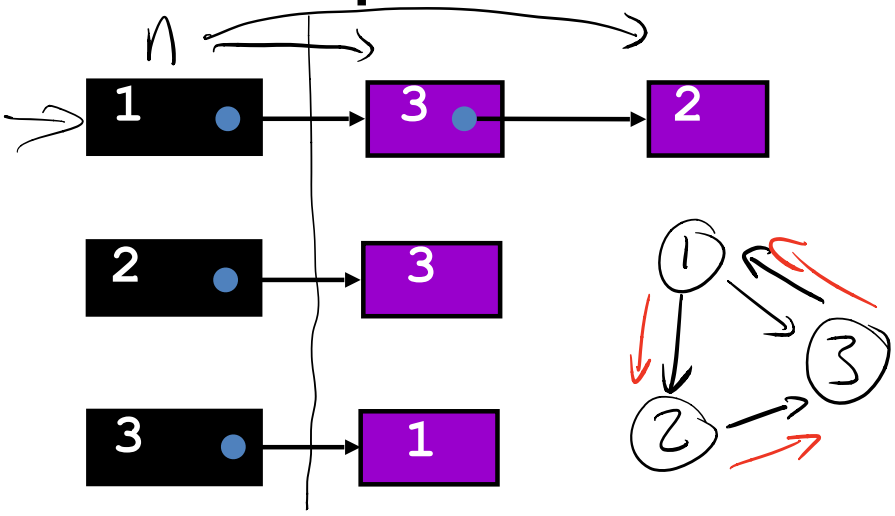
- A commonly-used flavor of graph: Directed Acyclic Graph (DAG).
- Definition: A graph that is directed and acyclic.

Breaking DAG

Which of the following two graphs are DAGs?

Directed Acyclic Graph

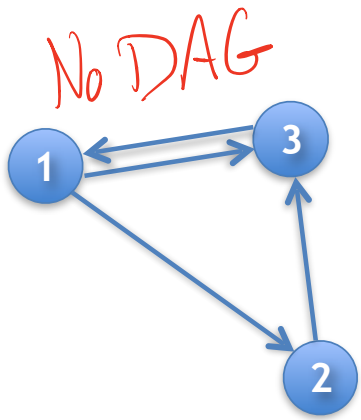
Graph 1:

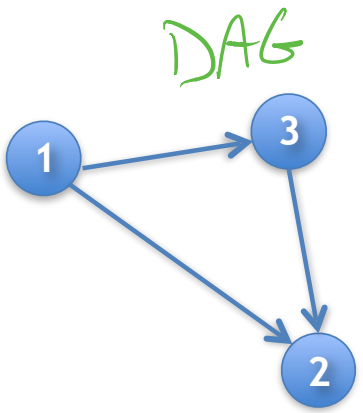


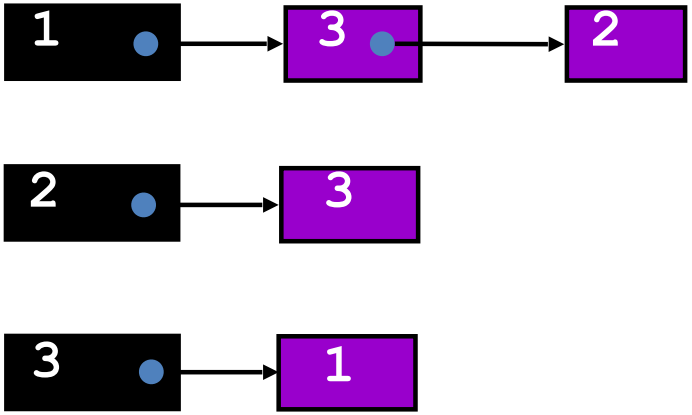
Graph 2:

	1	2	3
1	0	1	1
2	0	0	0
3	0	1	0

Breaking DAG







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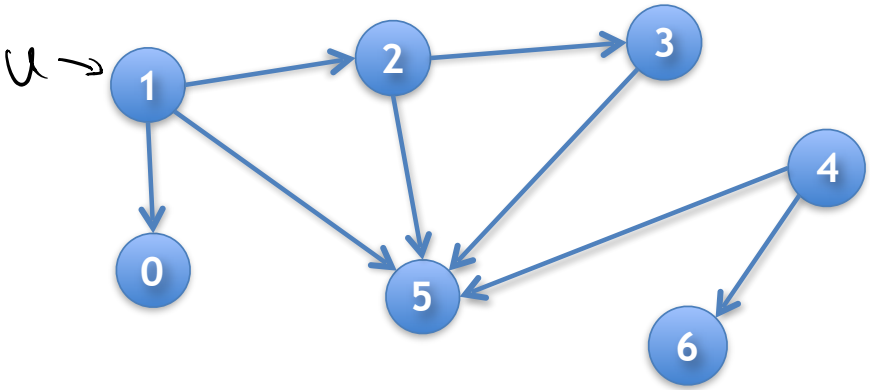
Depth-First Search

- Given a graph and one of its nodes u
  (say node 1 below)



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- We want to "visit" each node reachable from u

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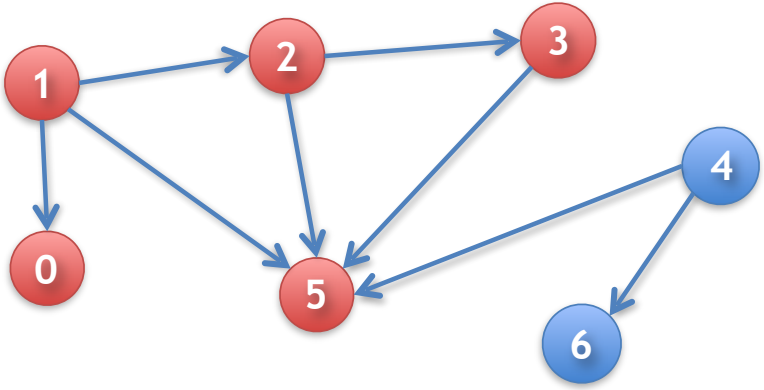

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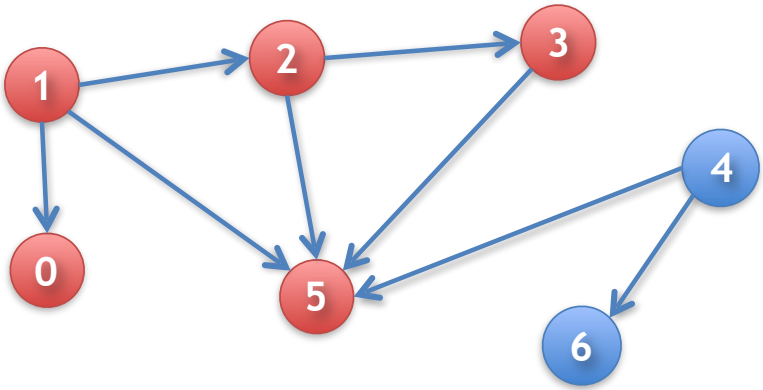
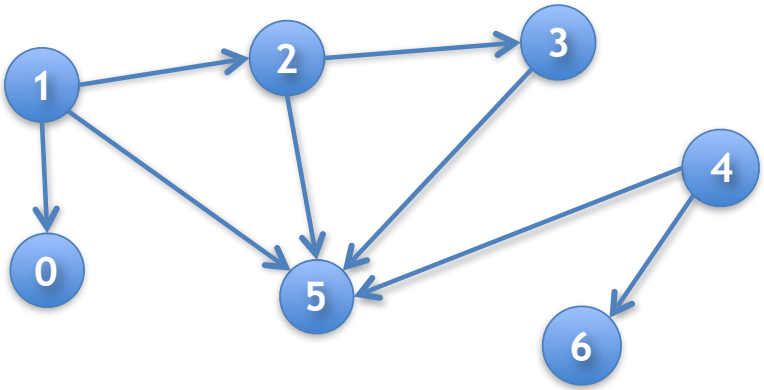


There are many paths to some nodes.

How do we visit all nodes efficiently, without doing extra work?

Depth-First Search

boolean[ ] visited;



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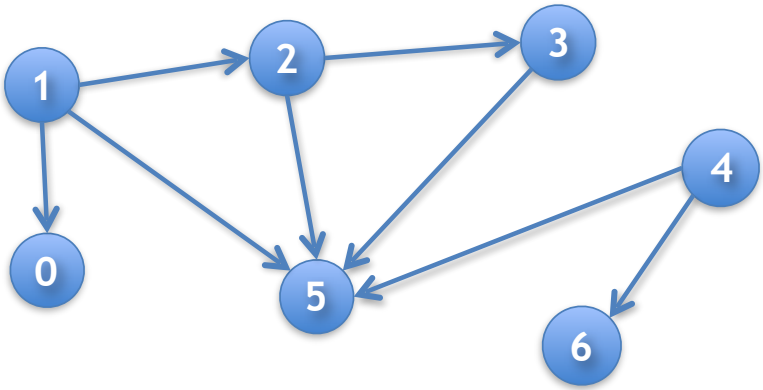

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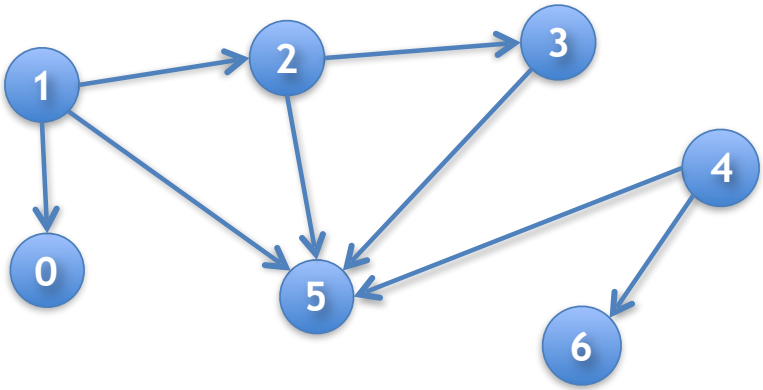

