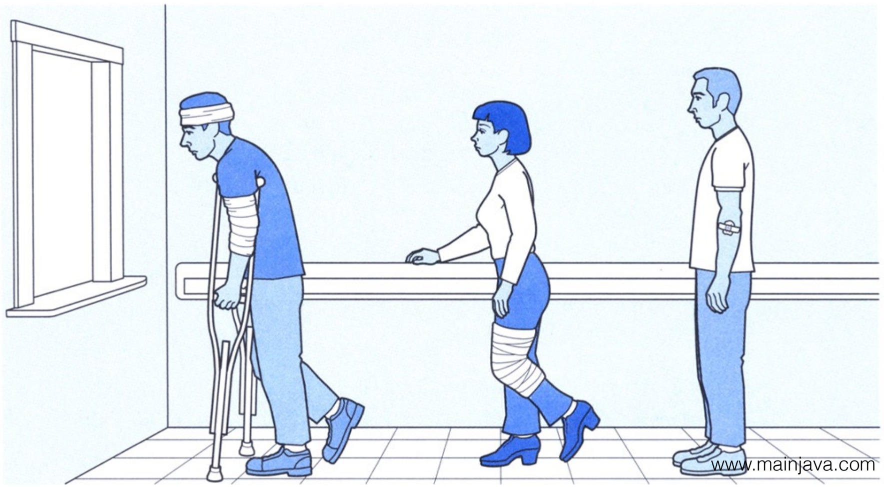


CSCI 241

Lecture 15:
Priority Queues
Heaps

There will be Socrative today!

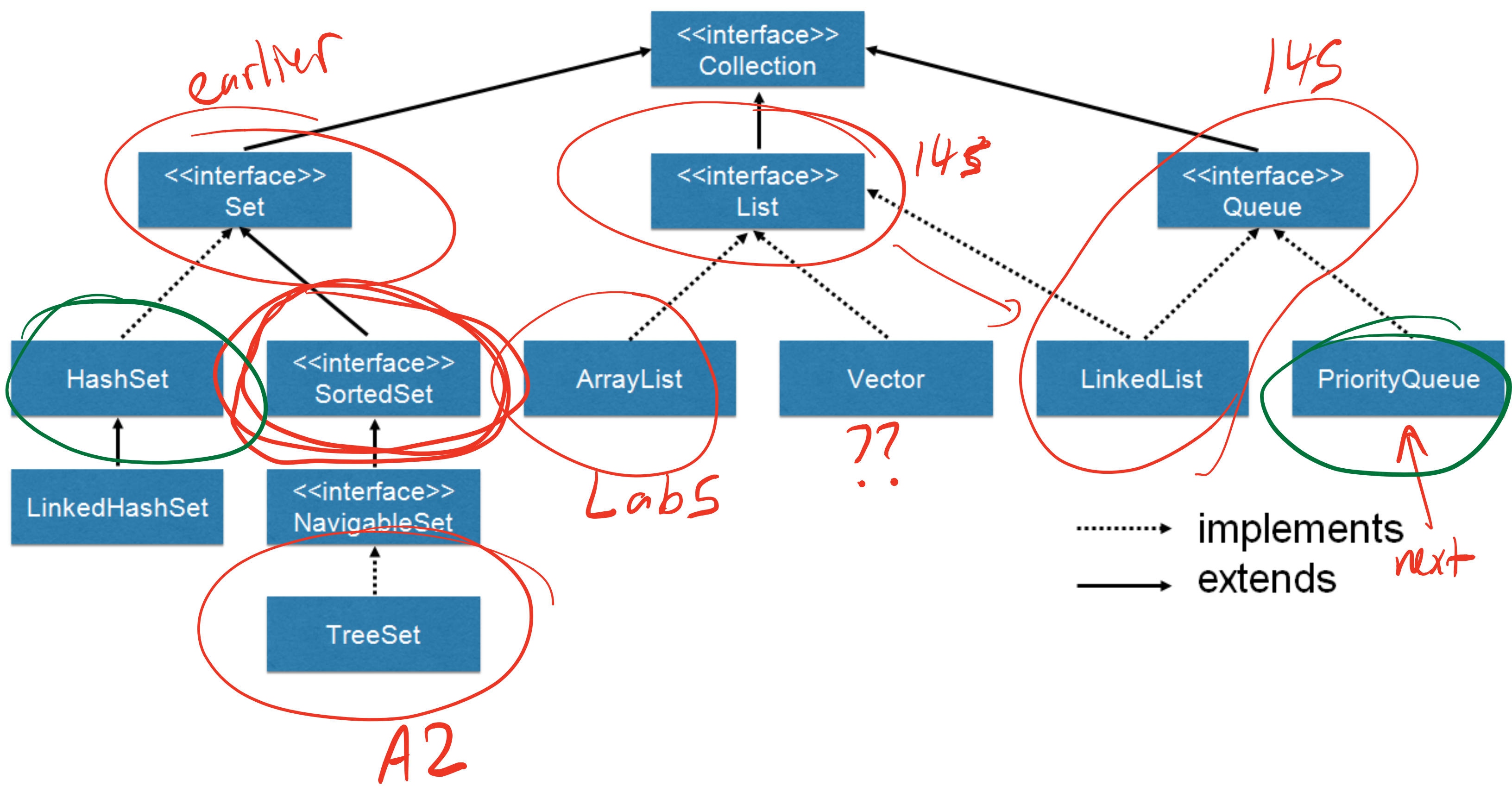
Announcements

- Quiz today: the usual
- A2 is due Monday night

Goals

- Understand the purpose and interface of the Priority Queue ADT.
- Know the definition and properties of a heap.
- Know how heaps are stored in practice.
- Know how to perform (on paper) and implement (in code) add, peek, and poll.

