CSCI 241

Lecture 17
Map ADT
~~A3 Overview~~
Hash Functions, Hash Tables, Hash Sets, Hashtags

Today's exercises: on paper and Socrative - have paper+pencil ready!

CSCI 241

Lecture 17

Map ADT

A3 Overview

Hash Functions, Hash Tables, Hash Sets~~, Hashtags~~

Announcements

- A3 is out!
- Exam is Friday!
- A3 video is posted (A3.mp4)

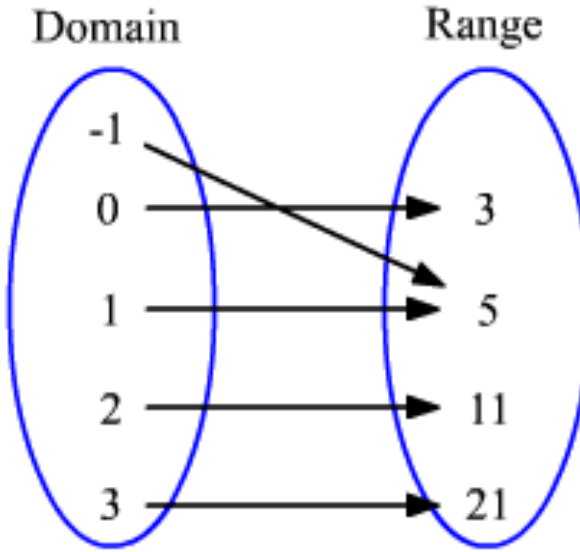
Goals

- Know the purpose and operations of the Map ADT
- Know the purpose, definition, and properties of hash functions.
- Know how to use a hash function to implement a hash table.
- Know how to use modular arithmetic to construct a basic hash function on integers.
- Know how to use chaining for collision resolution.
- Know the definition of load factor in a hash table.

The Map ADT

- In math, a **map** is a function.
- What is a function, anyway?

The Map ADT



- In math, a **map** is a function.

- If F is a map then
  F(a) —> b
  means that a maps to b.

- F has a:

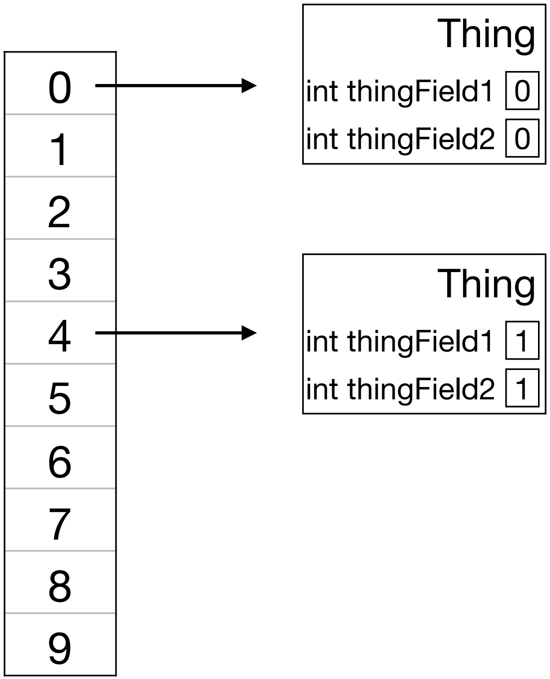
  - **domain** - the set of values F maps **from**

  - **range** - the set of values that F maps a domain element **to**

  - **codomain** - the set of **all** possible values in the range's type, regardless of whether any element in the domain maps to it

The Map ADT

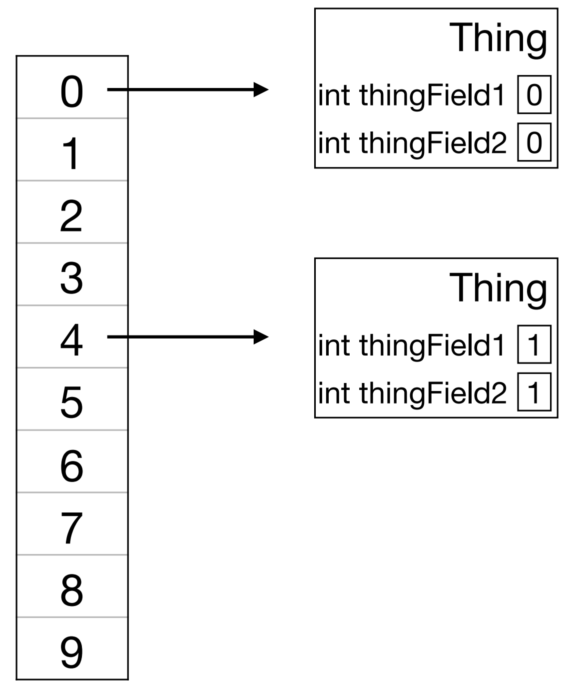
Thing[] a = new Thing[10];



