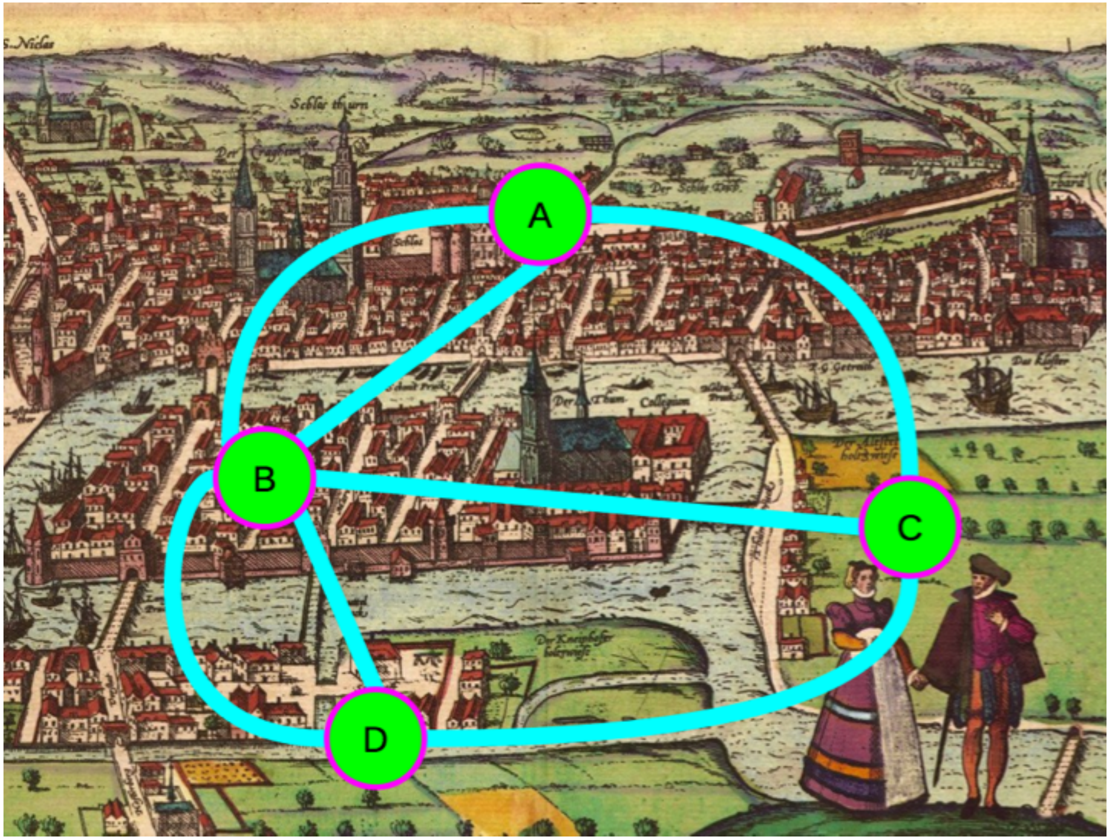
CSCI 241

Lecture 19
Introduction to Giraffes



CSCI 241

Lecture 19

Intro to Graphs: Terminology, Representation

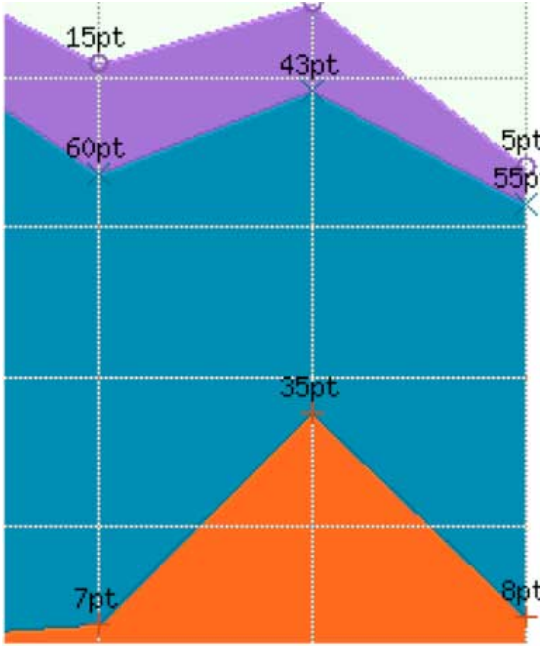
Announcements

- Lab 7 is (finally) out!
  - No code, just some stuff to read and some written problems to answer in a .txt file.

Goals

- Know the definition of a graph and basic associated terminology:
  - Node/vertex; edge/arc; directed, undirected; adjacent; (in/out-)degree; path; cycle;
- Understand how to represent a graph using:
  - adjacency list
  - adjacency matrix
- Be able to implement and analyze the runtime of simple graph operations on adjacency matrices and adjacency lists.
- Know how to implement breadth-first and depth-first graph traversals.

Next time.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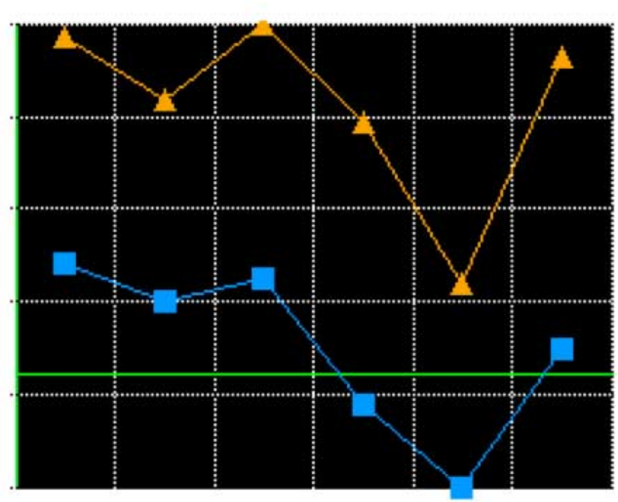
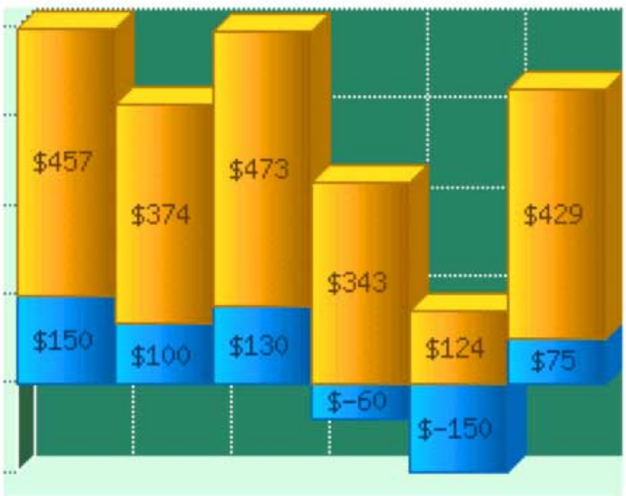