Collection Interface

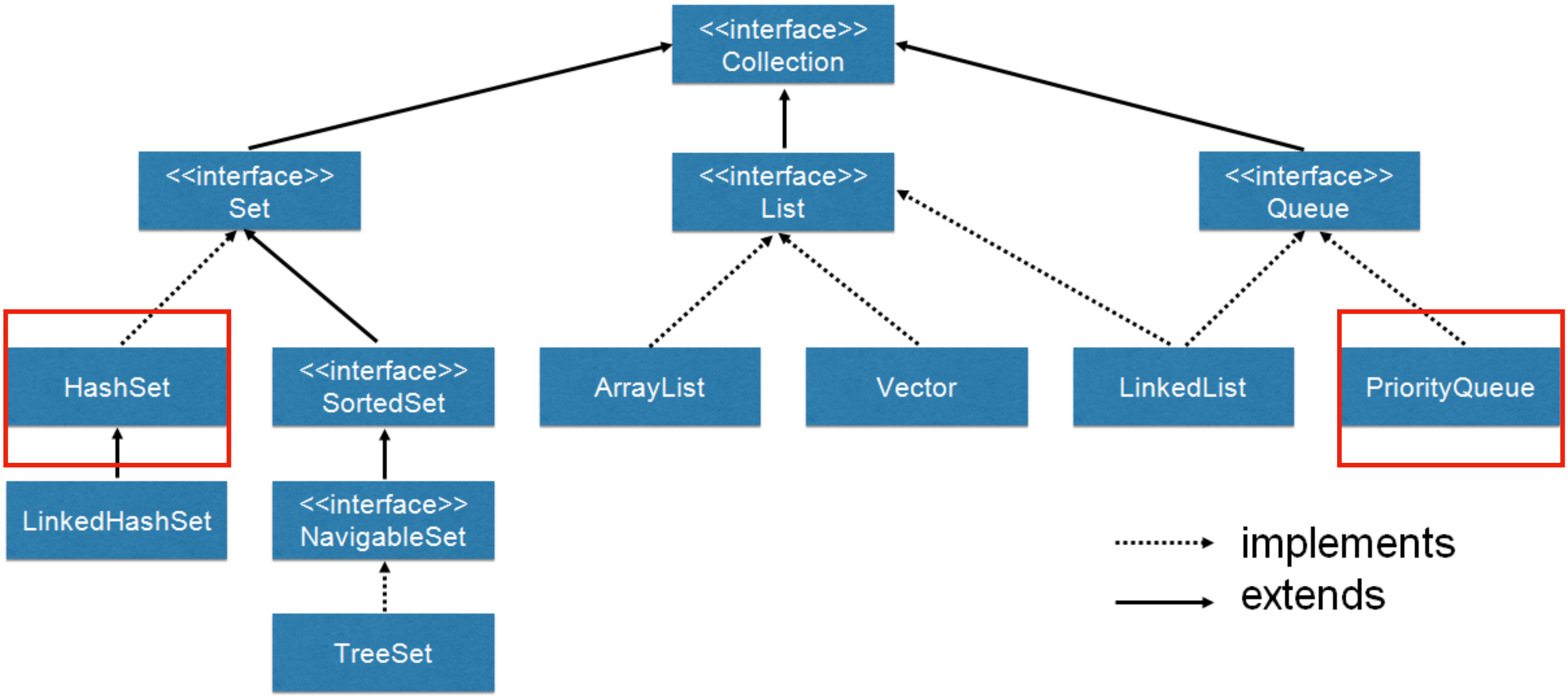


Abstract Data Types

- **interface** List defines an “abstract data type”
- It has public methods: add, get, remove, …
- Various classes **implement** List:

Class:	ArrayList	LinkedList
Backing storage:	array	chained nodes
add(i, val)	O(n)	O(n)
add(0, val)	O(n)	O(1)
add(n, val)	O(1)	O(1)
get(i)	O(1)	O(n)
get(0)	O(1)	O(1)
get(n)	O(1)	O(1)