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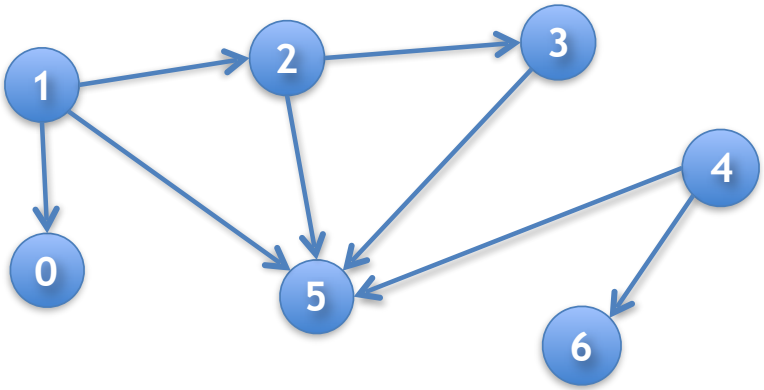

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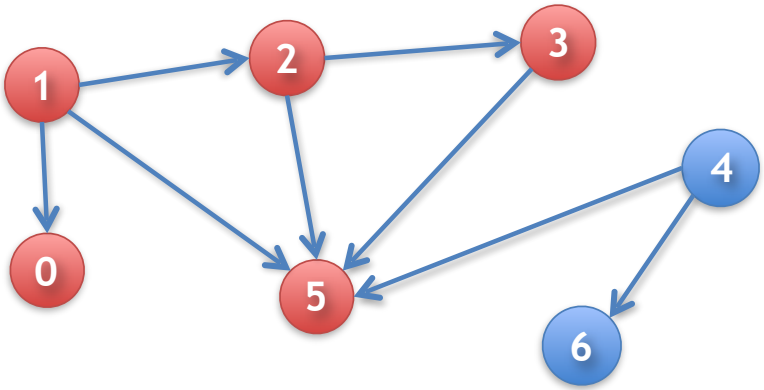

Suppose all nodes are unvisited.

Nodes **explorable** from node **1**:
{1, 0, 2, 3, 5}

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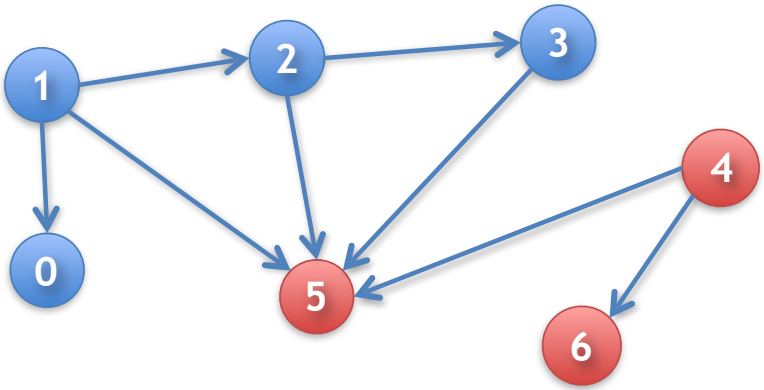

Suppose all nodes are unvisited.

Nodes **explorable** from node **1**: {1, 0, 2, 3, 5}

Nodes **explorable** from **4**: {4, 5, 6}

Depth-First Search

boolean[ ] visited;

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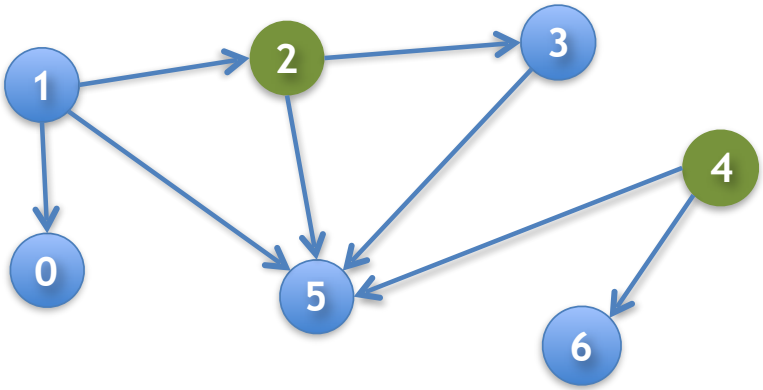

Green: visited
Blue: unvisited

Depth-First Search

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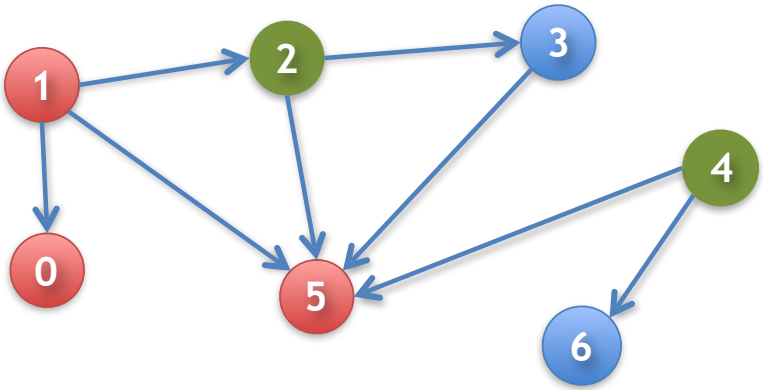

Green: visited
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Nodes **explorable** from node **1**:
{1, 0, 5}

Depth-First Search

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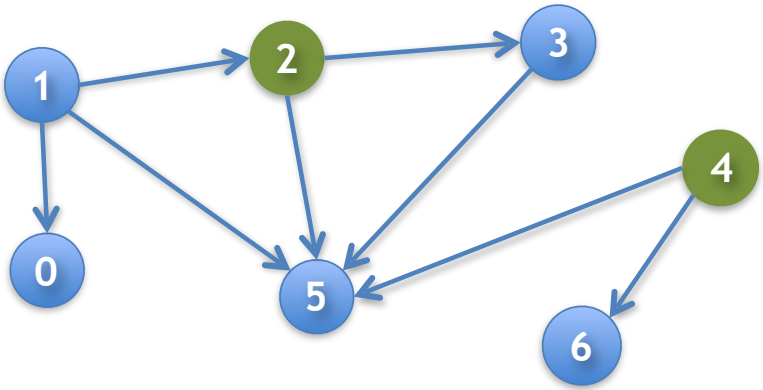

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Nodes **explorable** from node **1**:
{1, 0, 5}

Nodes **explorable** from **4**: none

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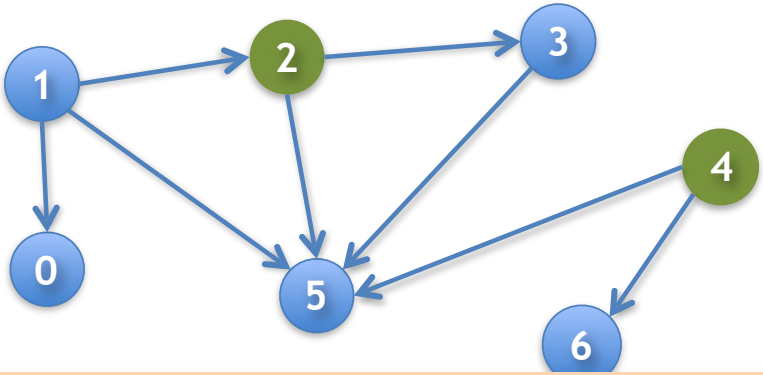

Green: visited
Blue: unvisited

Nodes **explorable** from node **1**:
{1, 0, 5}

Nodes **explorable** from **4**: none

Not even **4** itself, because it's already been visited

Depth-First Search

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explorable from u. Precondition: u is
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public static void dfs(int u) {



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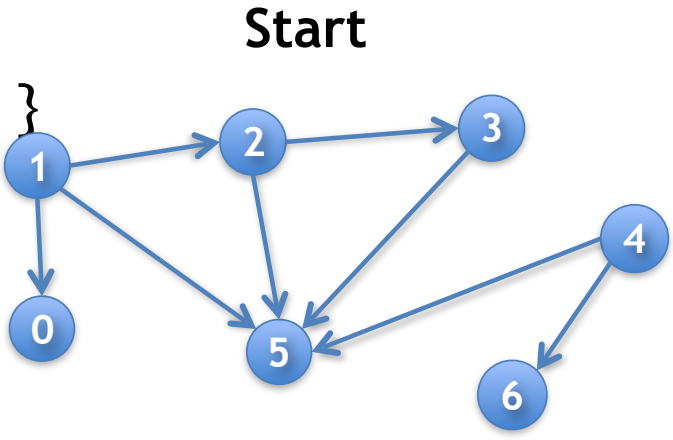