$12,000
$30,000
$23
$30,000

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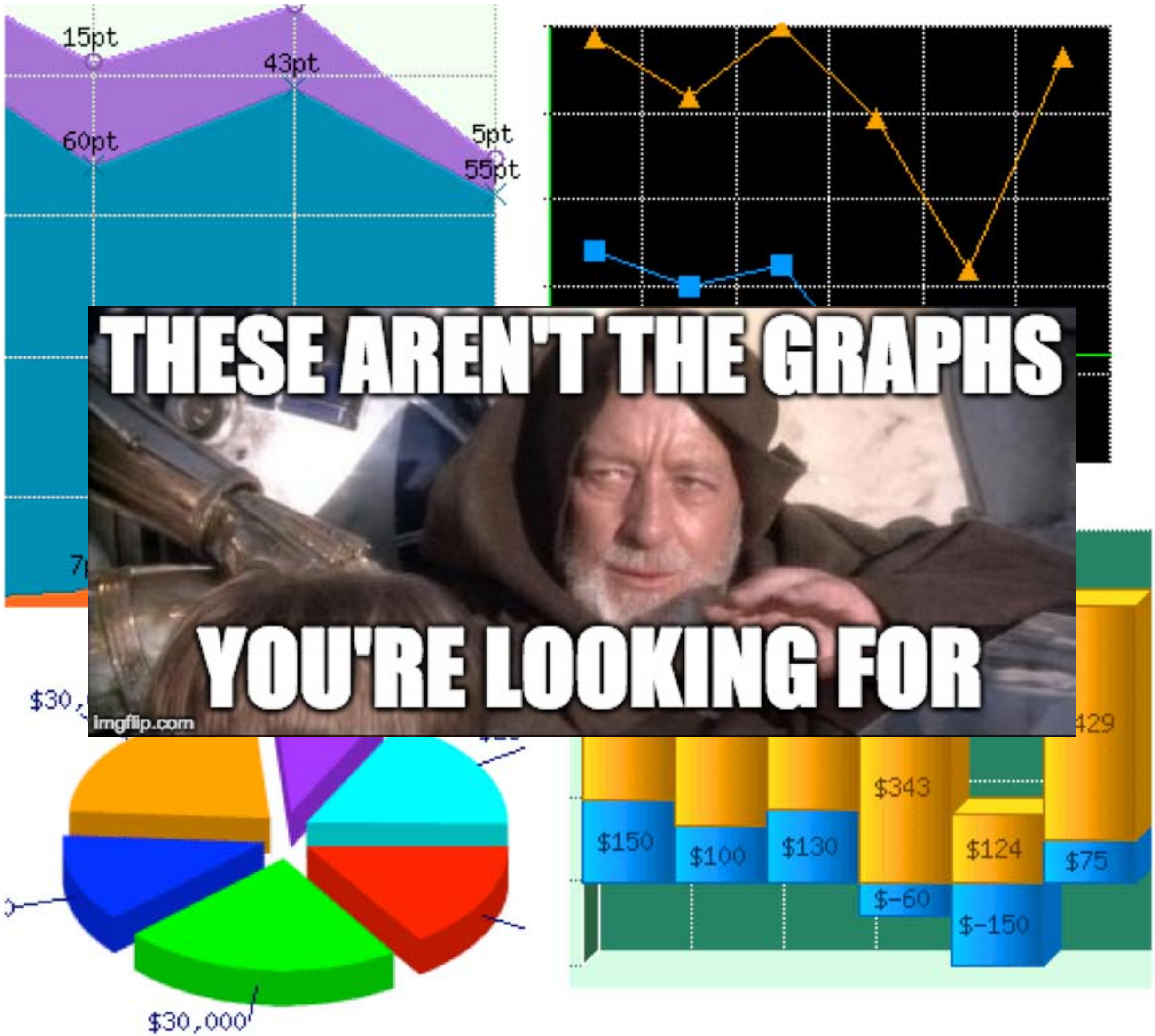
THESE AREN'T THE GRAPHS
YOU'RE LOOKING FOR
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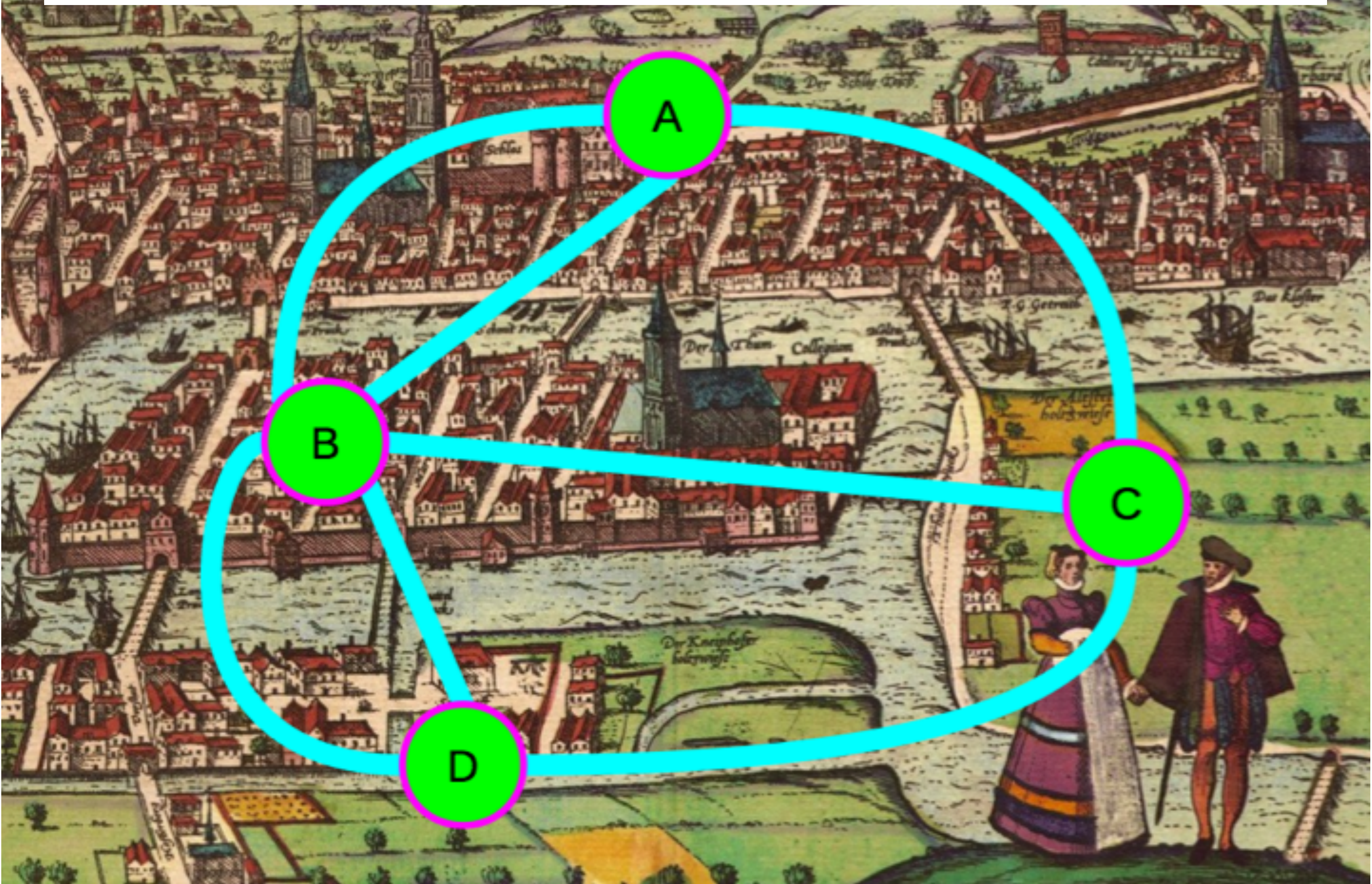
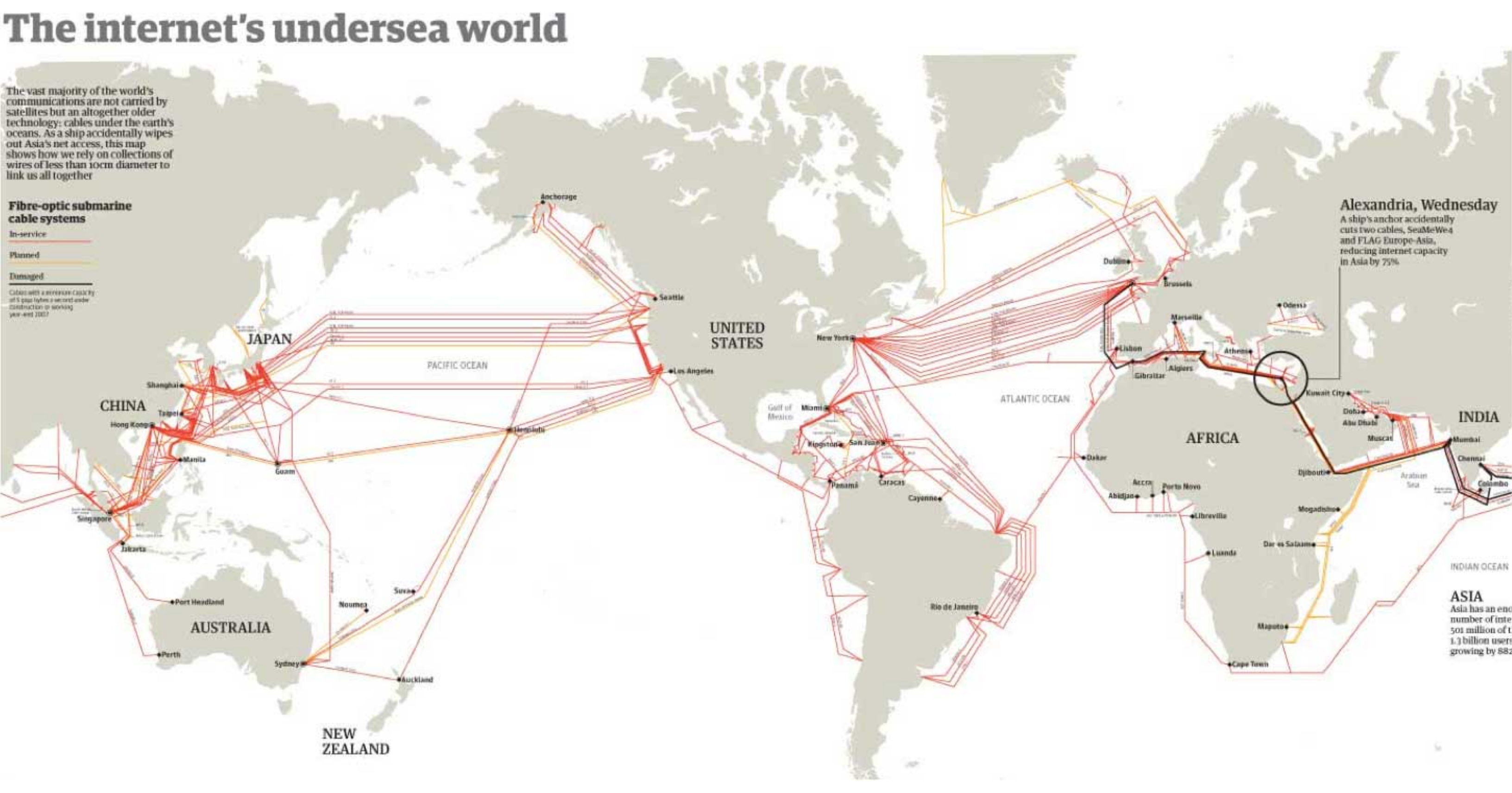
429