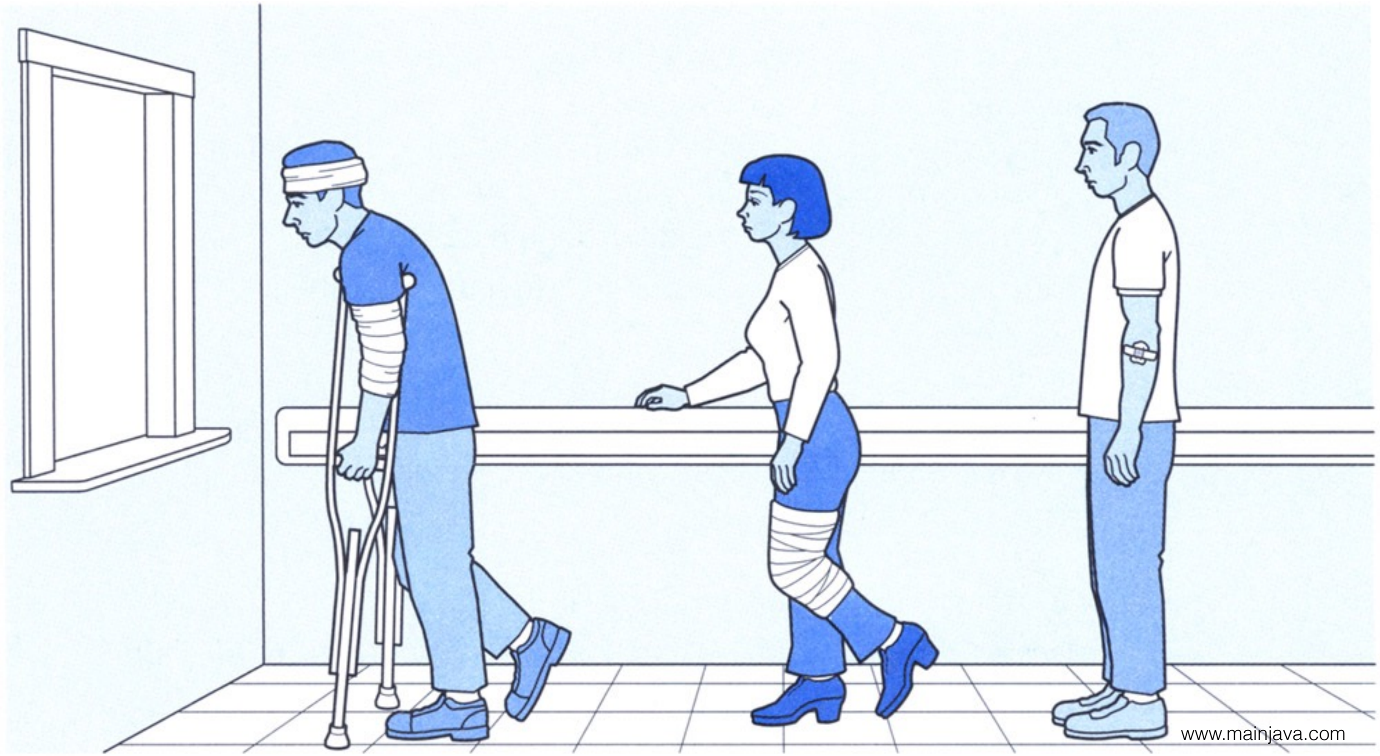
Collection Interface



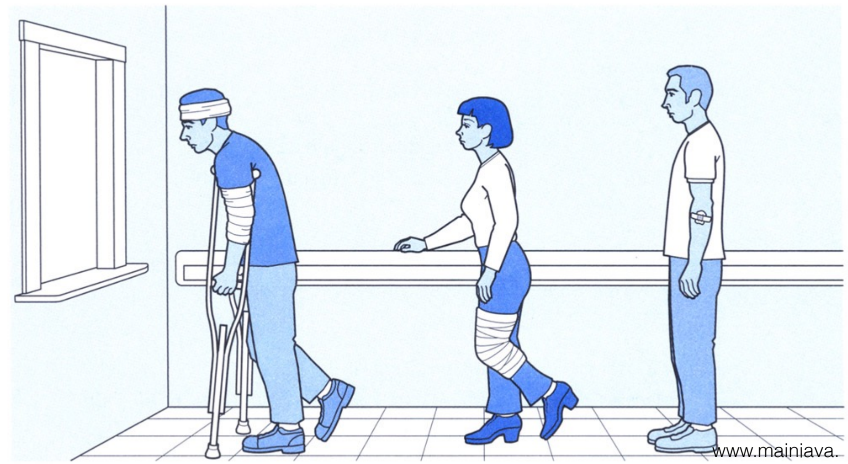
Our next two topics (and the subject of A3):

- **Priority Queues**
- Hashing, HashSets, **HashMaps**

Priority Queues



Queue vs Priority Queue



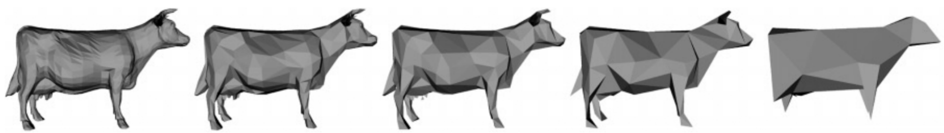
add (enqueue):
inserts an item into the queue

remove (dequeue):
removes the first item to be inserted (FIFO)

add (enqueue):
inserts an item into the queue

remove (poll):
remove the **highest-priority** item from the queue

Uses for Priority Queues



- Computer Graphics: mesh simplification
- Graph algorithms: shortest paths, spanning trees
- Statistics: maintain largest M values in a sequence
- Graphics and simulation: "next time of contact" for colliding bodies
- AI Path Planning: A* search (e.g., Map directions)
- Operating systems: load balancing, interrupt handling
- Discrete optimization: bin packing, scheduling

Priority Queues

Like a Queue, but:

- Each item in the queue has an associated **priority** which is some type that implements **Comparable**
- **remove()** returns item with the "highest priority"
  - or, the element with the "**smallest**" associated priority value
  - Ties are broken arbitrarily

```
interface PriorityQueue<E> {
 boolean add(E e); // insert e
 E peek(); // return min element
 E poll(); // remove/return min element
 void clear();
 boolean contains(E e);
 boolean remove(E e);
 int size();
 Iterator<E> iterator();
}
```

Priority Queue: LinkedList implementation

An unordered list:

- **add()** - new element at front of list - O(1)
- **poll()** - requires searching the list -
- **peek()** - requires searching the list -

An ordered list:

- **add()** - requires searching the list - O(n)
- **poll()** - min element is kept at front -
- **peek()** - min element is kept at front -

Exercise: fill in all the runtimes.