The Map ADT

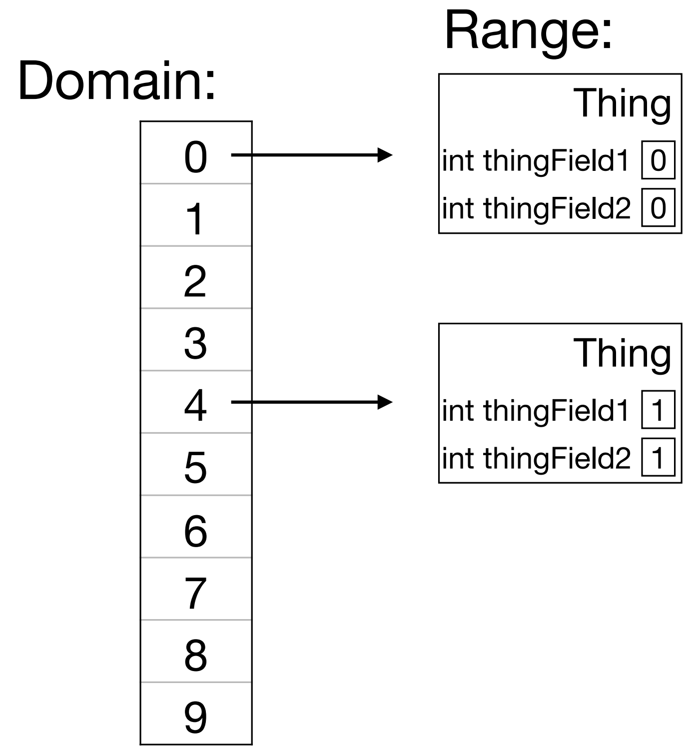
`Thing[] a = new Thing[10];`

- Arrays are great!
  - **Domain**: 0..a.length
  - **Range**: all elements in the array
  - **Codomain**: the array's **type**



The Map ADT

Thing[] a = new Thing[10];



We get to choose the **codomain.**

Codomain: Thing objects.

The Map ADT

- Arrays are great!
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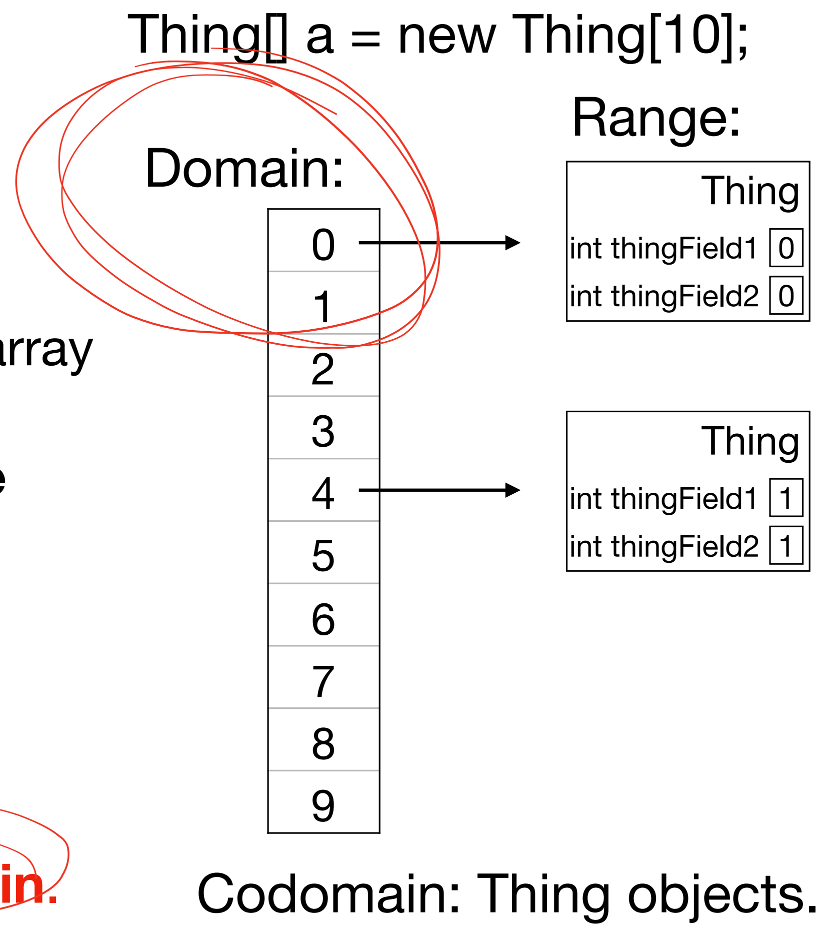


We get to choose the **codomain**.