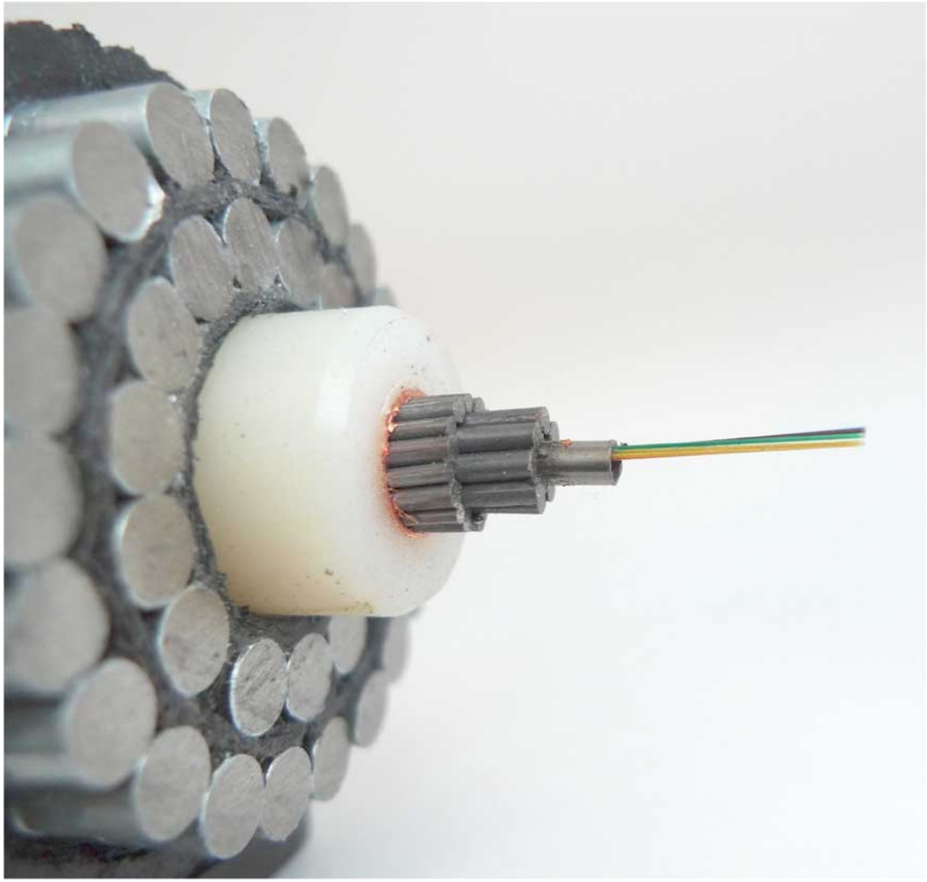
Graph: a bunch of points connected by lines. The lines may have directions, or not.



This is a graph:

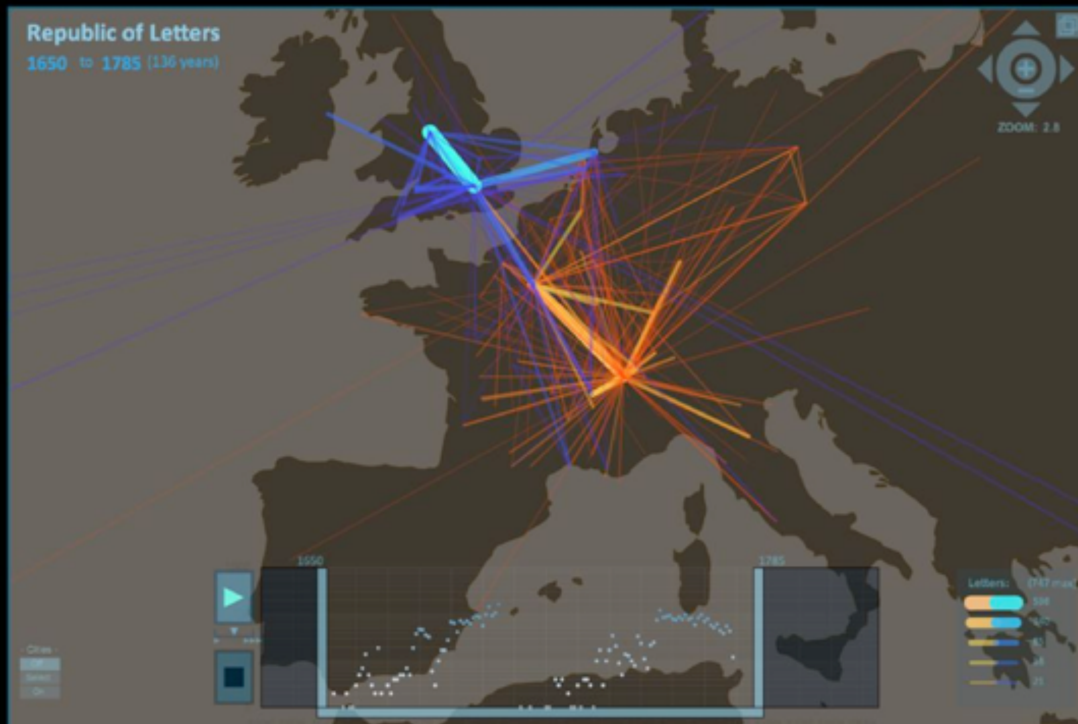


The edges are made of these:



Social Networks

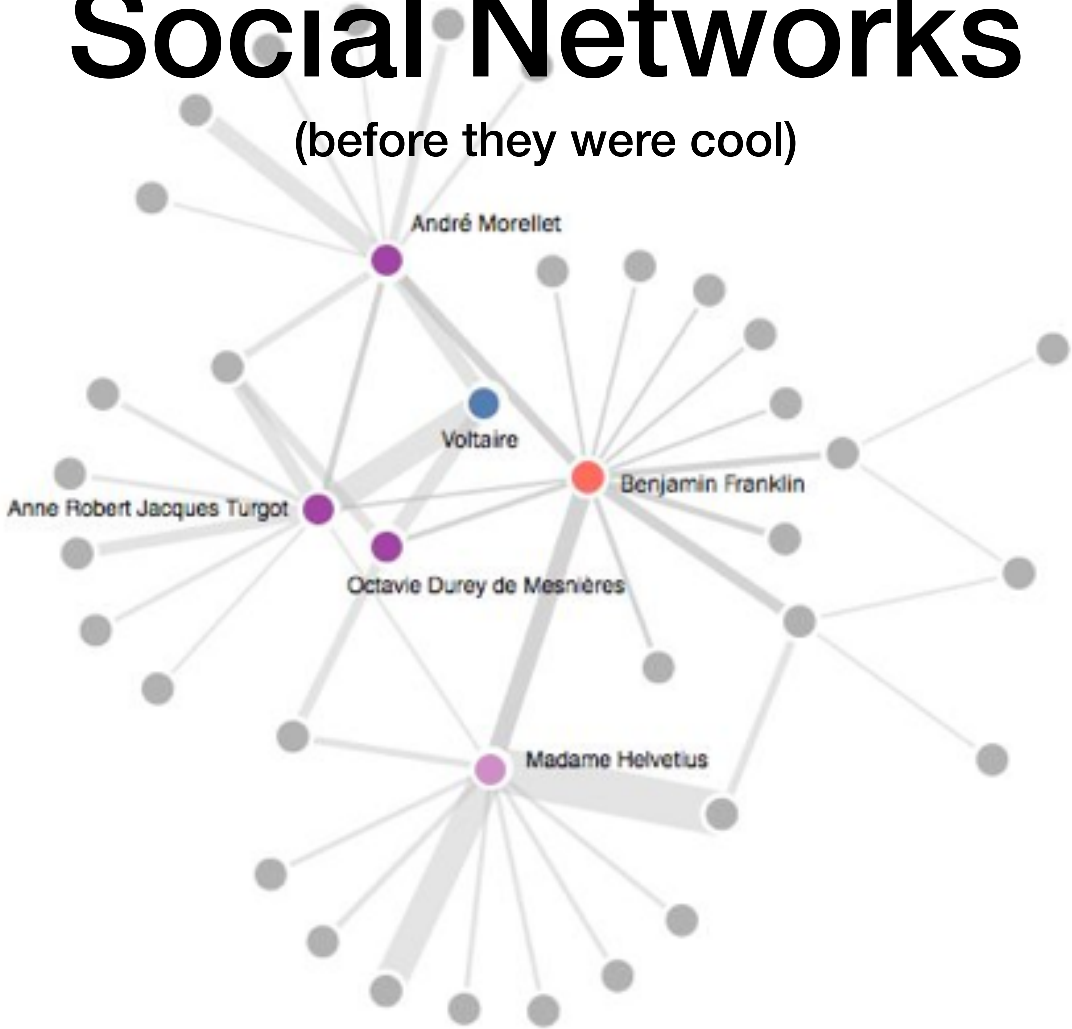
(before they were cool)



Locke's (blue) and Voltaire's (yellow) correspondence.
Only letters for which complete location information is available are shown.
Data courtesy the Electronic Enlightenment Project, University of Oxford.

Social Networks

(before they were cool)



MANHATTAN
East River
RIKERS ISLAND
HUNTS POINT
MOTT HAVEN
HARLEM
EAST HARLEM
UPPER EAST SIDE
CENTRAL PARK
METROPOLITAN MUSEUM OF ART
RANDALLS ISLAND
ROOSEVELT ISLAND
ASTORIA
LONG ISLAND CITY
JACKSON HEIGHTS
CORONA
FLUSHING
FLUSHING MEADOWS CORONA PARK
REGO PARK
JUNIPER VALLEY PARK
MIDDLE VILLAGE
GLENDALE
MASPETH
LAGUARDIA AIRPORT
LaGuardia Link Q70 SBS
M60 SBS
Q47
Q48
Q72
145 St
A·B·C·D
135 St
B·C
125 St
A·B·C·D
M60 SBS
LGA Airport
116 St
B·C
Cathedral Pkwy (110 St)
B·C
103 St
B·C
96 St
B·C
86 St
B·C
81 St-Museum of Natural History
B·C
72 St
B·C
59 St
Columbus Circle
A·B·C·D·1
50 St
C·E
Port Authority Bus Terminal
A·C·E
Times Sq-42 St
N·Q·R·S·W·1·2·3·7
34 St
Penn Station
34 St Hudson Yards
7
Harlem 148 St
3
145 St
3
135 St
2·3
125 St
2·3 · M60 SBS
LGA Airport
116 St
2·3
Central Park North (110 St)
2·3
96 St
1·2·3
86 St
1
79 St
1
72 St
1·2·3
66 St Lincoln Center
1
50 St
1
138 St-Grand Concourse
4·5
4 skips rush hour peak direction
rush hour peak direction express
weekday peak direction express
3 Av-149 St
2·5
3 Av 138 St
6
Brook Av
6
Cypress Av
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E 143 St St Mary's St
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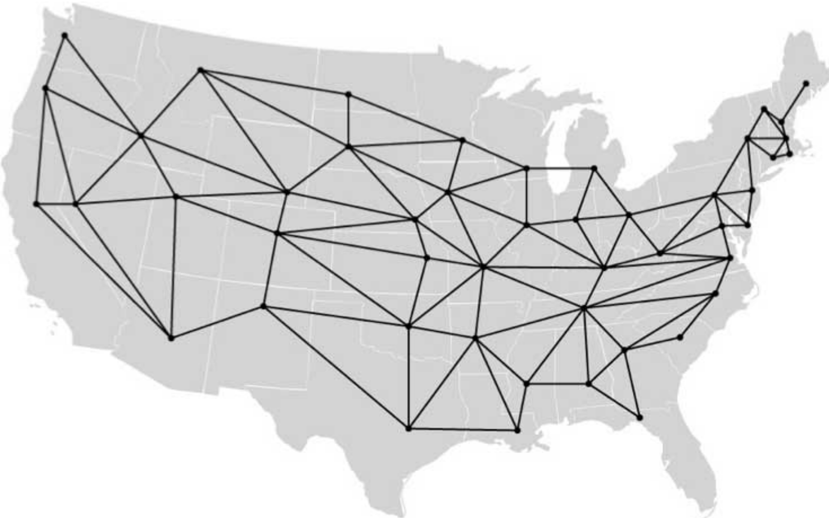
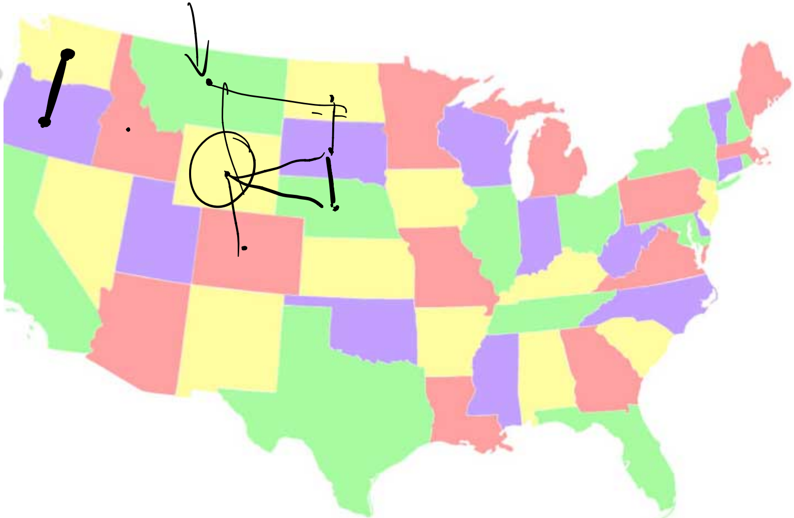
E 149 St
6
125 St
4·5·6
116 St
6
110 St
6
103 St
6
96 St
6
86 St
4·5·6
77 St
6
68 St Hunter College
6
59 St
4·5·6
51 St
6
Grand Central 42 St
S·4·5·6·7·Metro-North
33 St
6
96 St
Q
86 St
Q
72 St
Q
Lexington Av/63 St
F·Q
Lexington Av/59 St
N·R·W
5 Av/59 St
N·R·W
57 St-7 Av
N·Q·R·W
57 St
F
7 Av
B·D·E
49 St
N·R·W
5 Av/53 St
E·M
Lexington Av/53 St
E·M
47-50 Sts Rockefeller Ctr
B·D·F·M
42 St Bryant Pk
B·D·F·M
5 Av
7
34 St Herald Sq
B·D·F·M
TRAMWAY
QUEENSBORO BRIDGE
QUEENS MIDTOWN TUNNEL
Roosevelt Island
F
21 St Queensbridge
F
Queensboro Plaza
N·W·7
Court Sq-23 St
E·M
Court Sq
G·7
21 St
G
Vernon Blvd Jackson Av
7
Hunters Point Av
7·LIRR
Astoria Ditmars Blvd
N·W
Astoria Blvd
M60 SBS N·W
LGA Airport
30 Av
N·W
Broadway
N·W
36 Av
N·W
39 Av
N·W
36 St
M·R
Queens Plaza
E·M·R
Steinway St
M·R
46 St
M·R
Northern Blvd
M·R
65 St
M·R
Jackson Hts Roosevelt Av
E·F·M·R
74 St-Broadway
7
82 St-Jackson Hts
7
90 St-Elmhurst Av
7
Junction Blvd
7·Q72 LGA Airport
103 St-Corona Plaza
7
111 St
7·Q48 LGA Airport
Mets-Willets Point
7·Q48 LGA Airport
Flushing Main St
7
Elmhurst Av
M·R
Grand Av Newtown
M·R
Woodhaven Blvd
M·R
63 Dr-Rego Park
M·R·Q72 LGA Airport
67 Av
M·R
69 St
7
61 St Woodside
7·LIRR
52 St
7
46 St Bliss St
7
40 St Lowery St
7
33 St-Rawson St
7
Middle Village Metropolitan Av
M
Fresh Pond Rd
M
Forest Av
M