Priority Queue: LinkedList implementation

An unordered list:

- **add()** - new element at front of list - O(1)
- **poll()** - requires searching the list - O(n)
- **peek()** - requires searching the list - O(n)

An ordered list:

- **add()** - requires searching the list - O(n)
- **poll()** - min element is kept at front - O(1)
- **peek()** - min element is kept at front - O(1)

Question to ponder:

What would be the runtime of add, peek, and poll if you implement a Priority Queue using a BST?

What about an AVL tree?

Priority Queue: heap implementation

- A heap is a **concrete** data structure that can be used to **implement** a Priority Queue

- Better runtime complexity than either list implementation:
  - **peek()** is O(1)
  - **poll()** is O(log n)
  - **add()** is O(log n)

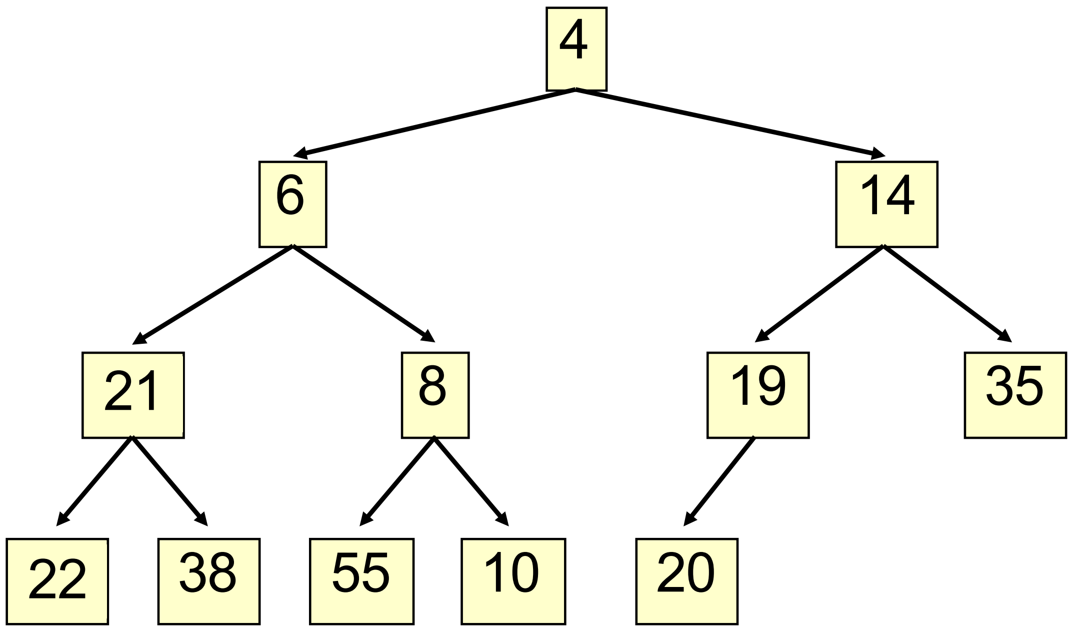
log n	n
0	1
1	10
2	100
3	1000

- Not to be confused with *heap memory*, where the Java virtual machine allocates space for objects – different usage of the word heap.

A heap is a special binary tree with two additional properties.

A heap is a special binary tree.

1. **Heap Order Invariant:**
Each element ≥ its parent.



A heap is a special binary tree.

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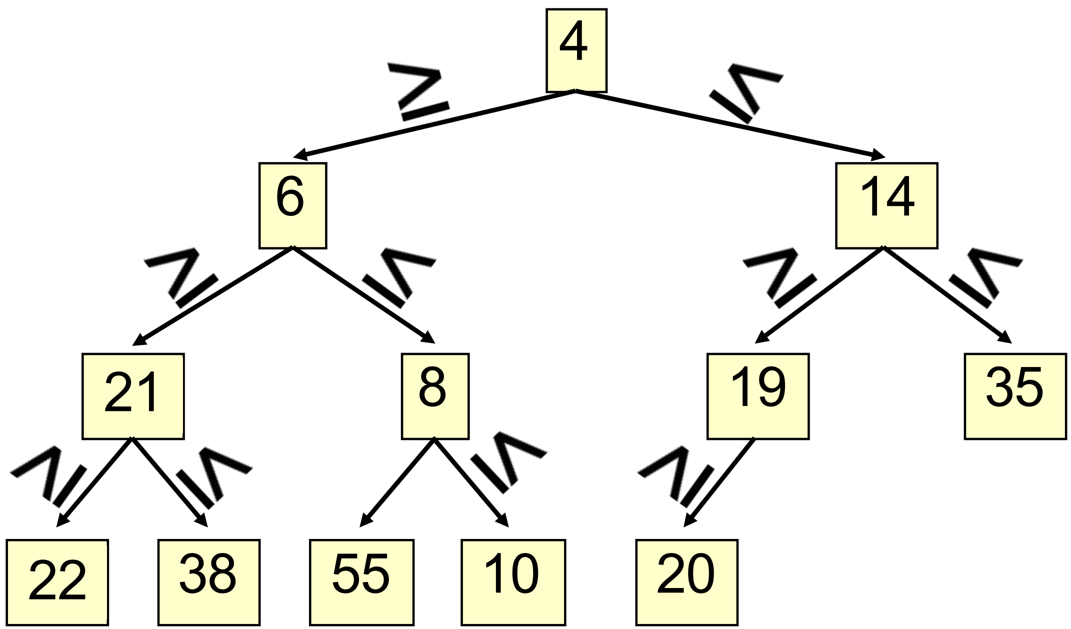