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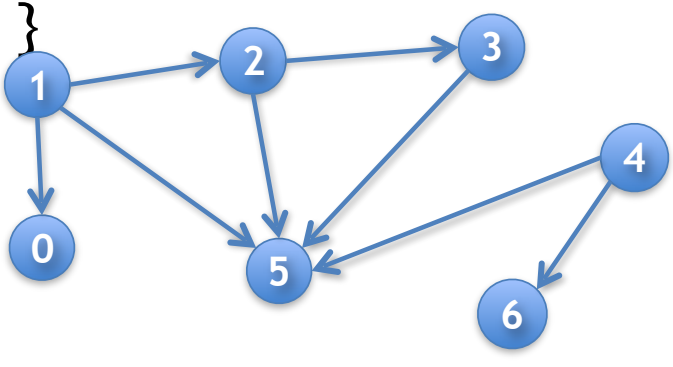
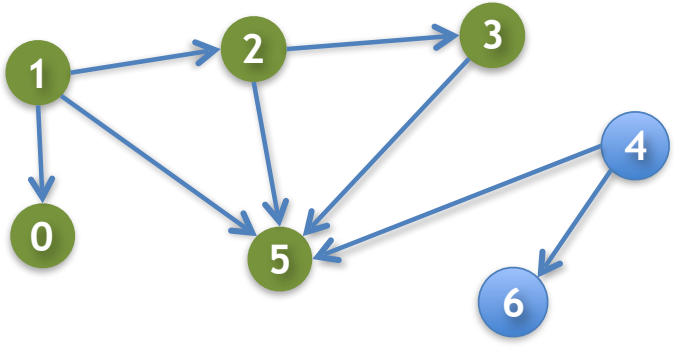
Let u be **1**

The nodes **explorable** from 1 are 1, 0, 2, 3, 5

Start



End



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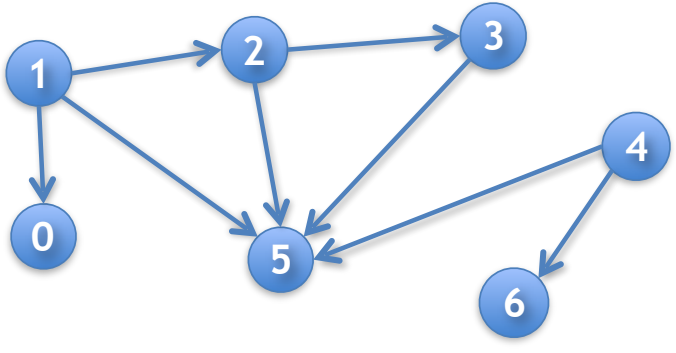

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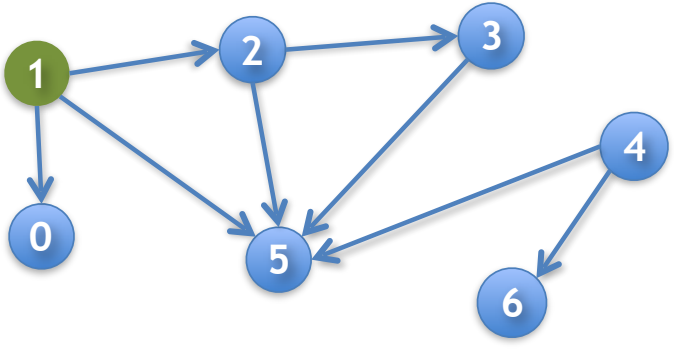

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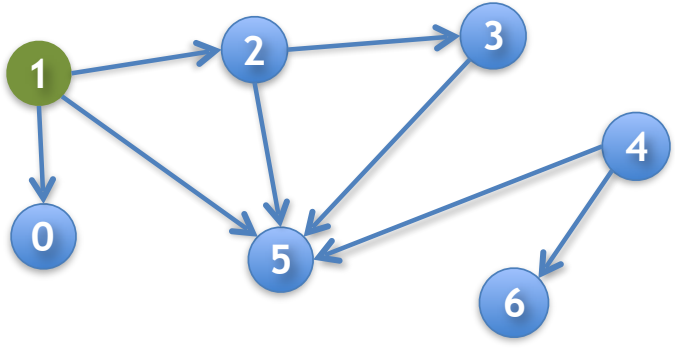

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The nodes to be visited are 0, 2, 3, 5

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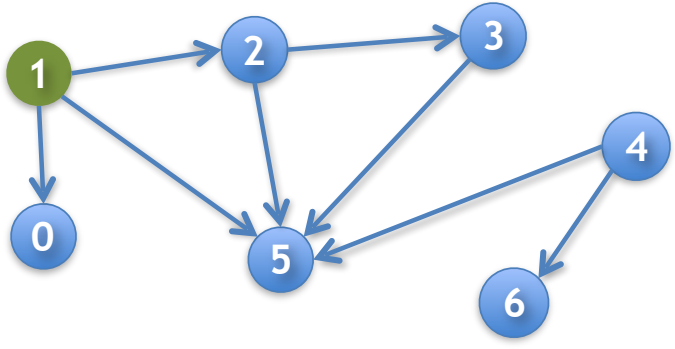

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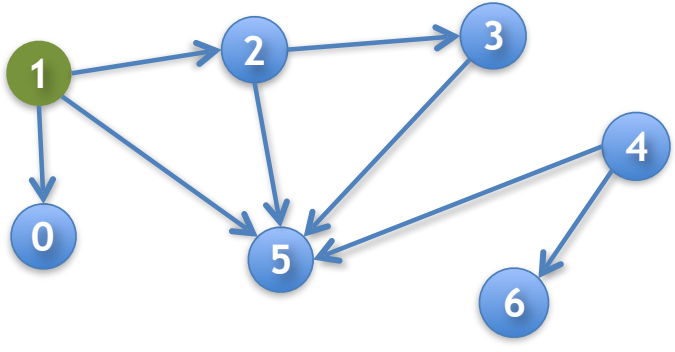

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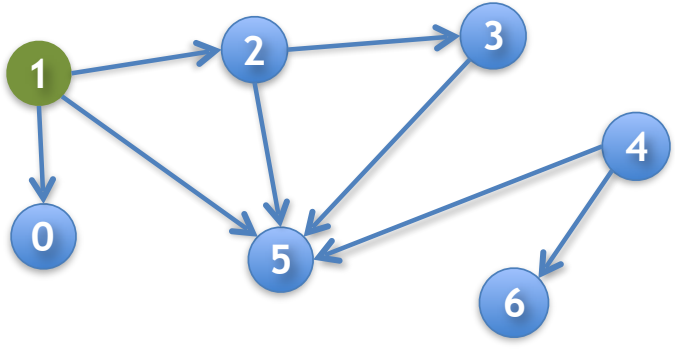

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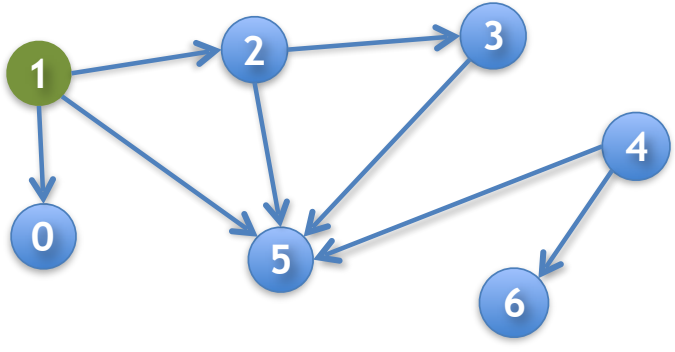

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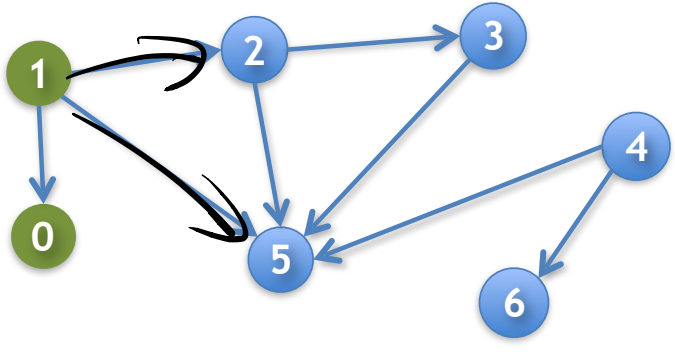

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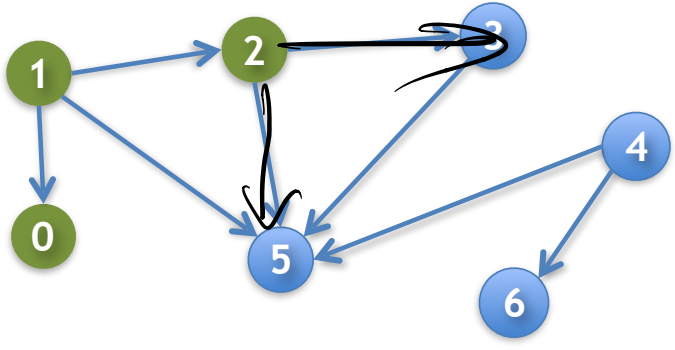

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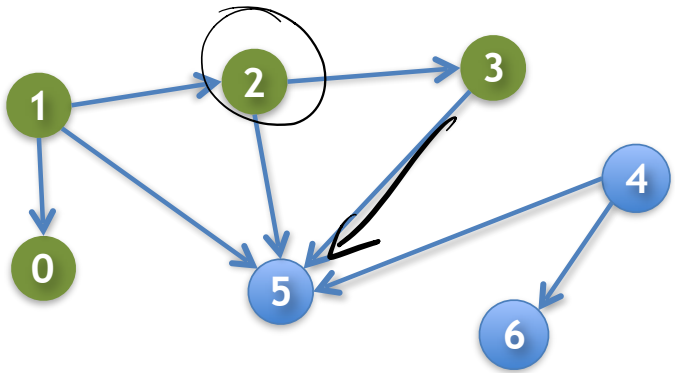

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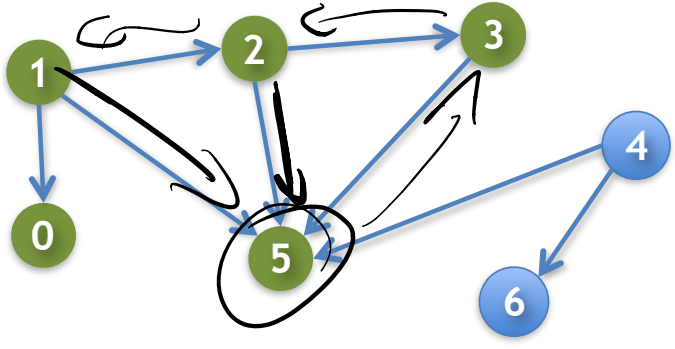