The Map ADT

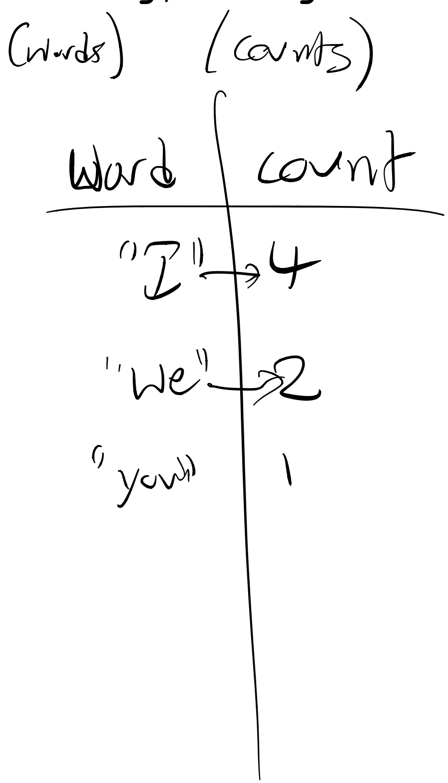
- Arrays are great!
- We get to choose the codomain - type of the array.
- Wouldn’t it be nice to choose the domain as well?
- The Map ADT represents a mapping from keys to values.
    - we get to choose the type of the **keys** (domain) AND the **values** (codomain)

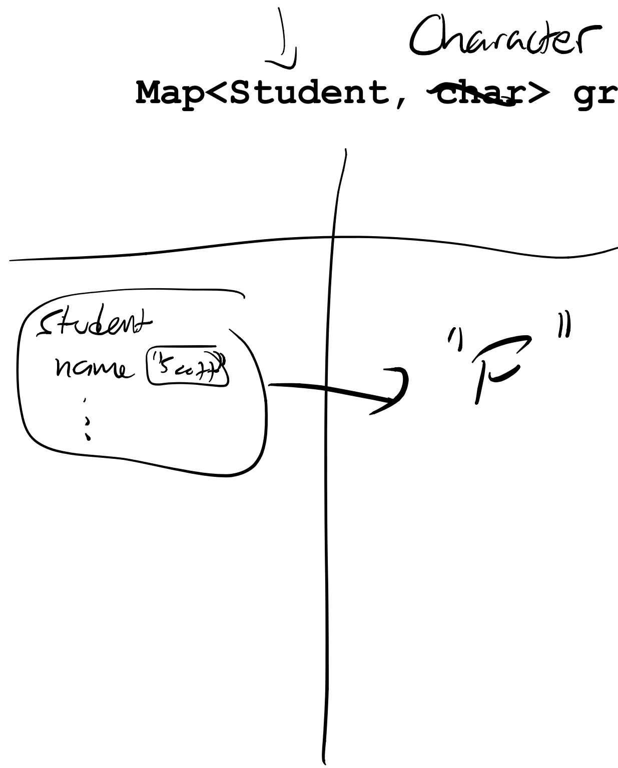
The Map Interface

```
public interface Map<K,V> {
    /** Returns the value to which the specified key
      * is mapped, or null if this map contains no
      * mapping for the key. */
    V get(Object key);

    /** Associates the specified value with the
      * specified key in this map */
    V put(K key, V value);

    /** Removes the mapping for a key from this map
      * if it is present */
    V remove(Object key);

    // more methods
}
```

Example Uses of Maps

```
Map<String, Integer> wordCounts;          Map<Student, char> grades;
```





Reminder: The **Set** ADT

- A **Set** maintains a collection of **unique** things.

- Java has this ADT built in as an interface:

  `java.util.Set`
- Some methods from `java.util.Set:`
  - `boolean` **`add`**`(Object ob)`
  - `boolean` **`contains`**`(Object ob)`
  - `boolean` **`remove`**`(Object ob)`

Reminder: The **Set** ADT

- A **Set** maintains a collection of **unique** things.