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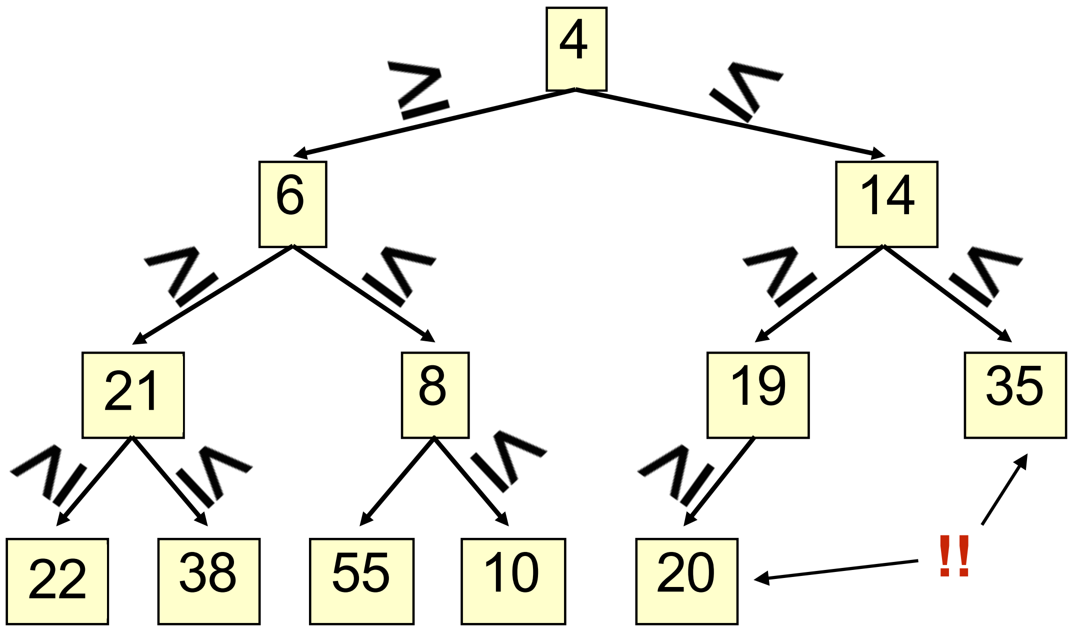
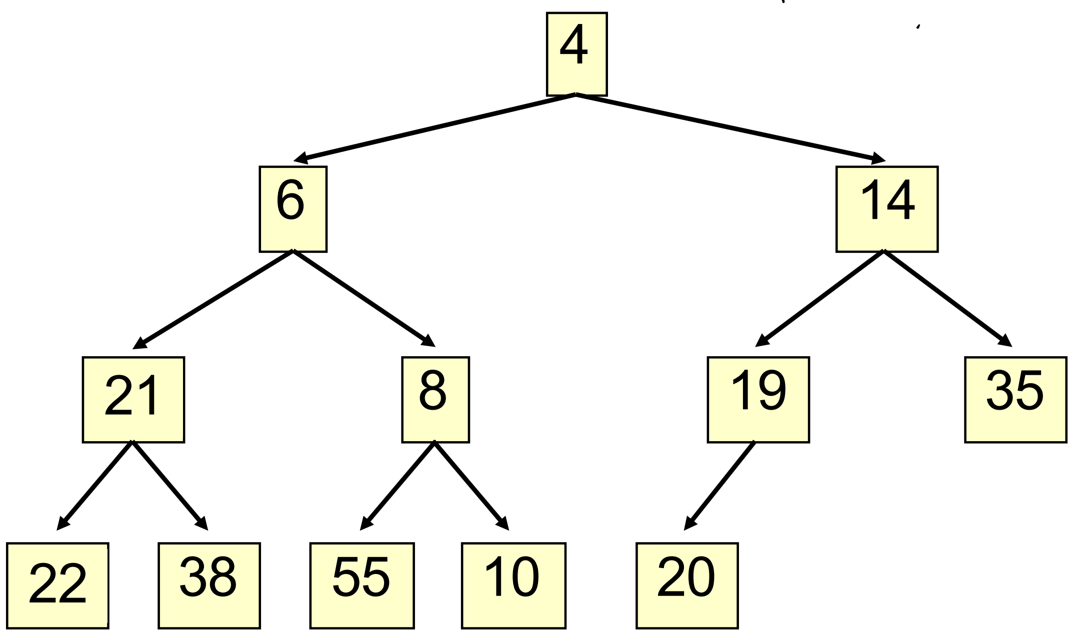


A heap is a special binary tree.

2. **Complete:** no holes!

- All levels except the last are full.
- Nodes in last level are as far left as possible.