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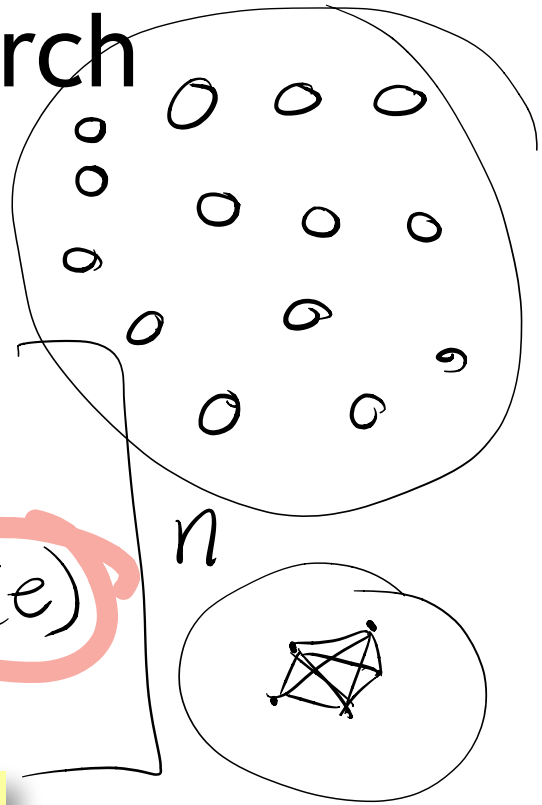

Suppose n nodes are explorable along e edges (in total). What is

- Worst-case runtime?
- Worst-case space?

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Suppose n nodes are explorable along e edges (in total). What is

- Worst-case runtime? O(n+e)
- Worst-case space? O(n)

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That's all there is to basic DFS. You may have to change it to fit a particular situation.

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That's all there is to basic DFS. You may have to change it to fit a particular situation.

Example: Use different way (other than array visited) to know whether a node has been visited

Depth-First Search

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That's all there is to basic DFS. You may have to change it to fit a particular situation.

Example: Use different way (other than array visited) to know whether a node has been visited

Example: We really haven't said what data structures are used to implement the graph

Depth-First Search

```
/** Visit all nodes that are
explorable from u. Precondition: u is
unvisited. */
public static void dfs(int u) {
    visited[u] = true;
    for all edges (u, v) leaving u:
        if v is unvisited then dfs(v);
}
```

That's all there is to basic DFS. You may have to change it to fit a particular situation.

If you don't have this spec and you do something different, it's probably wrong.

Example: Use different way (other than array visited) to know whether a node has been visited

Example: We really haven't said what data structures are used to implement the graph

Depth-First Search in OO fashion

```
public class Node {
    boolean visited;
    List<Node> neighbors;
```

Each node of the graph is an object of type Node

Depth-First Search in OO fashion

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public class Node {
    boolean visited;
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    /** Visit all nodes that are explorable
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        visited= true;
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Each node of the graph is an object of type Node

No need for a parameter. The object is the node.

Depth-First Search in OO fashion

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public class Node {
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    public void dfs() {
        visited= true;
        for (Node n: neighbors) {
            if (!n.visited) n.dfs();
        }
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Each node of the graph is an object of type Node

No need for a parameter. The object is the node.

Depth-First Search written iteratively

```
/** Visit all nodes explorable from u. Pre: u is unvisited. */
public static void dfs(int nodeID) {
    Stack s = (nodeID);     // Not Java!
    // inv: all nodes that have to be visited are
    //         explorable from some node in s
    while (                              ) {
```

Depth-First Search written iteratively

```
/** Visit all nodes explorable from u. Pre: u is unvisited. */
public static void dfs(int nodeID) {
    Stack s = (nodeID);     // Not Java!
    // inv: all nodes that have to be visited are
    //         explorable from some node in s
    while (s is not empty      ) {
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Depth-First Search written iteratively

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public static void dfs(int nodeID) {
    Stack s = (nodeID);     // Not Java!
    // inv: all nodes that have to be visited are
    //         explorable from some node in s
    while (s is not empty      ) {
      u = s.pop();    // Remove top stack node, put in u
      if (u has not been visited) {
```

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```
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    Stack s = (nodeID);     // Not Java!
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            visit u;
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            for each edge (u, v) leaving u:
                s.push(v);
        }
    }
}
```
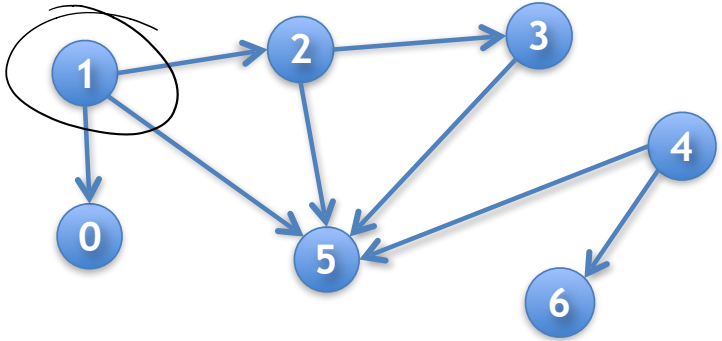

Call dfs(1)



Stack s

Depth-First Search written iteratively

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public static void dfs(int u) {
    Stack s= (u);
    while (s is not empty) {
        u= s.pop();
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            visit u;
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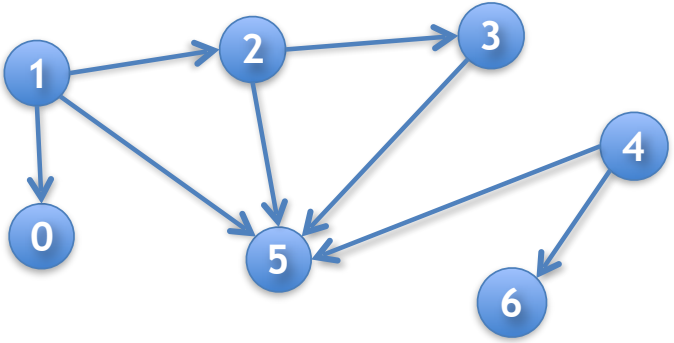

1

Stack s

Depth-First Search written iteratively

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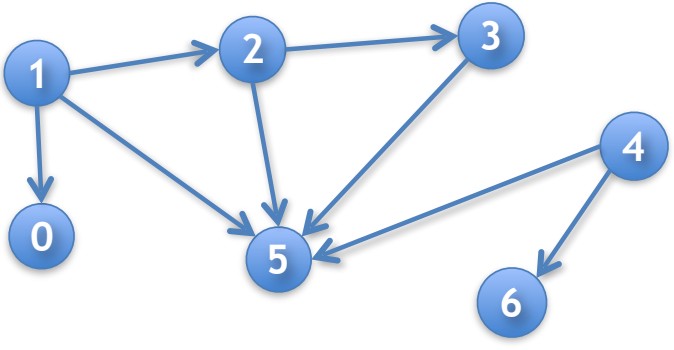
public static void dfs(int u) {
    Stack s= (u);
    while (s is not empty) {
        u= s.pop();
        if (u has not been visited) {
            visit u;
            for each edge (u, v) leaving u:
                s.push(v);
        }
    }
}
```

Call dfs(1)

Iteration 0



1

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        }
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}
```
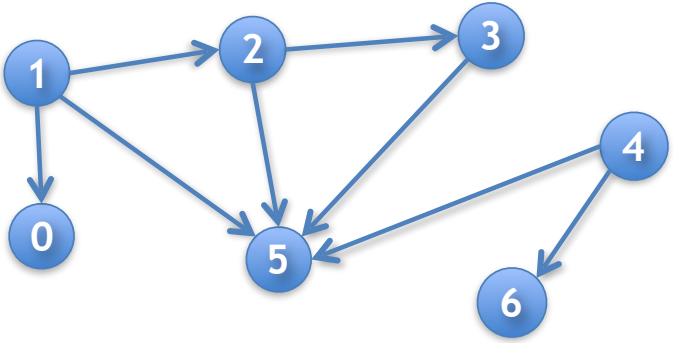

Call dfs(1)

Iteration 0



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Call dfs(1)

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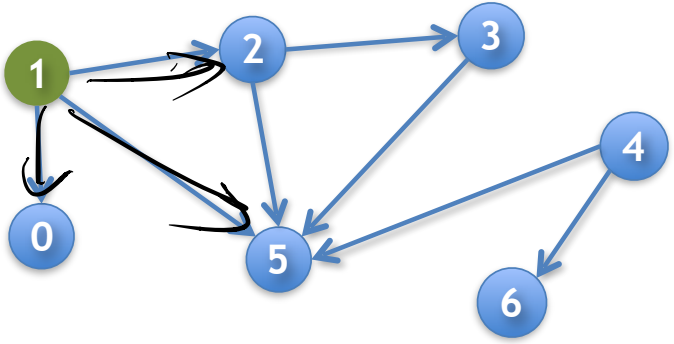

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Depth-First Search written iteratively

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            for each edge (u, v) leaving u:
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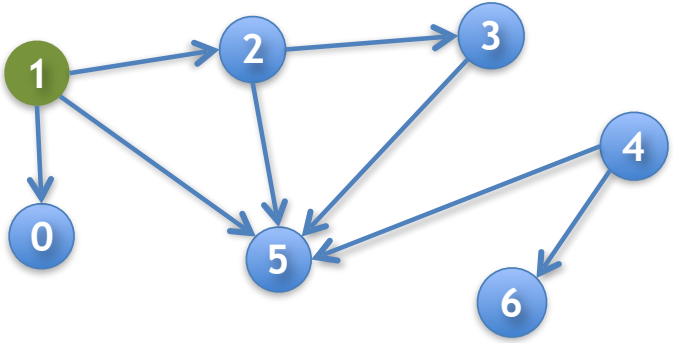
        }
    }
}
```