- Java has this ADT built in as an interface:

  `java.util.Set<T>`

- Some methods from `java.util.Set:`
  - `boolean` **`add`**`(T ob)`
  - `boolean` **`contains`**`(T ob)`
  - `boolean` **`remove`**`(T ob)`

Hashing: Motivation

- Consider implementations of the Set ADT:

	`add`	`contains`	`remove`
Unsorted Array or Linked List	O(1)	O(n)	O(n)
Sorted Linked List	O(n)	O(n)	O(n)
Sorted Array	O(n)	O(log n)	O(n)
AVL Tree	O(log n)	O(log n)	O(log n)
Magical Array	**O(1)***	**O(1)***	**O(1)***

How would you implement a Set that can only contain the digits 0..10?

Remember Radix Sort?

[07, 19, 61, 11, 14, 54, 01, 08]

0	1	2	3	4	5	6	7	8	9

Bukkits on 1’s place

insert(4)

boolean[] A:

0	F
1	F
2	F
3	F
4	F
5	F
6	F
7	F
8	F
9	F

insert(4)

boolean[] A:

0	F
1	F
2	F
3	F
4	T
5	F
6	F
7	F
8	F
9	F

insert(4)

insert(7)

boolean[] A:

0	F
1	F
2	F
3	F
4	T
5	F
6	F
7	F
8	F
9	F

insert(4)

insert(7)

boolean[] A:

0	F
1	F
2	F
3	F
4	T
5	F
6	F
7	T
8	F
9	F

insert(4)

insert(7)

insert(4)

boolean[] A:

0	F
1	F
2	F
3	F
4	T
5	F
6	F
7	T
8	F
9	F

Exercise

Write pseudocode for a **Set** that can only contain the digits 0..10.

```
public class DigitSet {
  boolean[] A[10];

  /** pre: 0 <= i < 10 */
  void insert(int i) {
    // your code
  }
  /** pre: 0 <= i < 10 */
  void contains(int i) {
    // your code
  }
```

0	F
1	F
2	F
3	F
4	T
5	F
6	F
7	T
8	F
9	F

Exercise

Write pseudocode for a **Set** that can only contain the digits 0..10.

```
public class DigitSet {
  boolean[] A[10];

  /** pre: 0 <= i < 10 */
  void insert(int i) {
        A[i]=true;
  }
  /** pre: 0 <= i < 10 */
  void contains(int i) {
       return A[i];
  }
```

0	F
1	F
2	F
3	F
4	T
5	F
6	F
7	T
8	F
9	F

Direct-Address Table

insert(4)

insert(7)

insert(4)

boolean[] A:

0	F
1	F
2	F
3	F
4	T
5	F
6	F
7	T
8	F
9	F

```
insert(i):
 A[i] = true

contains(i):
 return A[i]

remove(i):
 A[i] = false
```

Direct-Address Table

- This was easy because the Set contents came from a small, fixed space of possible values (0..10).

- Hash functions are the magic that lets us map *any* space of values onto a fixed space of integer values.

Reminder:
The Modulus Operator

`a % b` gives the remainder when dividing a by b:

```
12 % 8 => 4

24 % 10 => 4

4 % 10 => 4

28 % 14 => 0
```

Exercise

a % b gives the remainder when dividing a by b:

```
12 % 8   => 4
24 % 10  => 4
4 % 10   => 4
28 % 14  => 0
```

```
12 % 3  => 0
14 % 3  => 2
8 % 5   => 3
3 % 10  => 3
```

Hash Tables with Integers

How can we determine an index for **any** integer in a **fixed-sized** array?

- Modular arithmetic:
  store value k in the k % 10 bucket
  - (14 % 10) => 4

boolean[] A:

0	F
1	F
2	F
3	F
4	T
5	F
6	F
7	F
8	F
9	F

Hash Tables with Integers

How can we determine an index for **any integer** in a **fixed-sized** array?

- Modular arithmetic:
  store value k in the k % 10 bucket
  - (14 % 10) => 4
  - (10 % 10) => 0

boolean[] A:

0	T
1	F
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3	F
4	T
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6	F
7	F
8	F
9	F

Hash Tables with Integers

How can we determine an index for **any integer** in a **fixed-sized** array?

- Modular arithmetic:
  store value k in the k % 10 bucket
  - (14 % 10) => 4
  - (10 % 10) => 0
  - (1 % 10) => 1

boolean[] A:

0	T
1	T
2	F
3	F
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8	F
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Hash Tables with Integers

How can we determine an index for **any integer** in a **fixed-sized** array?