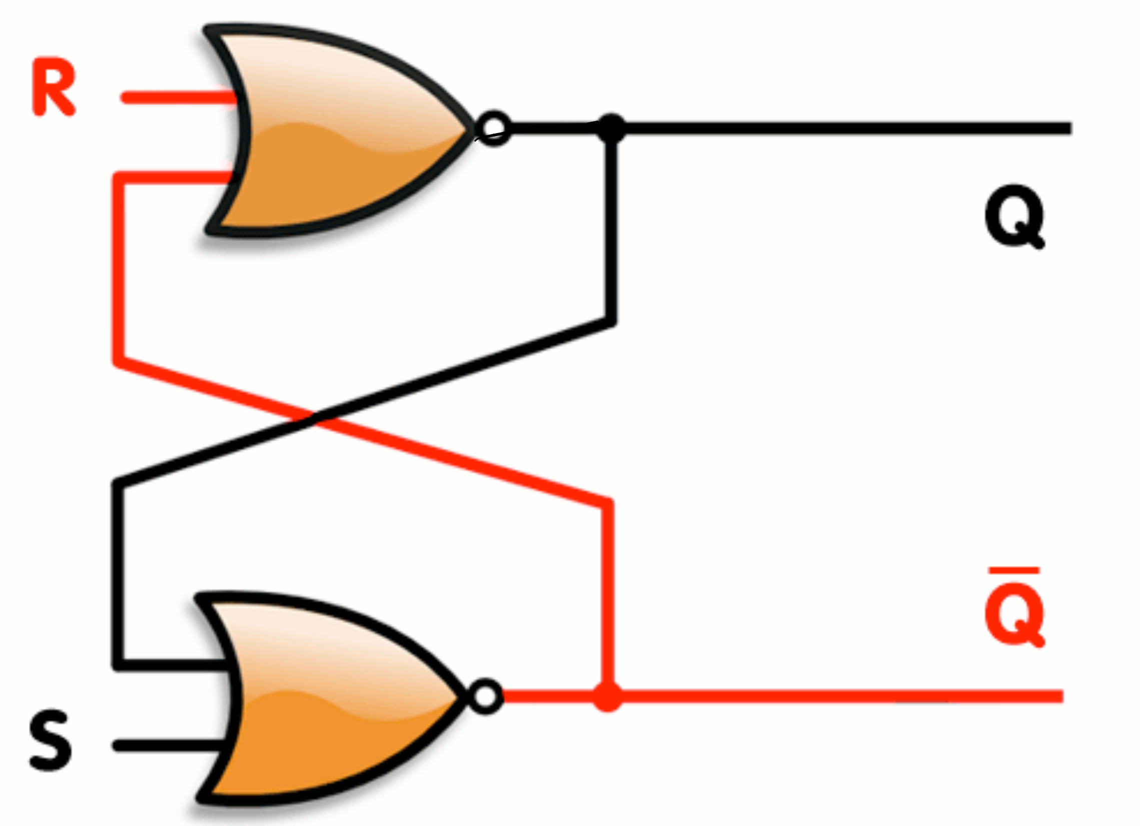
The USA as a graph:

- Neighboring states are connected by edges.