Call dfs(1)

Iteration 0



0
2
5

Stack s

Depth-First Search written iteratively

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public static void dfs(int u) {
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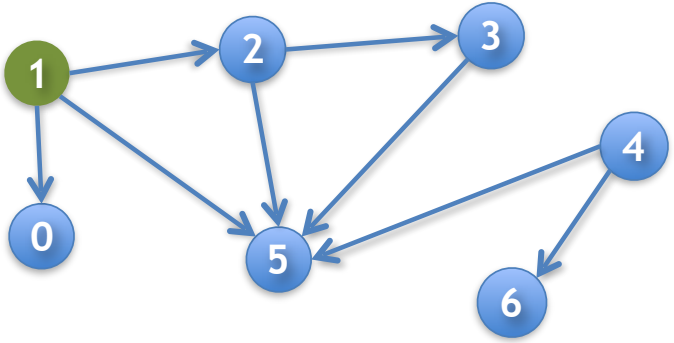
Call dfs(1)

Iteration 1



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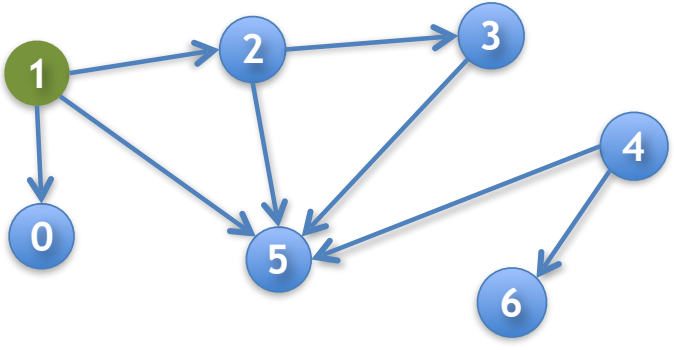
Call dfs(1)

Iteration 1



2
5

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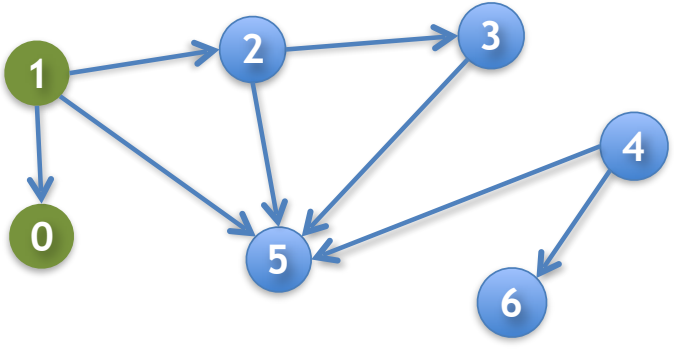

2
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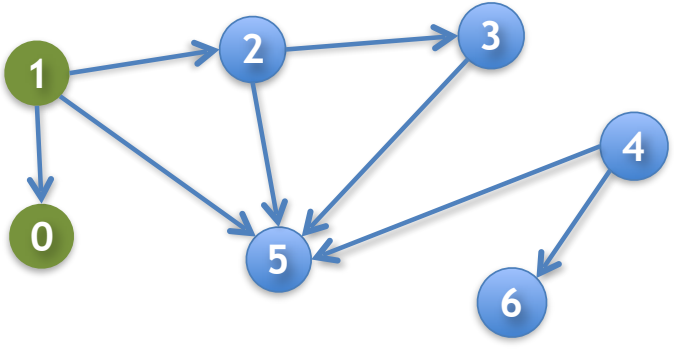
Call dfs(1)

Iteration 2



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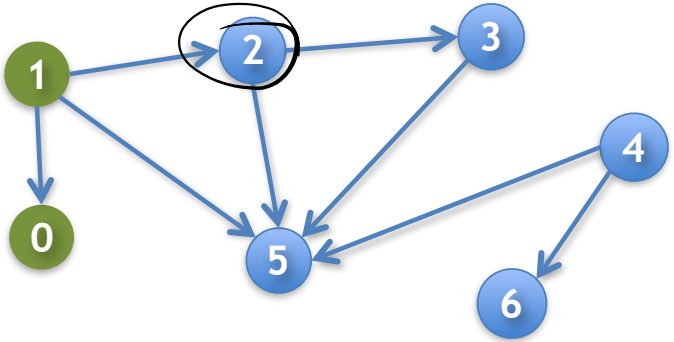
Iteration 2



5

Stack s

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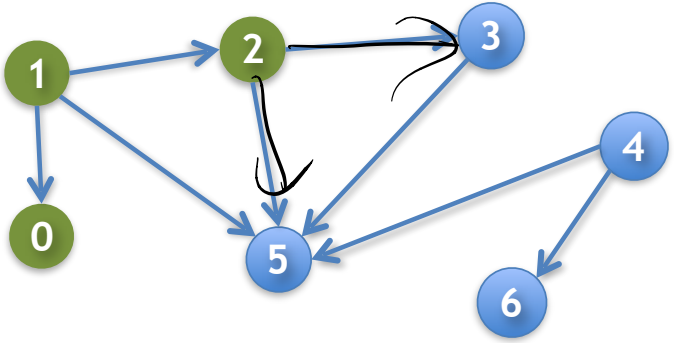
Call dfs(1)

Iteration 2



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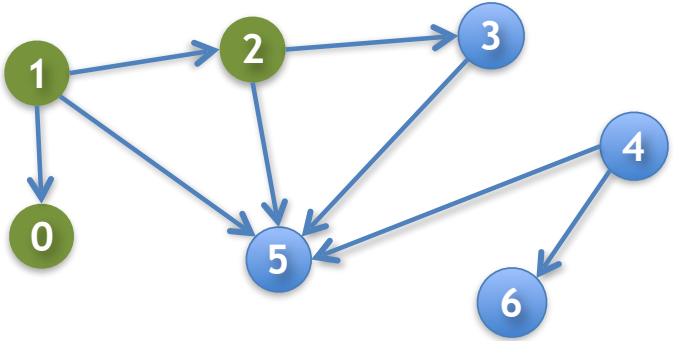
Iteration 2



3
5
5

Stack s

Depth-First Search written iteratively

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public static void dfs(int u) {
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    while (s is not empty) {
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            visit u;
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    }
}
```
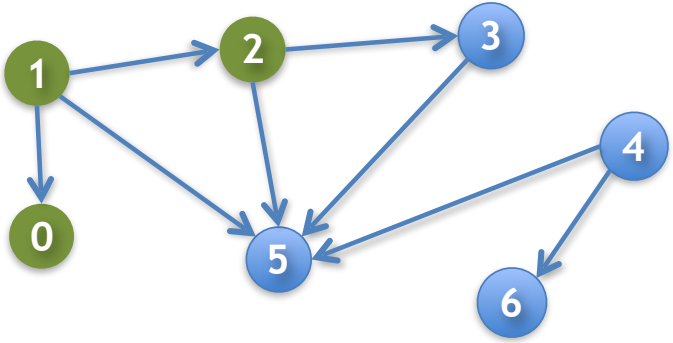

Call dfs(1)

Iteration 2

Yes, 5 is put on the stack twice, once for each edge to it. It will be visited only once.



3
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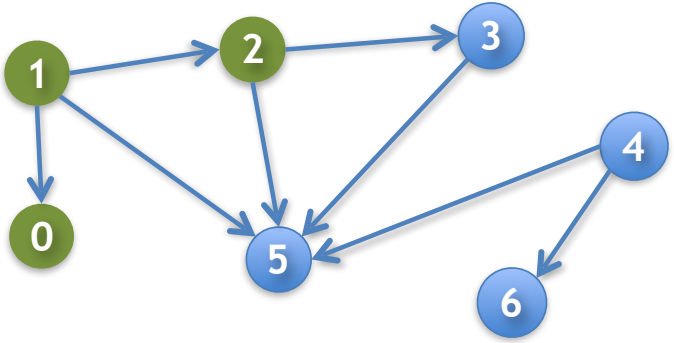
```

Call dfs(1)

Iteration 3



3
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Stack s

Depth-First Search written iteratively

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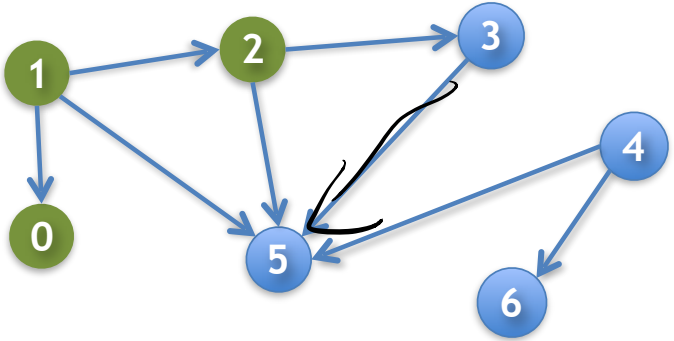
Call dfs(1)