- Modular arithmetic:
  store value k in the k % 10 bucket
  - (14 % 10) => 4
  - (10 % 10) => 0
  - (1 % 10) => 1
  - (11 % 10) => 1

boolean[] A:

0	T
1	T
2	F
3	F
4	T
5	F
6	F
7	F
8	F
9	F

Hash Tables with Integers

How can we determine an index for **any integer** in a **fixed-sized** array?

- Modular arithmetic:
  store value k in the k % 10 bucket
  - (14 % 10) => 4
  - (10 % 10) => 0
  - (1 % 10) => 1
  - (11 % 10) => 1

uh oh…

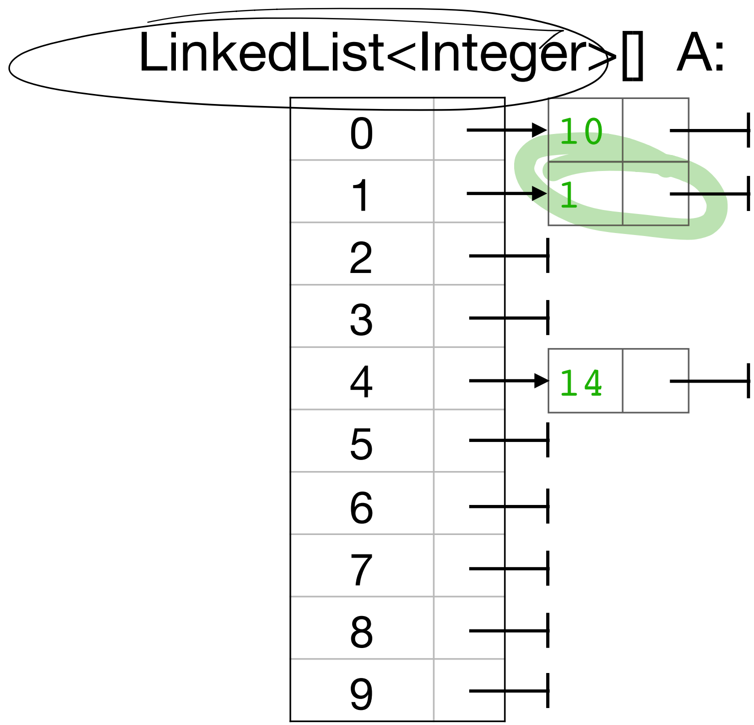
boolean[] A:

0	T
1	T
2	F
3	F
4	T
5	F
6	F
7	F
8	F
9	F

Hash Tables with Integers: Collisions

- Modular arithmetic:
  store value k in the k % 10 bucket

  - (14 % 10) => 4
  - (10 % 10) => 0
  - (1 % 10) => 1
  - (11 % 10) => 1



Hash Tables with Integers: Collisions

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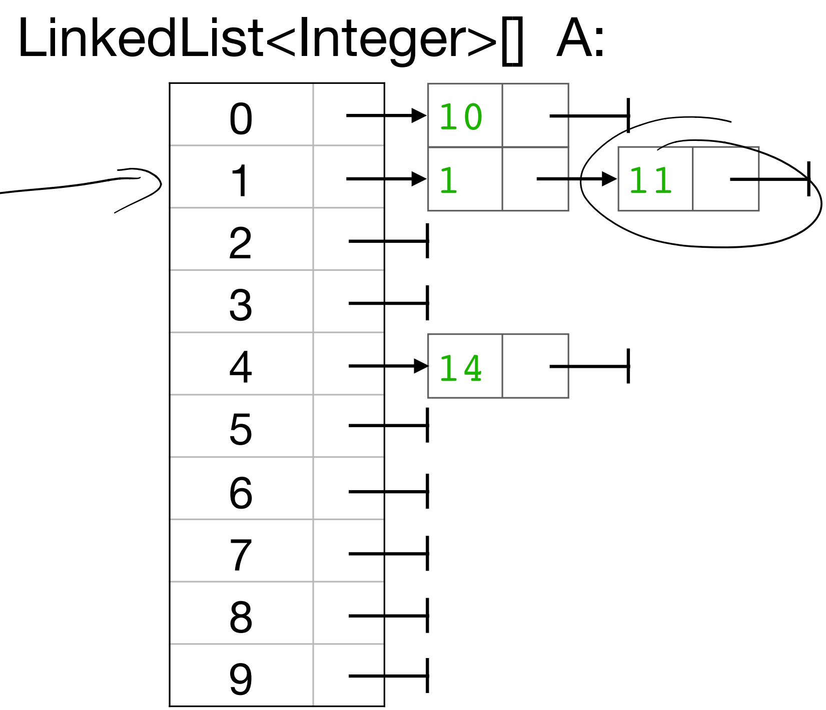
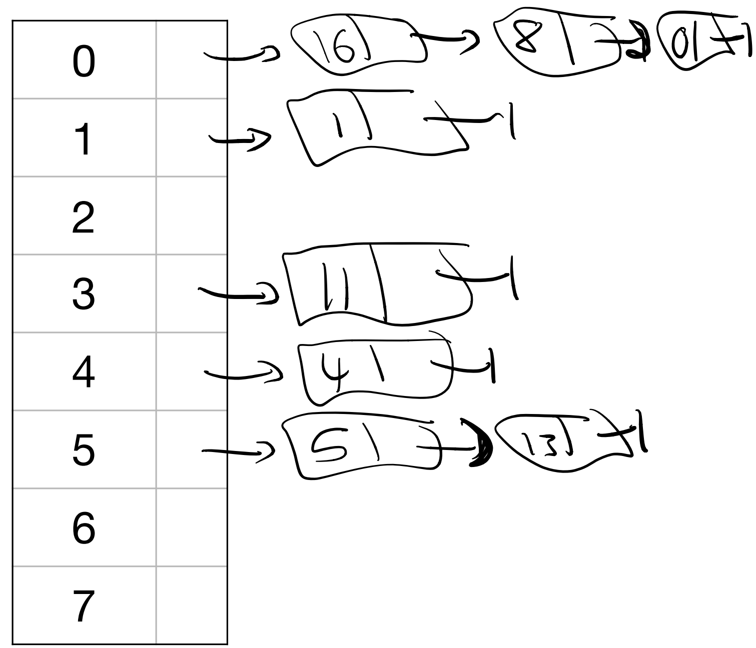


Exercise

Insert the following values into a table of size 8: `1, 11, 16, 4, 5, 8, 0, 13`

- Use h(k) = k % 8 as the hash function.
- Use chaining for collision resolution.

LinkedList<Integer>[] A:



HashSet<T>

```
/** insert value into the set. return false if the
value was already in the set, true otherwise */
boolean insert(T value) {
  int h = hash(value)
  search the list at A[h] for value
  if found:
     return false
  else:
     insert value into A[h] and return true
}

/** return true if value is in the set,
  * false otherwise */
boolean contains(T value) { … }

/** insert value into the set. return true if the
  * value was in the set, false otherwise */
boolean remove(T value) { … }
```

HashSet<T>: What's the runtime?

```
/** insert value into the set. return false if the
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boolean insert(T value) {
  int h = hash(value)
  search the list at A[h] for value
  if found:
     return false
  else:
     insert value into A[h] and return true
}
```

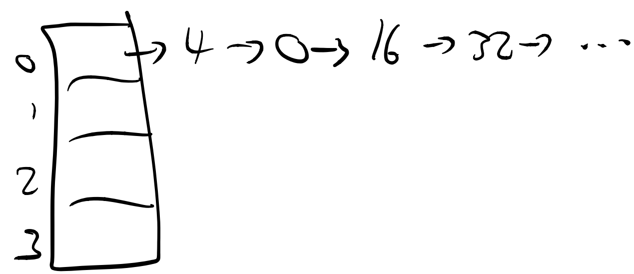
O(??) → O(1)

O(1)

O(1)

h(x) = x % 4

4, 0, 16, 32, 8, 12, 24, ...



HashSet<T>: What's the runtime?

```
/** insert value into the set. return false if the
value was already in the set, true otherwise */
boolean insert(T value) {
  int h = hash(value)                        O(1)
  search the list at A[h] for value          O(length of list)
  if found:
     return false                            O(1)
  else:
     insert value into A[h] and return true O(1)
}
```

Object.hashCode()

HashSet<T>: What's the runtime?

All operations require searching a single bucket and doing some other stuff that runs in O(1).

```
/** return true if value is in the set,
  * false otherwise */
boolean contains(T value) { … }

/** remove value from the set. return true if the
  * value was in the set, false otherwise */
boolean remove(T value) { … }
```

Hash Tables: Load Factor

Hash Tables: Load Factor

How full is your hash table?

Load factor $\lambda = \dfrac{\text{\# entries in table}}{\text{size of the array}}$

With a perfectly-behaved hash function, average bucket size is λ, so average-case runtime is $O(\lambda)$.

Exercise

- What is the load factor of the hash table you built, after all the insertions?

Let's talk about A3.

A3 has 4 phases.