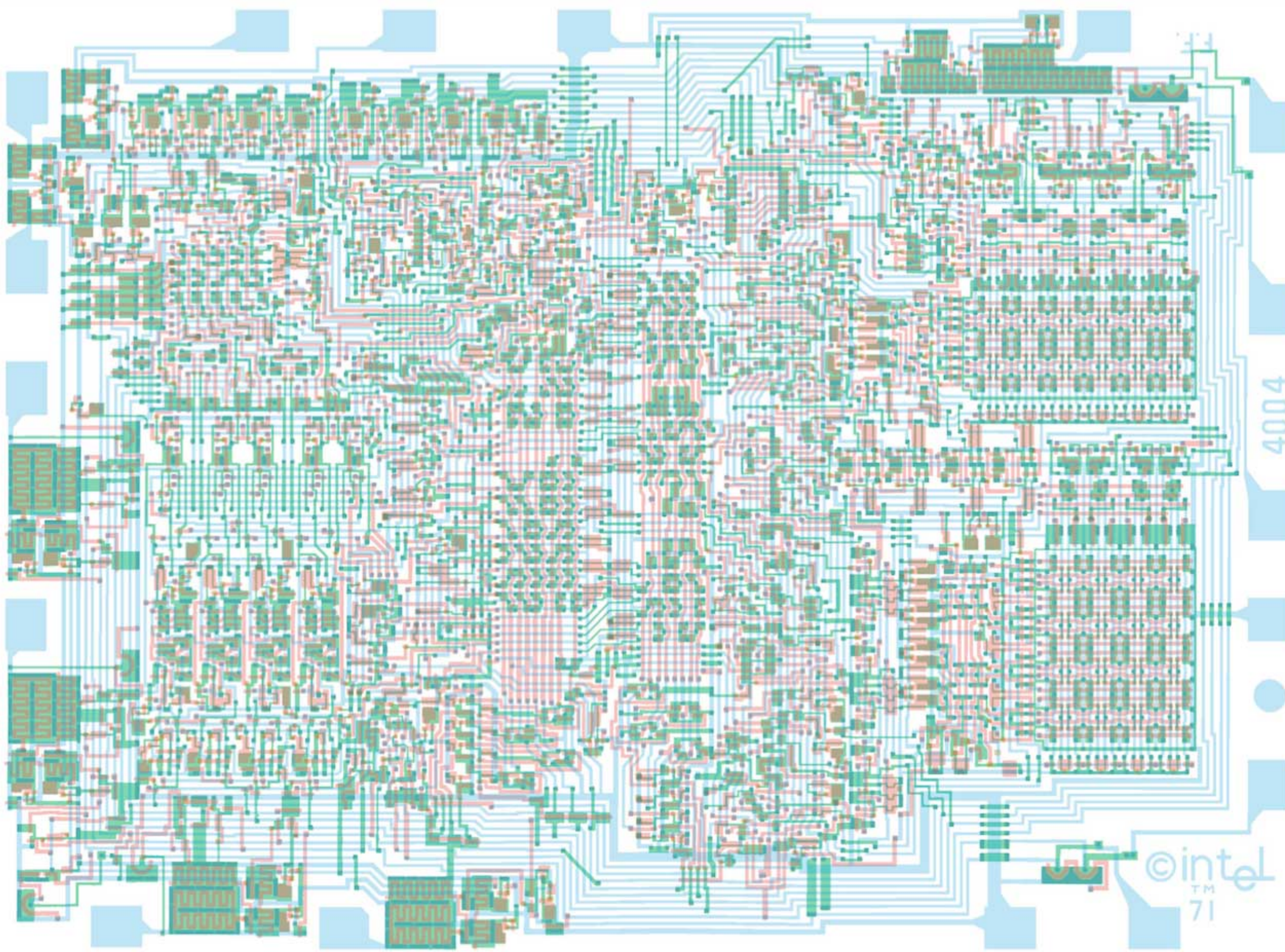




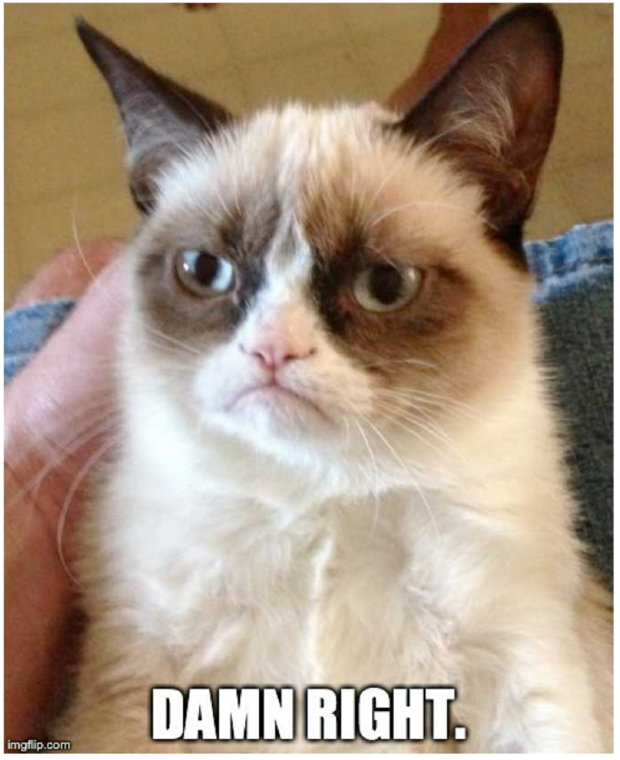
Electrical circuit



A bigger electrical circuit

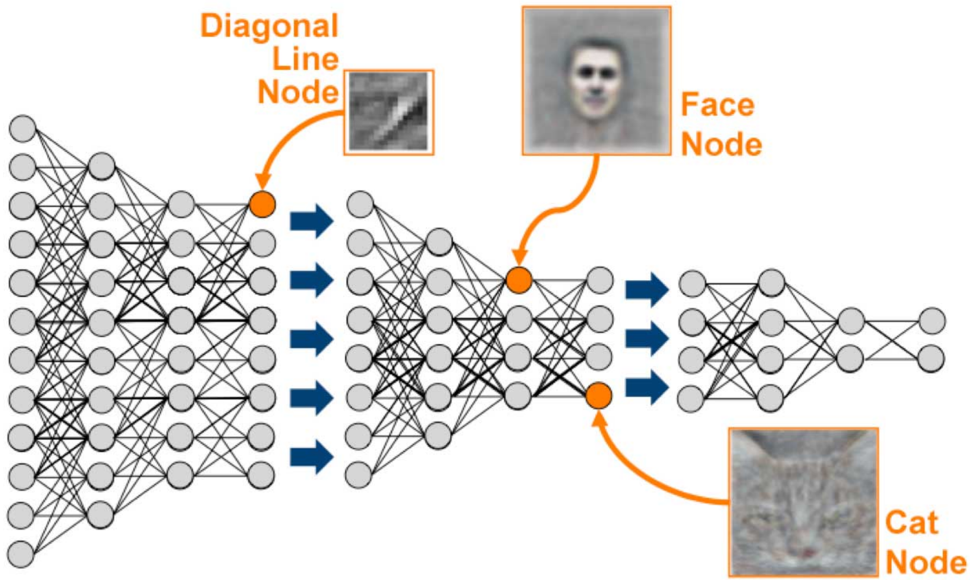


This is not a graph:



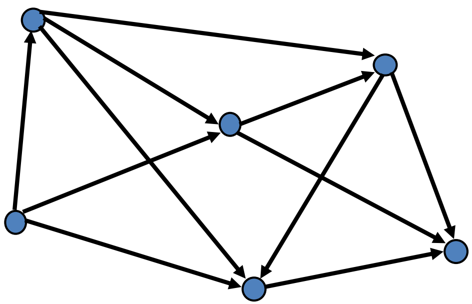
it is a cat.

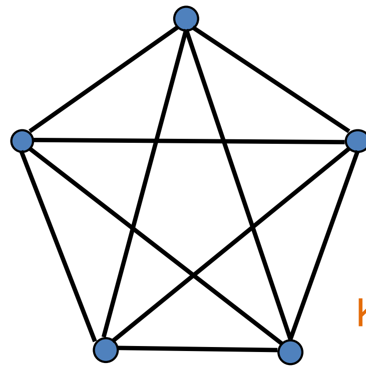
This is a graph



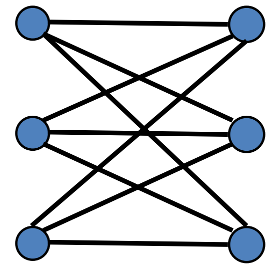
that can recognize cats.

Graphs: Abstract View

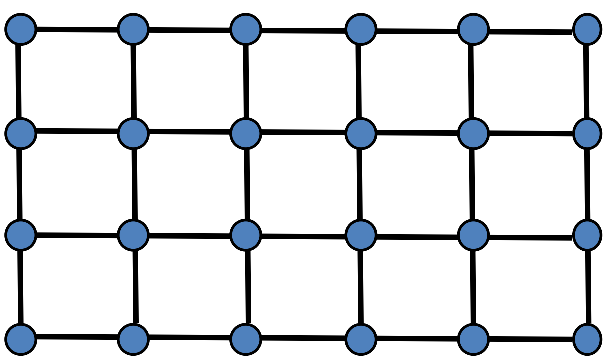


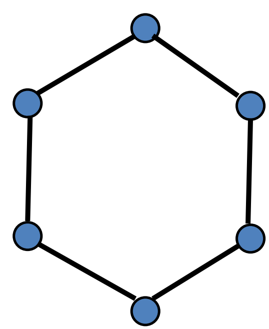


K_5



$K_{3,3}$





Graphs, Formally

- A directed graph (digraph) is a pair (V, E) where:
    - V is a (finite) set
    - E is a set of **ordered** pairs (u, v) where u, v are in V
    - Often (not always): u ≠ v (i.e. no edges from a vertex to itself)

- An element in V is called a vertex or node
- Elements in E are called edges or arcs
- |V| = size of V (traditionally called n)
- |E| = size of E (traditionally called m)

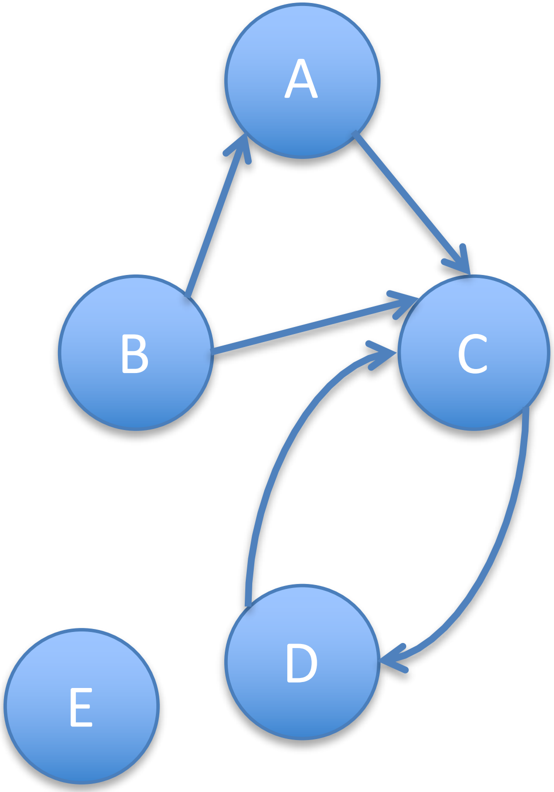
An example directed graph

$$V = \{A, B, C, D, E\}$$

$$E = \{(A, C), (B, A), (B, C), (C, D), (D, C)\}$$

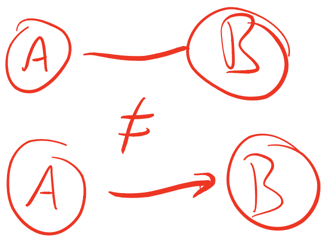
$$|V| = 5$$

$$|E| = 5$$



Graphs, Formally

- An **un**directed graph is a just like a digraph, but
  - E is a set of **un**ordered pairs (u, v) where u, v are in V

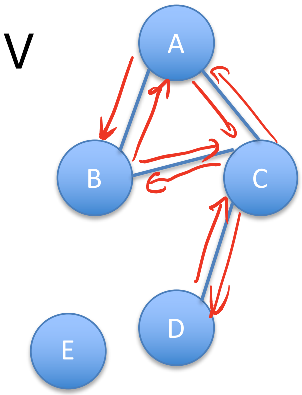


$\boldsymbol{V} = \{A, B, C, D, E\}$

$\boldsymbol{E} = \{\{A,C\}, \{B,A\}, \{B,C\}, \{C,D\}\}$

$|\boldsymbol{V}| = 5$

$|\boldsymbol{E}| = 4$



- An **un**directed graph can be converted to an equivalent **directed** graph:
  - Replace each undirected edge with two directed edges in opposite directions
- A **directed** graph can't always be converted to an **undirected** graph.