Iteration 3



5
5

Stack s

Depth-First Search written iteratively

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        u= s.pop();
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            for each edge (u, v) leaving u:
                s.push(v);
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}
```
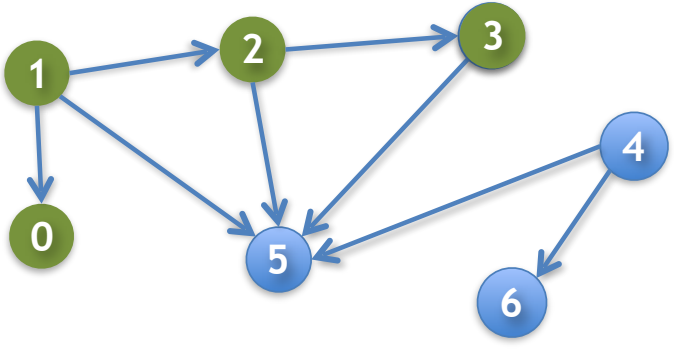

Call dfs(1)

Iteration 3



5
5

Stack s

Depth-First Search written iteratively

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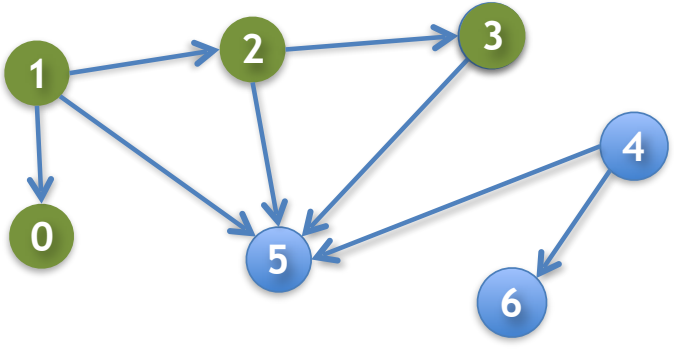
Call dfs(1)

Iteration 3



5
5
5

Stack s

Depth-First Search written iteratively

```
/** Visit all nodes explorable from u. Pre: u is unvisited. */
public static void dfs(int u) {
    Stack s= (u);
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        u= s.pop();
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            for each edge (u, v) leaving u:
                s.push(v);
        }
    }
}
```
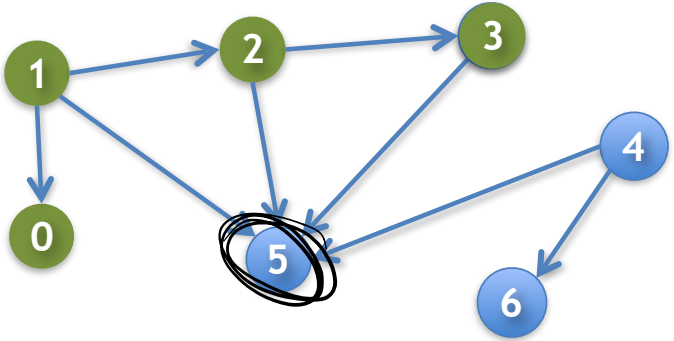

Call dfs(1)

Iteration 4



5
5

Stack s

Depth-First Search written iteratively

```
/** Visit all nodes explorable from u. Pre: u is unvisited. */
public static void dfs(int u) {
    Stack s= (u);
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        u= s.pop();
        if (u has not been visited) {
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            for each edge (u, v) leaving u:
                s.push(v);
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}
```
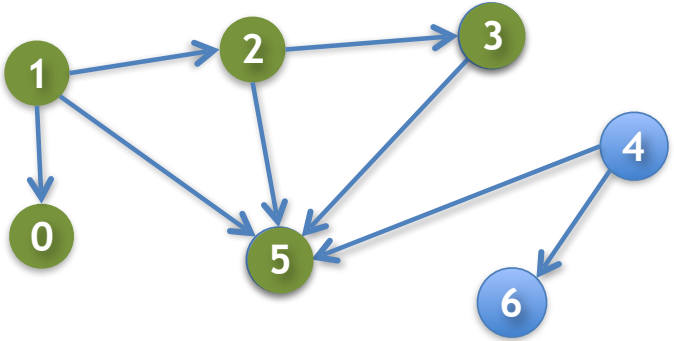

Call dfs(1)

Iteration 4



5
5

Stack s

Depth-First Search written iteratively

```
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    Stack s= (u);
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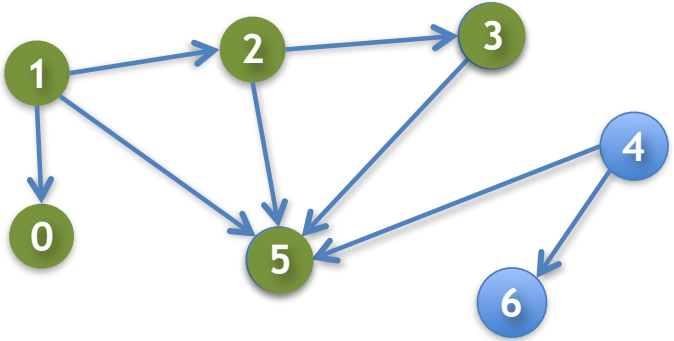
Call dfs(1)

Iteration 4



5
5

Stack s

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    Stack s= (u);
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        u= s.pop();
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                s.push(v);
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    }
}
```
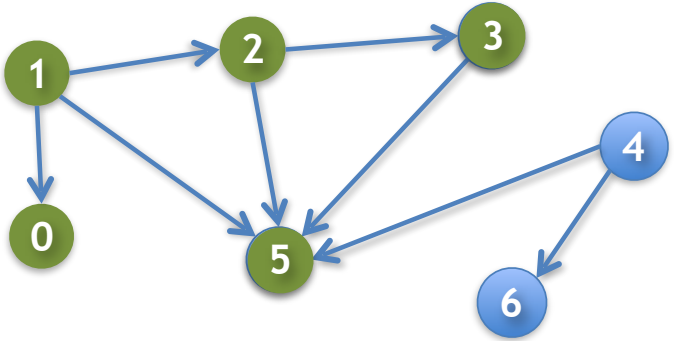

Call dfs(1)

Iteration 5



5

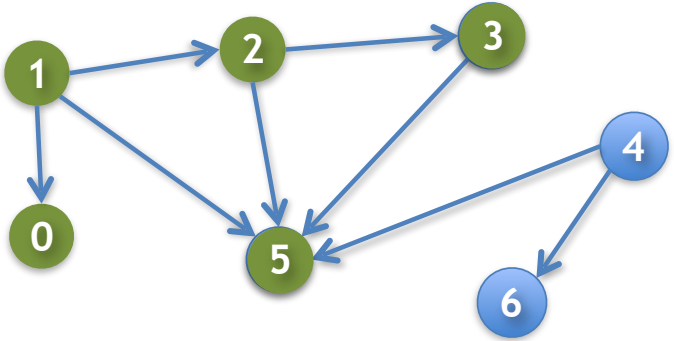
Stack s

Depth-First Search written iteratively

```
/** Visit all nodes explorable from u. Pre: u is unvisited. */
public static void dfs(int u) {
    Stack s= (u);
    while (s is not empty) {
        u= s.pop();
        if (u has not been visited) {
            visit u;
            for each edge (u, v) leaving u:
                s.push(v);
        }
    }
}
```

Call dfs(1)

Iteration 6



Stack s

Depth-First Search written iteratively

```
/** Visit all nodes explorable from u. Pre: u is unvisited. */
public static void dfs(int u) {
    Stack s= (u);
    while (s is not empty) {
        u= s.pop();
        if (u has not been visited) {
            visit u;
            for each edge (u, v) leaving u:
                s.push(v);
        }
    }
}
```

That's DFS!

```
/** Visit all nodes explorable from u. Pre: u is unvisited. */
public static void dfs(int u) {
    Stack s= (u);    // Not Java!
    // inv: all nodes that have to be visited are
    //        explorable from some node in s
    while (s is not empty      ) {
        u= s.pop();    // Remove top stack node, put in u
        if (u has not been visited) {
            visit u;
            for each edge (u, v) leaving u:
                s.push(v);
        }
    }
}
```

That's DFS!

```
/** Visit all nodes explorable from u. Pre: u is unvisited. */
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            visit u;
            for each edge (u, v) leaving u:
                s.push(v);
        }
    }
}
```

Want to see a magic trick?

Depth-First Search

```
/** Visit all nodes explorable from u. Pre: u is unvisited. */
public static void dfs(int u) {
    Stack s= (u);     // Not Java!
    // inv: all nodes that have to be visited are
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        if (u has not been visited) {
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        }
    }
}
```

Depth-First Search

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    Stack s= (u);     // Not Java!
    // inv: all nodes that have to be visited are
    //        explorable from some node in s
    while (s is not empty      ) {
        u= s.pop();    // Remove top stack node, put in u
        if (u has not been visited) {
            visit u;
            for each edge (u, v) leaving u:
                s.push(v);
        }
    }
}
```

Breadth-First Search

```
/** Visit all nodes explorable from u. Pre: u is unvisited. */
public static void bfs(int u) {
    Queue q= (u);     // Not Java!
    // inv: all nodes that have to be visited are
    //        explorable from some node in s
    while (q is not empty     ) {
        u= q.popFirst();    // Remove first node in queue, put
in u
        if (u has not been visited) {
            visit u;
            for each edge (u, v) leaving u:
                q.append(v);    // Add to end of queue
        }
    }
}
```

Breadth-First Search

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/** Visit all nodes explorable from u. Pre: u is unvisited. */
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        }
    }
}
```
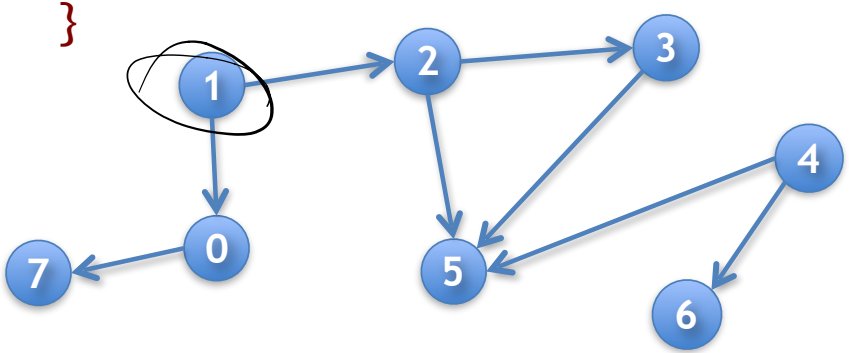

Call
bfs(1)



Queue q

Breadth-First Search