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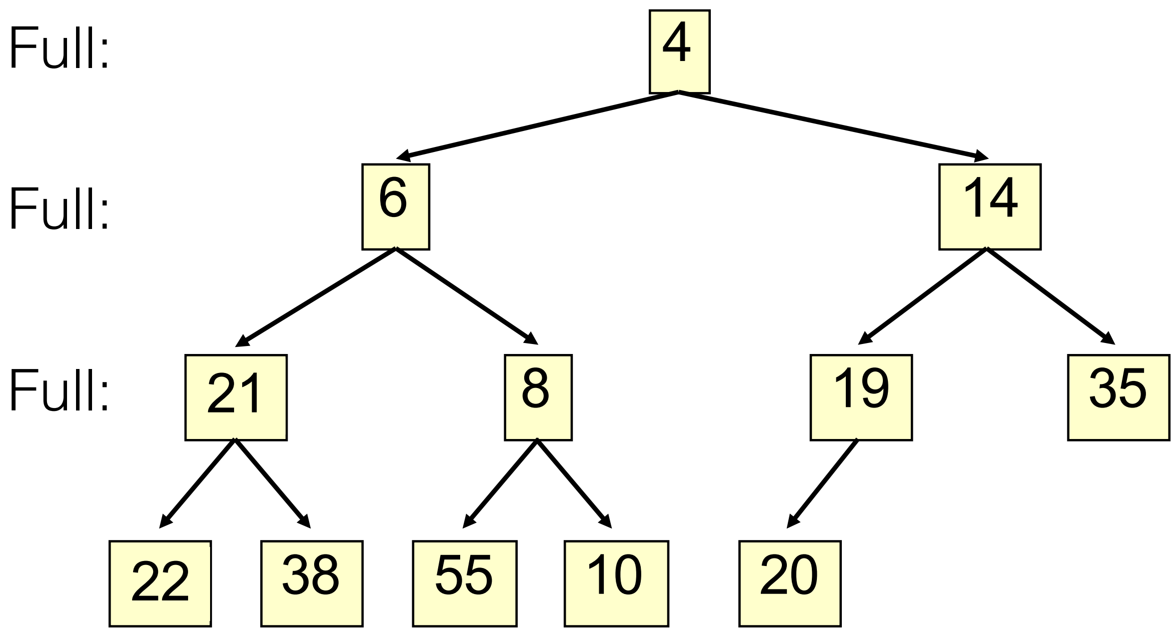

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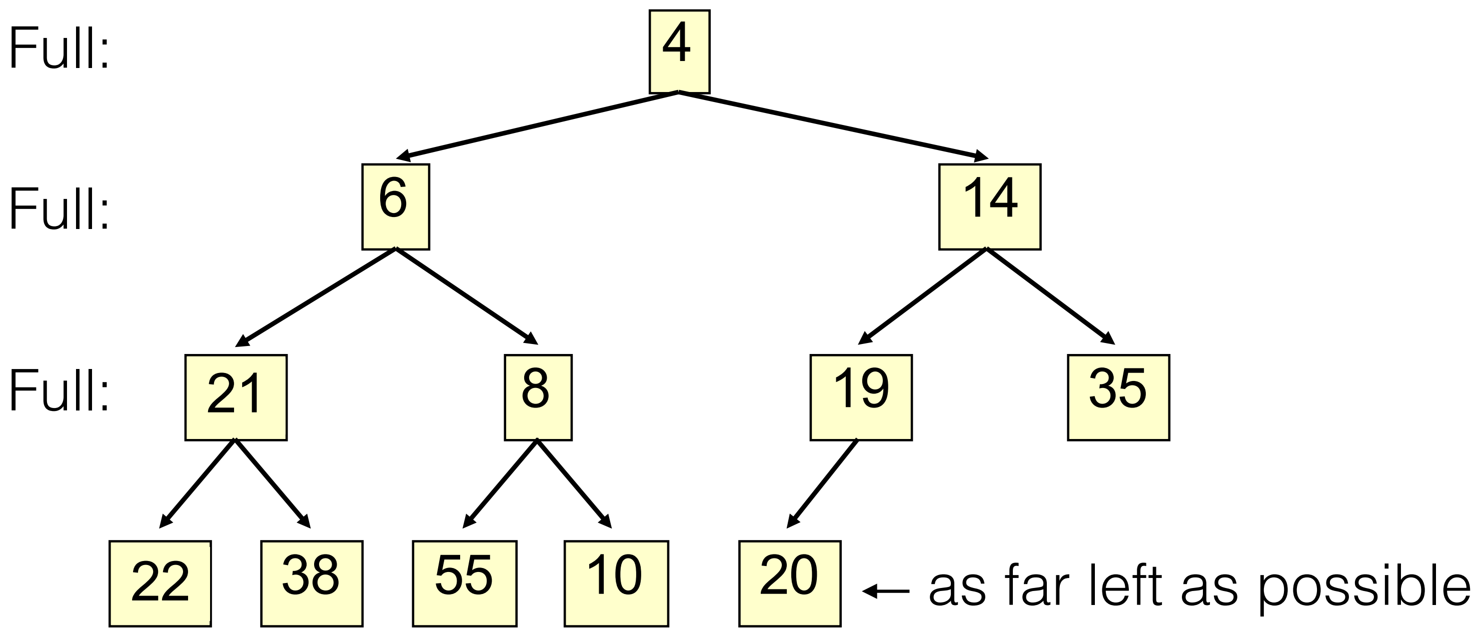
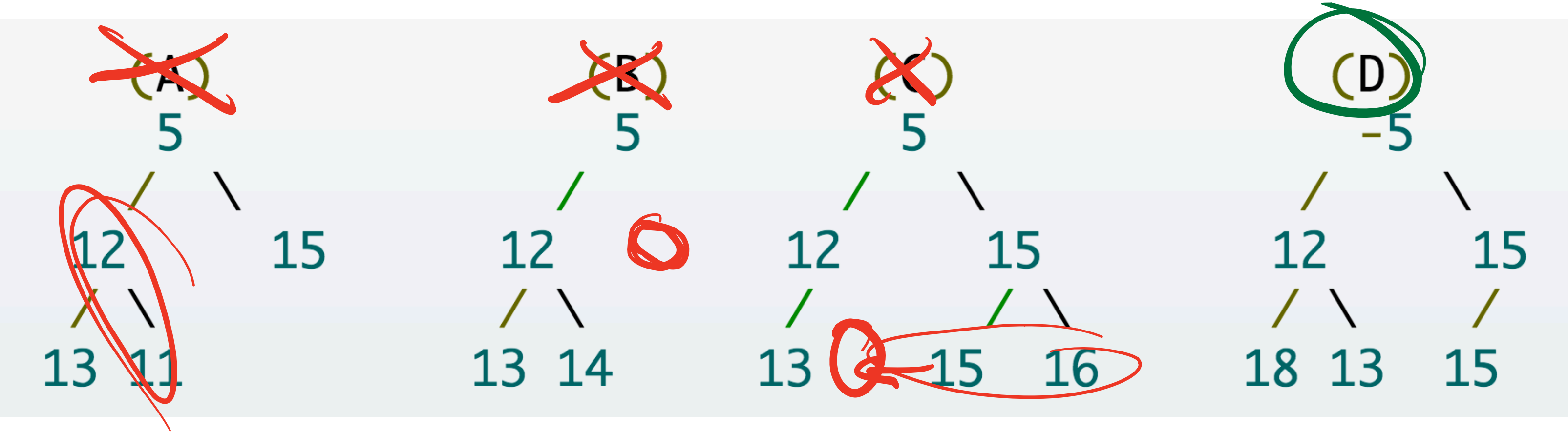