Graph Terminology: Adjacency, Degree

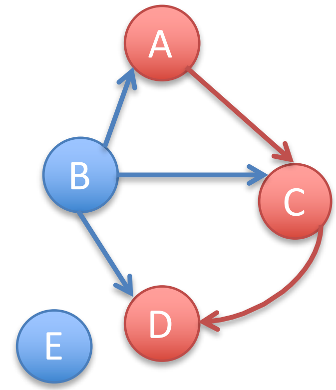
- Two vertices are adjacent if they are connected by an edge
- Nodes u and v are called the source and sink of the **directed** edge (u, v)

- Nodes u and v are endpoints of an edge (u, v) (directed or undirected)
- The outdegree of a vertex u in a **directed** graph is the number of edges for which u is the source
- The indegree of a vertex v in a **directed** graph is the number of edges for which v is the sink
- The degree of a vertex u in an **undirected** graph is the number of edges of which u is an endpoint

Graph Teminology: Paths, Cycles

- A path is a sequence of vertices where each consecutive pair are adjacent.

- In a directed graph, paths must follow the direction of the edges (nodes must be ordered source then sink).



Path A,C,D

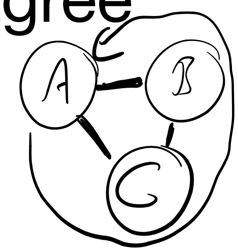
- A cycle is a path that ends where it started, e.g.: x, y, z, x

- A graph is acyclic if it has no cycles.

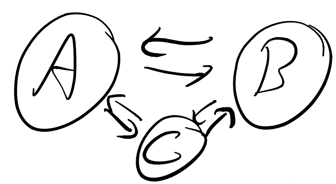
Social Networks

A social network can be modeled as a graph.

- Facebook model: both people must agree to be "friends".



- Twitter model: A can "follow B"; Independently, B may or may not follow A.



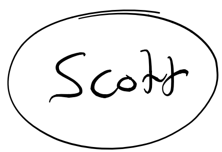
ABCD

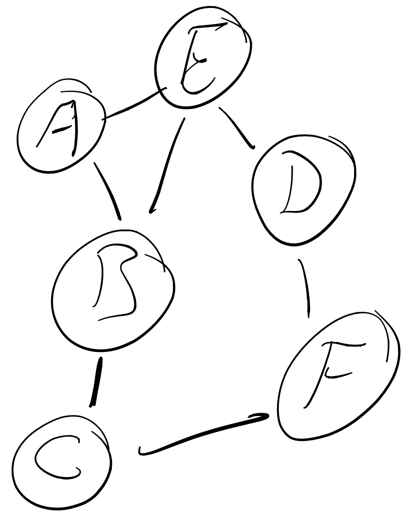
Facebook is a(n) undirected graph
Twitter is a(n) directed graph

A: directed / undirected
B: acyclic / not acyclic
C: not acyclic; acyclic
D: undirected / directed

Graph Terminology

What is the graph term for the number of facebook friends a person has?





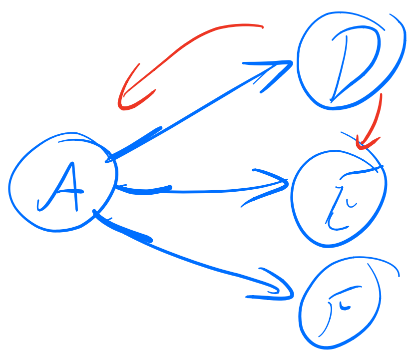
degree(Scott) = 0

$|E|$

Graph Terminology

What is the graph term for the number of people someone follows on Twitter?

outdegree = 3



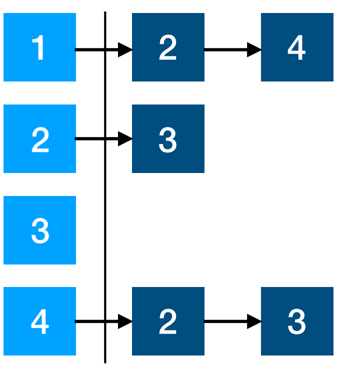
Representing Graphs: Adjacency Lists

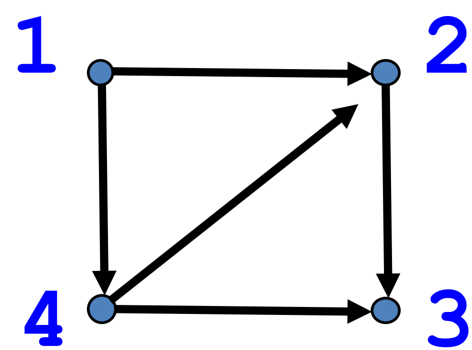
```
public class GraphNode {
    // fields storing information
    // about this node

    List<GraphNode> neighbors;
}
```

adjacent nodes

Node: Neighbors:



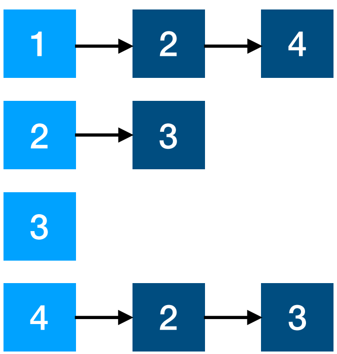


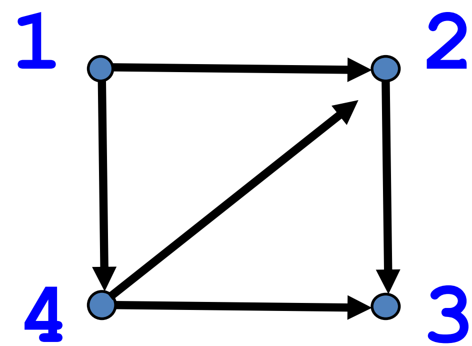
Representing Graphs: Adjacency Matrix

```
public class Graph {
    boolean[][] adjacent; // size n x n
}
```

Adjacency lists:

Node: Neighbors:





Representing Graphs: Adjacency Matrix

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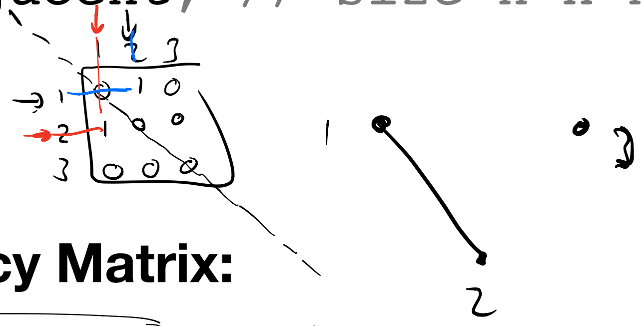
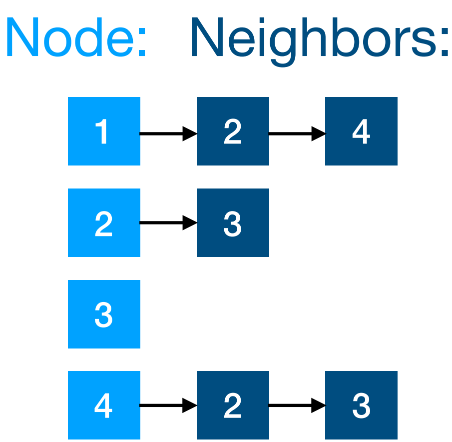
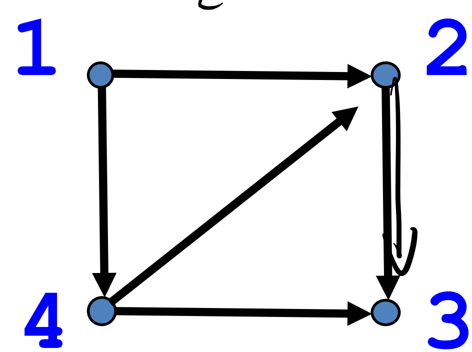


Adjacency lists:



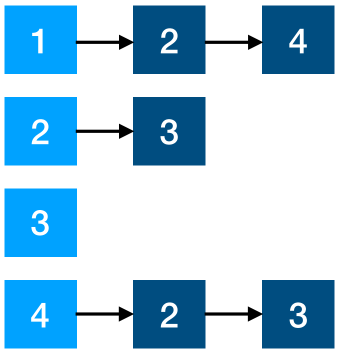
Adjacency Matrix:

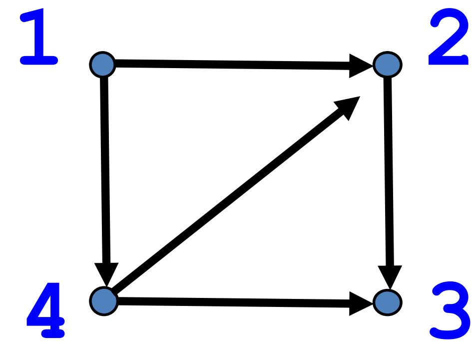
$$
\begin{array}{c|cccc}
 & \mathbf{1} & \mathbf{2} & \mathbf{3} & \mathbf{4} \\
\hline
\mathbf{1} & 0 & 1 & 0 & 1 \\
\mathbf{2} & 0 & 0 & 1 & 0 \\
\mathbf{3} & 0 & 0 & 0 & 0 \\
\mathbf{4} & 0 & 1 & 1 & 0
\end{array}
$$



Adjacency lists:

Node: Neighbors:





- Let n = |V| and m = |E|; let d(u) = degree of u

- ABCD: How much space does it take to store G as an adjacency list vs. adjacency matrix?

   A. List: $O(n^2+e)$; Matrix: $O(n^2)$
   B. List: $O(n+e)$; Matrix: $O(n + e)$
   C. List: $O(n^2)$; Matrix: $O(n + e^2)$
   D. List: $O(n+e)$; Matrix: $O(n^2)$

Adjacency lists: **Adjacency Matrix:**

Node: Neighbors:

1 → 2 → 4

2 → 3

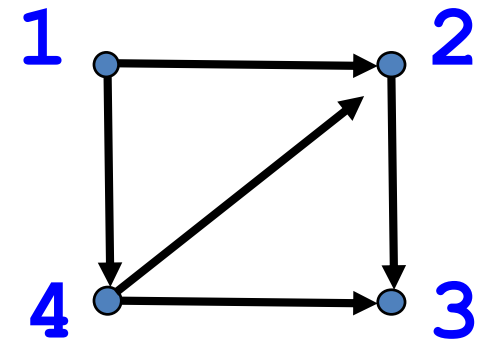
3

4 → 2 → 3

n (2)m

O(n + m)

$$
\begin{array}{c|cccc}
 & \multicolumn{4}{c}{\xrightarrow{\quad n \quad}} \\
 & 1 & 2 & 3 & 4 \\
\hline
1 & 0 & 1 & 0 & 1 \\
2 & 0 & 0 & 1 & 0 \\
3 & 0 & 0 & 0 & 0 \\
4 & 0 & 1 & 1 & 0
\end{array}
\quad n \downarrow
$$



- Let n = |V| and m = |E|; let d(u) = degree of u

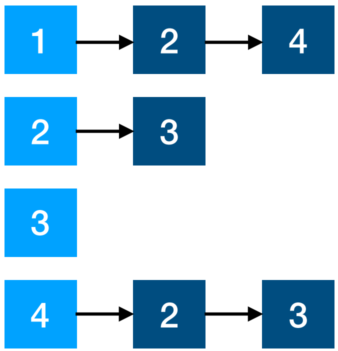
  e = |E|

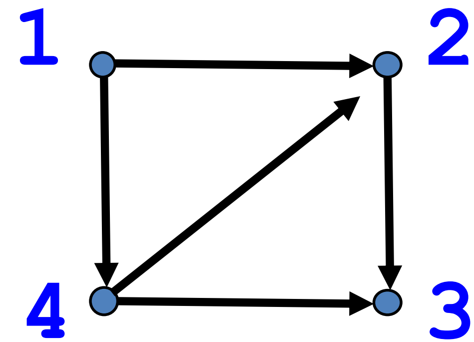
- ABCD: How much space does it take to store G as an adjacency list vs. adjacency matrix?

  A. List: $O(n^2+\overset{m}{e})$; Matrix: $O(n^2)$
  B. List: $O(n+\overset{m}{e})$; Matrix: $O(n + e)$
  C. List: $O(n^2)$; Matrix: $O(n + e^2)$
  D. List: $O(n+\overset{m}{e})$; Matrix: $O(n^2)$

Adjacency lists:

Node: Neighbors:





- Let n = |V| and m = |E|; let d(u) = degree of u

- ABCD: What's the runtime of iterating over all edges?
  A. List: $O(n^2)$; Matrix: $O(n^2)$
  B. List: $O(n+e)$; Matrix: $O(n^2)$
  C. List: $O(n + e)$; Matrix: $O(n + e)$
  D. List: $O(n+e)$; Matrix: $O(n^2 + e)$

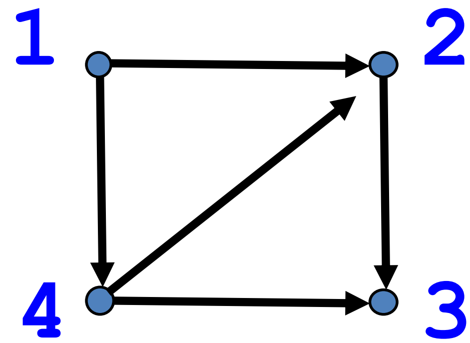
Adjacency lists:

Node: Neighbors:

1 → 2 → 4
2 → 3
3
4 → 2 → 3

Adjacency Matrix:

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 0 & 1 \\ 2 & 0 & 0 & 1 & 0 \\ 3 & 0 & 0 & 0 & 0 \\ 4 & 0 & 1 & 1 & 0 \end{array}$$



- Let n = |V| and $\overset{e}{m}$ = |E|; let d(u) = degree of u
- ABCD: What's the runtime of iterating over all edges?
  A. List: O(n^2); Matrix: O(n^2)
  B. List: O(n+e); Matrix: O(n^2)
  C. List: O(n + e); Matrix: O(n + e)
  D. List: O(n+e); Matrix: O(n^2 + e)

Adjacency Matrix vs Adjacency List

- Reminder: $n = |V|$ and $m = |E|$; let $d(u)$ = degree of u
- Adjacency matrix:
  - Storage space: $O(n^2)$
  - Iterate over edges: $O(n^2)$ time
  - Check if there's an edge from u to v: O(1)
  - Good for dense graphs
    - e.g., if n^2 is close to n^2, you need n^2 storage anyway.

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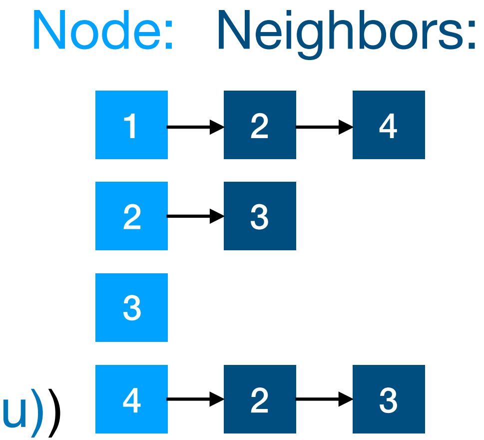
$$
\begin{array}{c|cccc}
 & \mathbf{1} & \mathbf{2} & \mathbf{3} & \mathbf{4} \\
\hline
\mathbf{1} & 0 & 1 & 0 & 1 \\
\mathbf{2} & 0 & 0 & 1 & 0 \\
\mathbf{3} & 0 & 0 & 0 & 0 \\
\mathbf{4} & 0 & 1 & 1 & 0
\end{array}
$$

Adjacency Matrix vs Adjacency List

- Reminder: n = |V| and m = |E|; let d(u) = degree of u
- Adjacency list:
  - Storage space: O(n + e)
  - Iterate over edges: O(n + e) time
  - Check if there's an edge from u to v: O(d(u))
  - Good for more sparse graphs:
    - e.g., if |E| is close to n, n + e ~= 2n, which is O(n)



Graph Algorithms

You can take entire graduate-level courses on graph algorithms. In this class:

- Search/traversal: search for a particular node or traverse all nodes (Lab 8)
  - Breadth-first
  - Depth-first
- Shortest Paths (A4)
- Spanning trees

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You can take entire graduate-level courses on graph algorithms. In this class:

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