```
/** Visit all nodes explorable from u. Pre: u is unvisited. */
public static void bfs(int u) {
    Queue q= (u);
    while q is not empty) {
        u= q.popFirst();
        if (u has not been visited) {
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```
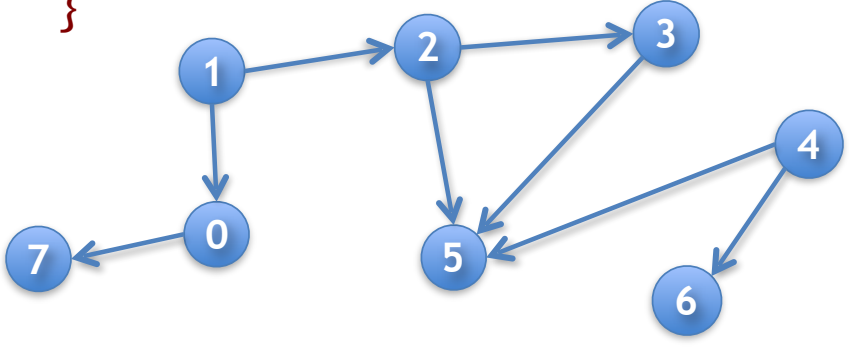

Call
bfs(1)



1

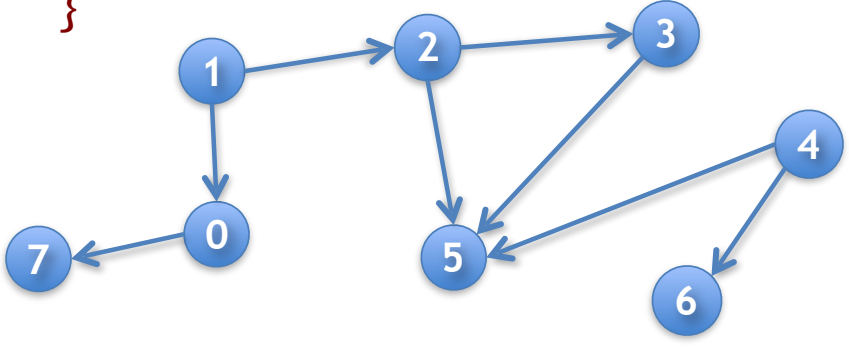
Queue q

Breadth-First Search

```
/** Visit all nodes explorable from u. Pre: u is unvisited. */
public static void bfs(int u) {
    Queue q= (u);
    while q is not empty) {
        u= q.popFirst();
        if (u has not been visited) {
            visit u;
            for each edge (u, v) leaving u:
                q.append(v);
        }
    }
}
```

Call bfs(1)

Iteration 0



1

Queue q

Breadth-First Search

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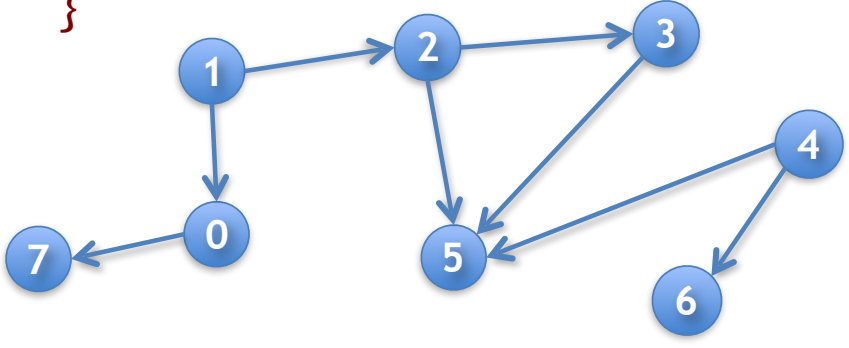
}
```

Call bfs(1)

Iteration 0



Queue q

Breadth-First Search

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            visit u;
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```
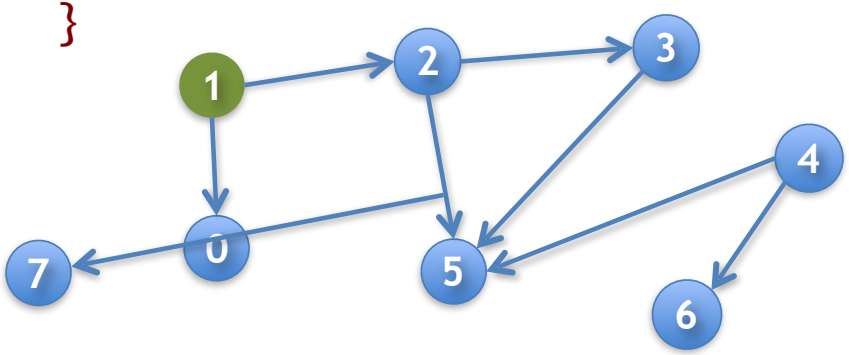

Call bfs(1)

Iteration 0



Queue q

Breadth-First Search

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/** Visit all nodes explorable from u. Pre: u is unvisited. */
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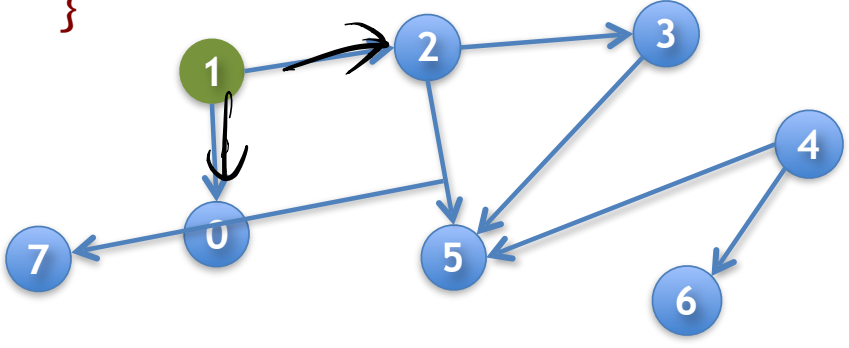
```

Call bfs(1)

Iteration 0



0 2

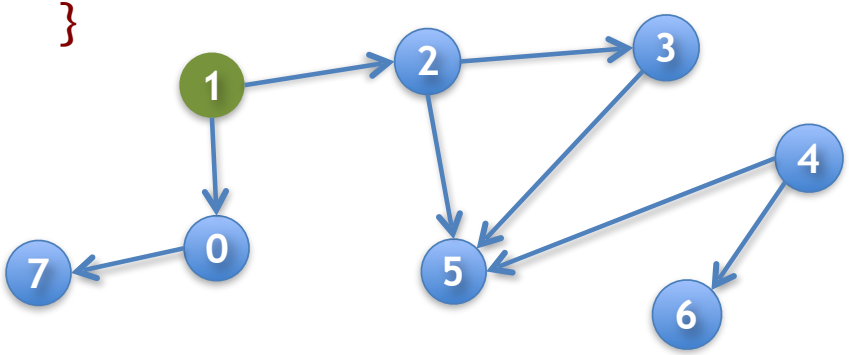
Queue q

Breadth-First Search

```
/** Visit all nodes explorable from u. Pre: u is unvisited. */
public static void bfs(int u) {
    Queue q= (u);
    while q is not empty) {
        u= q.popFirst();
        if (u has not been visited) {
            visit u;
            for each edge (u, v) leaving u:
                q.append(v);
        }
    }
}
```

Call bfs(1)

Iteration 1



0 2

Queue q

Breadth-First Search

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        }
    }
}
```
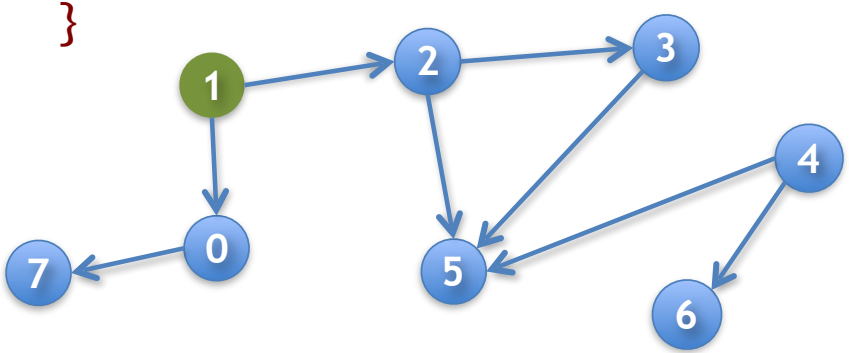

Call bfs(1)

Iteration 1



2

Queue q

Breadth-First Search