Heap it real



Which of these are valid heaps?

1. Each element >= its parent.
2. The tree is complete

Heap it real.

```
   (A)              (B)            (C)              (D)
    5                5              5               -5
   / \              /              / \             /   \
 12     15        12             12     15       12     15
 / \              / \            /      / \      / \    /
13 11            13 14          13     15  16   18 13  15
```

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Heap it real.

```
   (A)
    5
   /  \
 12     15
 / \
13 11
```

```
   (B)
    5
   /
 12
 / \
13 14
```

```
     (C)
      5
    /   \
  12     15
  /      / \
13     15   16
```

```
      (D)
       -5
     /    \
   12      15
   / \     /
 18  13  15
```

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13 11
```

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      5
     /
   12
  / \
13  14
```

```
      (C)
       5
     /   \
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  /      / \
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       5
      /
    12
    / \
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      (C)
       5
    /     \
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  /        / \
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       -5
    /      \
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  / \       /
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       5
      /
    12
   / \
  13 14
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      / \
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   /      / \
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      -5
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```

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Heap operations

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interface PriorityQueue<E> {
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 E peek(); // return min element
 E poll(); // remove/return min element
 void clear();
 boolean contains(E e);
 boolean remove(E e);
 int size();
 Iterator<E> iterator();
}
```

```
void add(E e);
```

Algorithm:

- Add e in the wrong place
- While e is in the wrong place
  - move e towards the right place

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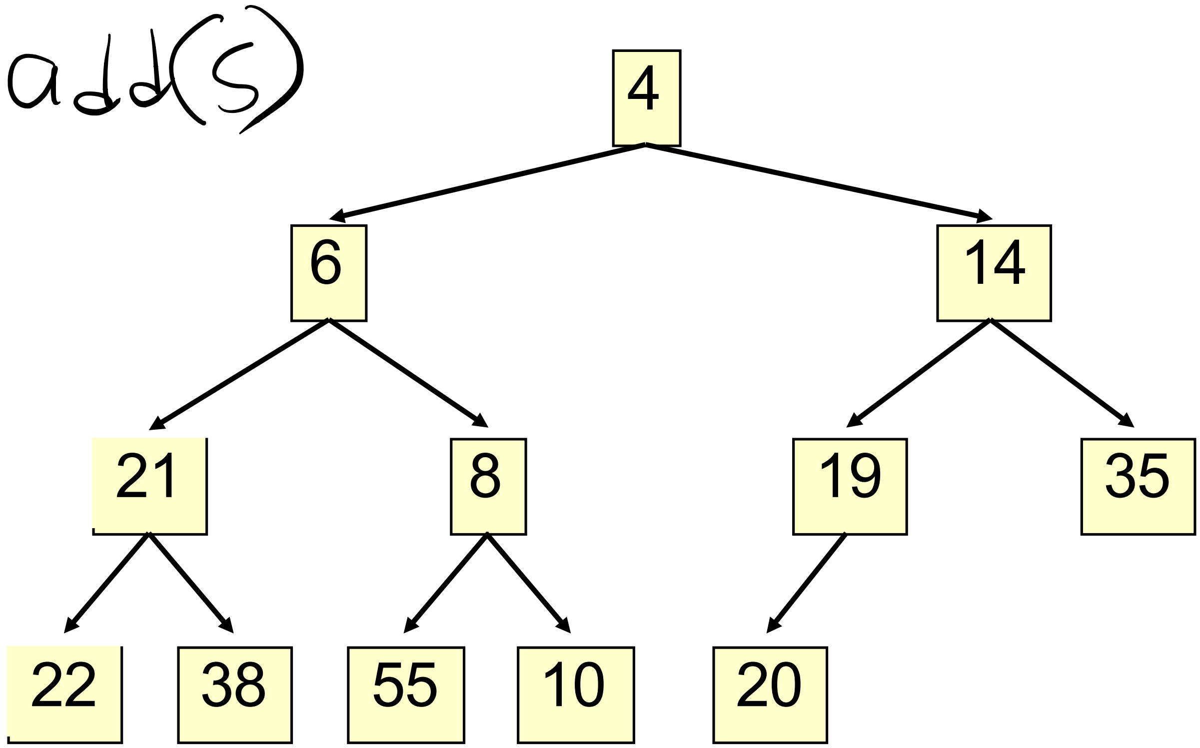