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A3 has 4 phases.

It isn't so bad:

- total lines of code is probably <= A2
- nothing here is as tricky as AVL rebalance
- you're given unit tests

A3 has 4 phases.

0. Write an ArrayList clone

A3 has 4 phases.

0. Write an ArrayList clone

   (done in Lab 5!)

A3 has 3 phases.

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1. Write a min-heap to implement a priority queue with operations:
   - `boolean add(V value, P priority)`
   - `V peek();`
   - `V poll();`

A3 has 3 phases.

use AList to handle growing the array!

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A3 has 3 phases.

use AList to handle growing the array!

1. Write a min-heap to implement a priority queue with operations:
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   - `V poll();`
2. Write a hash table implementation of Map.

A3 has 3 phases.

use AList to handle growing the array!

1. Write a min-heap to implement a priority queue with operations:
   - `boolean add(V value, P priority)`
   - `V peek();`
   - `V poll();`
2. Write a hash table implementation of Map.
3. Use the Map to augment the heap, making the following operations efficient:
   - `boolean contains(V v);`
   - `void changePriority(V v, P newP);`

A3 has 3 phases.

use AList to handle growing the array!

1. Write a min-heap to implement a priority queue with operations:
   - `boolean add(V value, P priority)`
   - `V peek();`
   - `V poll();`

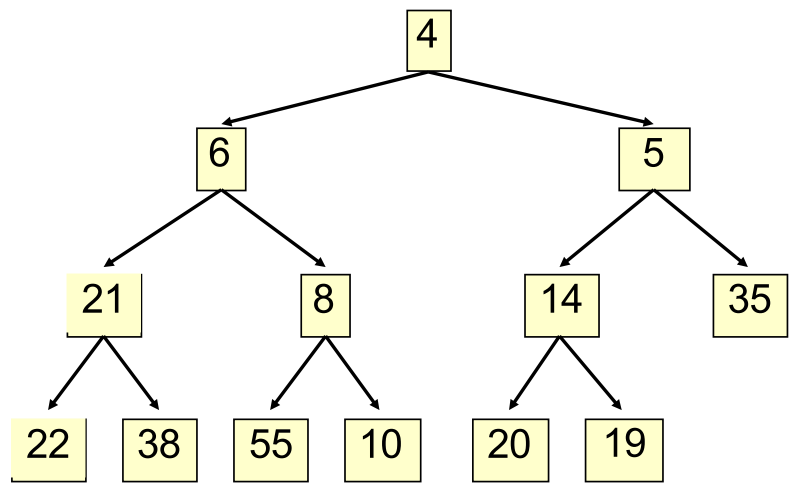
(**not** using AList to handle growing the array)

2. Write a hash table implementation of Map.
3. Use the Map to augment the heap, making the following operations efficient:
   - `boolean contains(V v);`
   - `void changePriority(V v, P newP);`

Phase 3 - Hash your Heap

In Phase 1 Heap:

- `contains` requires searching the whole tree.
- `changePriority` requires searching the whole tree, then bubbling down or up.



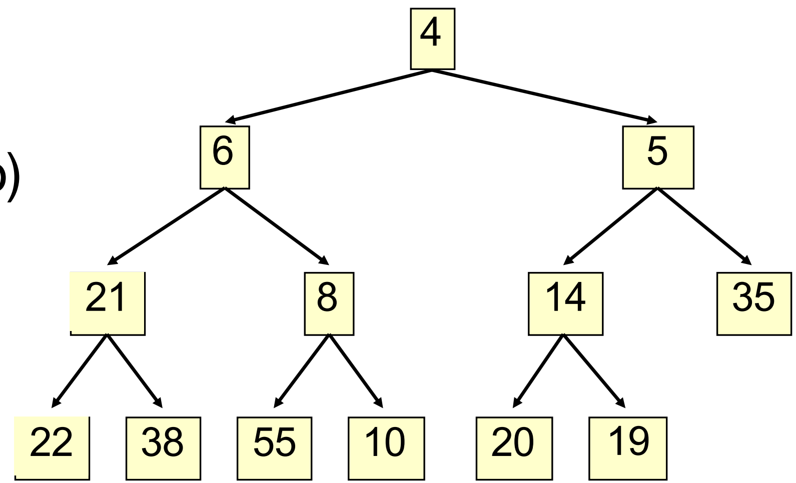
Phase 3 - Hash your Heap

In Phase 3 Heap:

- Each heap value is stored in the heap **and** in a HashTable that tracks its index in the heap.