```
/** Visit all nodes explorable from u. Pre: u is unvisited. */
public static void bfs(int u) {
    Queue q= (u);
    while q is not empty) {
        u= q.popFirst();
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}
```
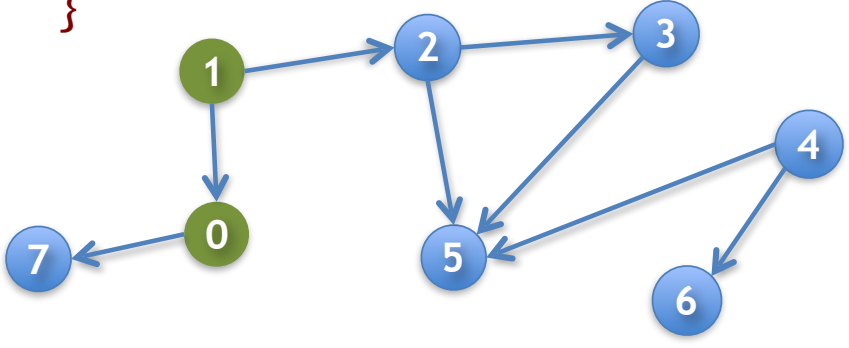

Call bfs(1)

Iteration 1



2

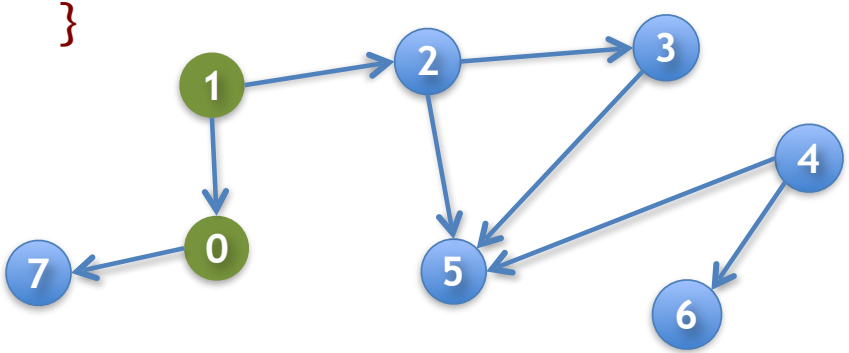
Queue q

Breadth-First Search

```
/** Visit all nodes explorable from u. Pre: u is unvisited. */
public static void bfs(int u) {
    Queue q= (u);
    while q is not empty) {
        u= q.popFirst();
        if (u has not been visited) {
            visit u;
            for each edge (u, v) leaving u:
                q.append(v);
        }
    }
}
```

Call bfs(1)

Iteration 1



2 7

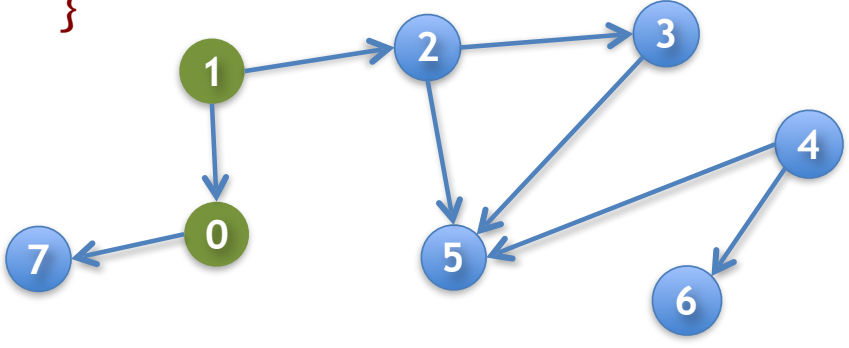
Queue q

Breadth-First Search

```
/** Visit all nodes explorable from u. Pre: u is unvisited. . */
public static void bfs(int u) {
    Queue q= (u);
    while q is not empty) {
        u= q.popFirst();
        if (u has not been visited) {
            visit u;
            for each edge (u, v) leaving u:
                q.append(v);
        }
    }
}
```

Call bfs(1)

Iteration 2



2 7

Queue q

Breadth-First Search

```
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    while q is not empty) {
        u= q.popFirst();
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                q.append(v);
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}
```
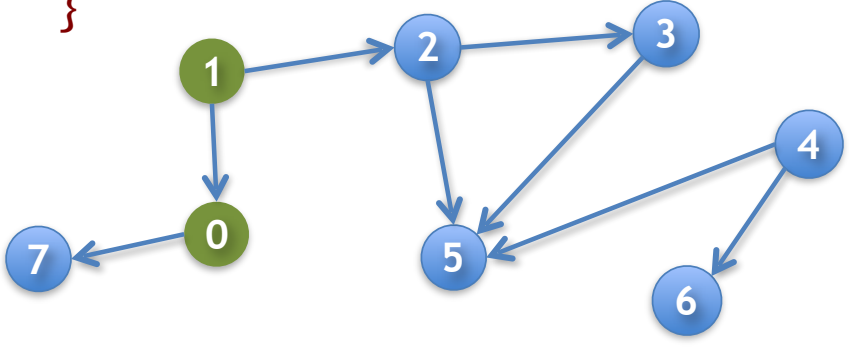

Call bfs(1)

Iteration 2



7

Queue q

Breadth-First Search

```
/** Visit all nodes explorable from u. Pre: u is unvisited. */
public static void bfs(int u) {
    Queue q= (u);
    while q is not empty) {
        u= q.popFirst();
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            for each edge (u, v) leaving u:
                q.append(v);
        }
    }
}
```
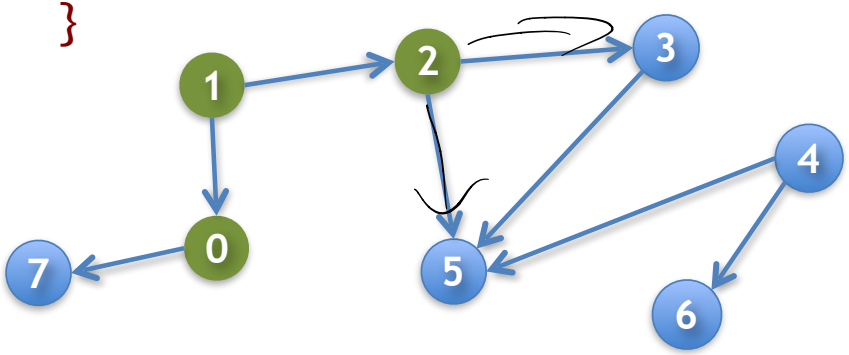

Call bfs(1)

Iteration 2



7

Queue q

Breadth-First Search

```
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```
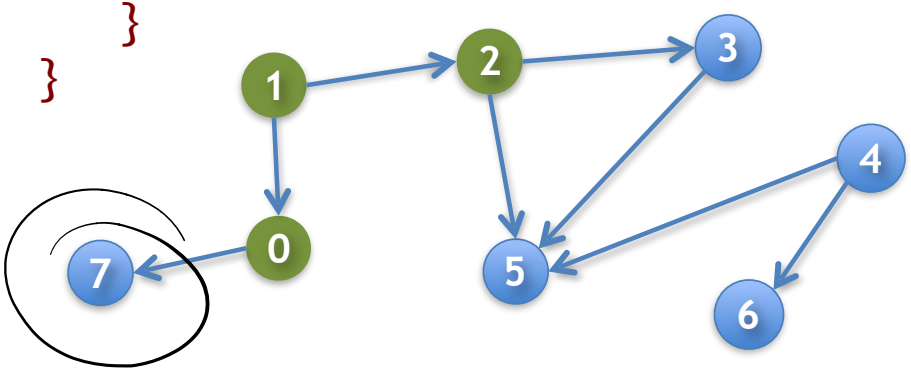

Call bfs(1)

Iteration 2



7 3 5

Queue q

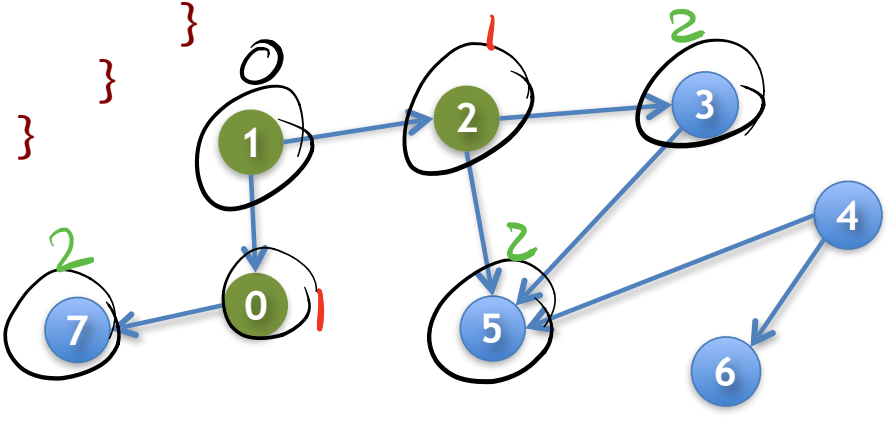
Breadth-First Search

```
/** Visit all nodes explorable from u. Pre: u is unvisited. */
public static void bfs(int u) {
    Queue q= (u);
    while q is not empty) {
        u= q.popFirst();
        if (u has not been visited) {
            visit u;
            for each edge (u, v) leaving u
                q.append(v);
        }
    }
}
```

Call bfs(1)

Iteration 2

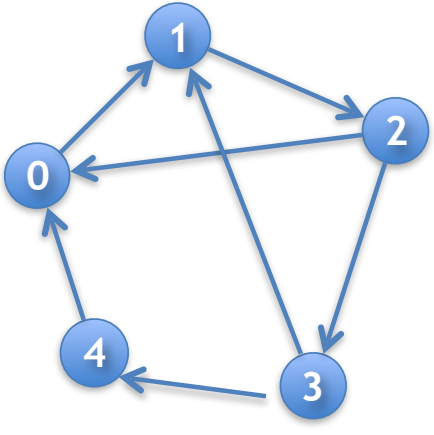
Breadth first:
(1) Node u
(2) All nodes 1 edge from u
(3) All nodes 2 edges from u
(4) All nodes 3 edges from u
…



7 3 5
Queue q

Some working code for DFS

- https://codeboard.io/projects/97448
- Sample graph constructed by the code:



- Suggested exercises:
  - Run DFS by hand
  - Run BFS by hand
  - Code BFS

Questions to Ponder

- BFS(root) on a tree corresponds to which tree traversal?
- Write out the order nodes are visited in this undirected graph, when calling:
  - BFS(5)
  - DFS(5)
  - DFS(0)

(if there are ties, visit the lower # first)