HashTable<V, Integer>:

value	i (index in heap)
4	0
8	4
6	1
38	8
35	6
21	3
10	10
[illegible]	[illegible]



0 1 2 3 4 5 6 7 8 9 10 11 12

Heap: [4 6 5 21 8 14 35 22 38 55 10 20 19]

Phase 3 - Hash your Heap

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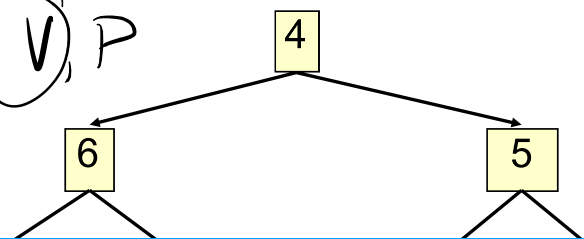


To maximize confusion:
- The hash table maps Heap *values* to heap *indices*.
- The hash table's *keys* are the heap's *values*

	0	1	2	3	4	5	6	7	8	9	10	11	12	
Heap: [	4	6	5	21	8	14	35	22	38	55	10	20	19	]

Phase 3 - Hash your Heap

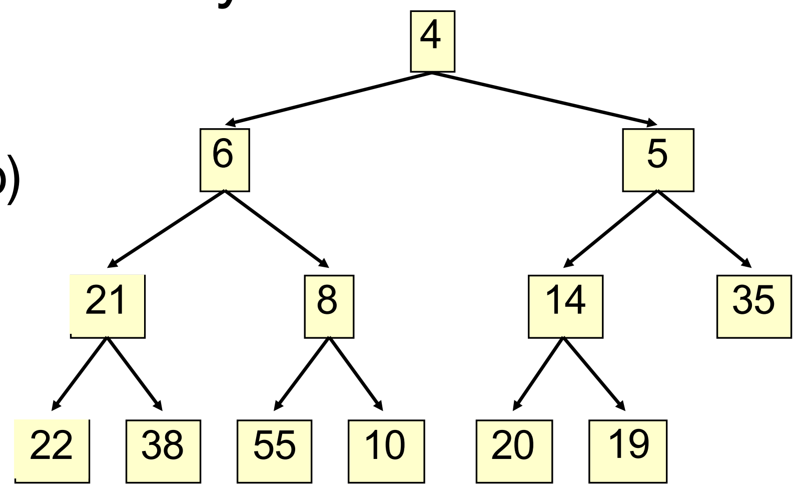
In Phase 3 Heap:

`boolean contains(V v):`

true iff map contains key v

HashTable<V, Integer>:

value	i (index in heap)
4	0
8	4
6	1
38	8
35	6
21	3
10	10
19	12



0 1 2 3 4 5 6 7 8 9 10 11 12

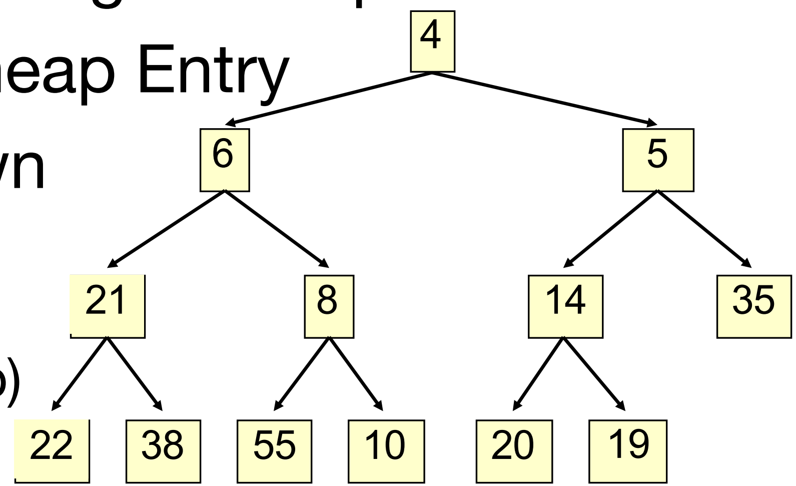
Heap: [4 6 5 21 8 14 35 22 38 55 10 20 19]

Phase 3 - Hash your Heap

In Phase 3 Heap:

`void changePriority(V v, P newP):`

- find where `v` lives using the map
- change priority of heap Entry
- bubble it up or down



HashTable<V, Integer>:

value	i (index in heap)
4	0
8	4
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	0	1	2	3	4	5	6	7	8	9	10	11	12
Heap: [	4	6	5	21	8	14	35	22	38	55	10	20	19]