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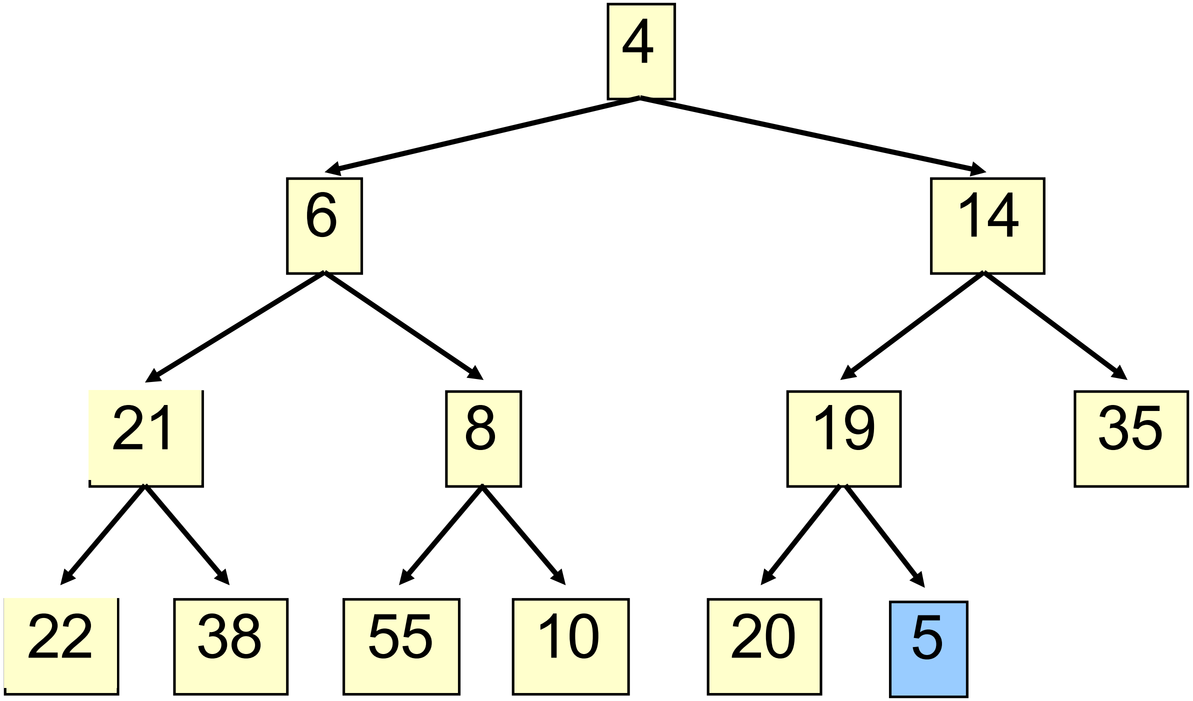

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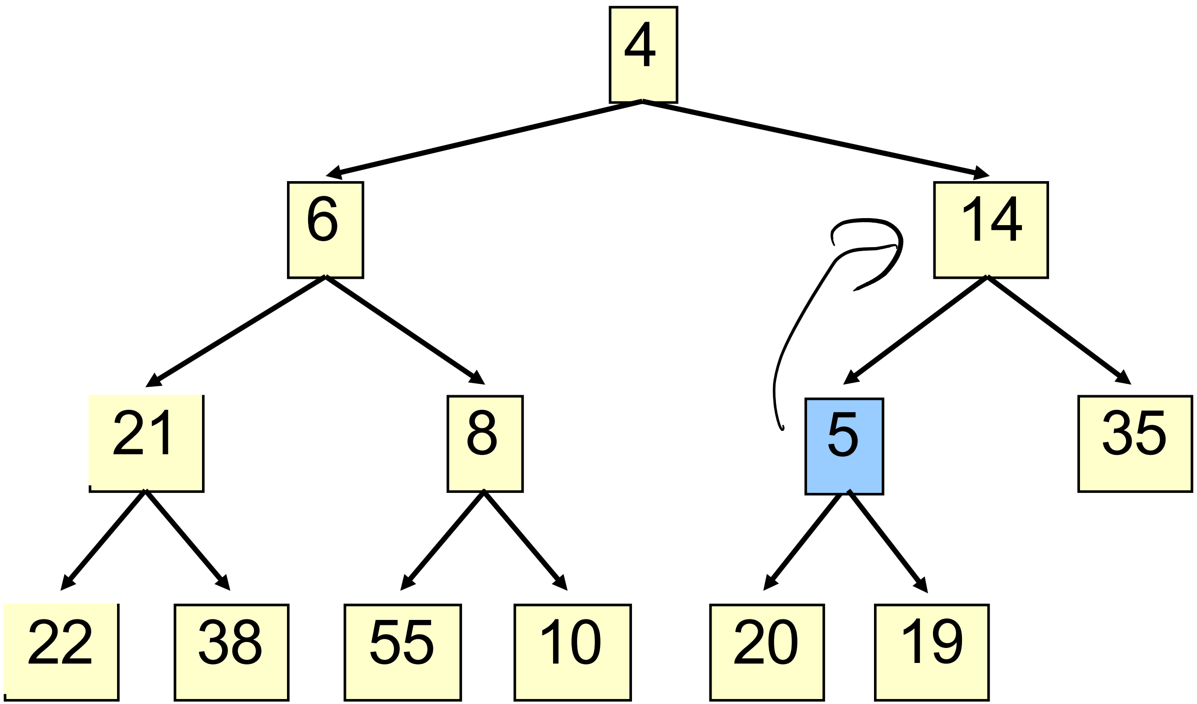

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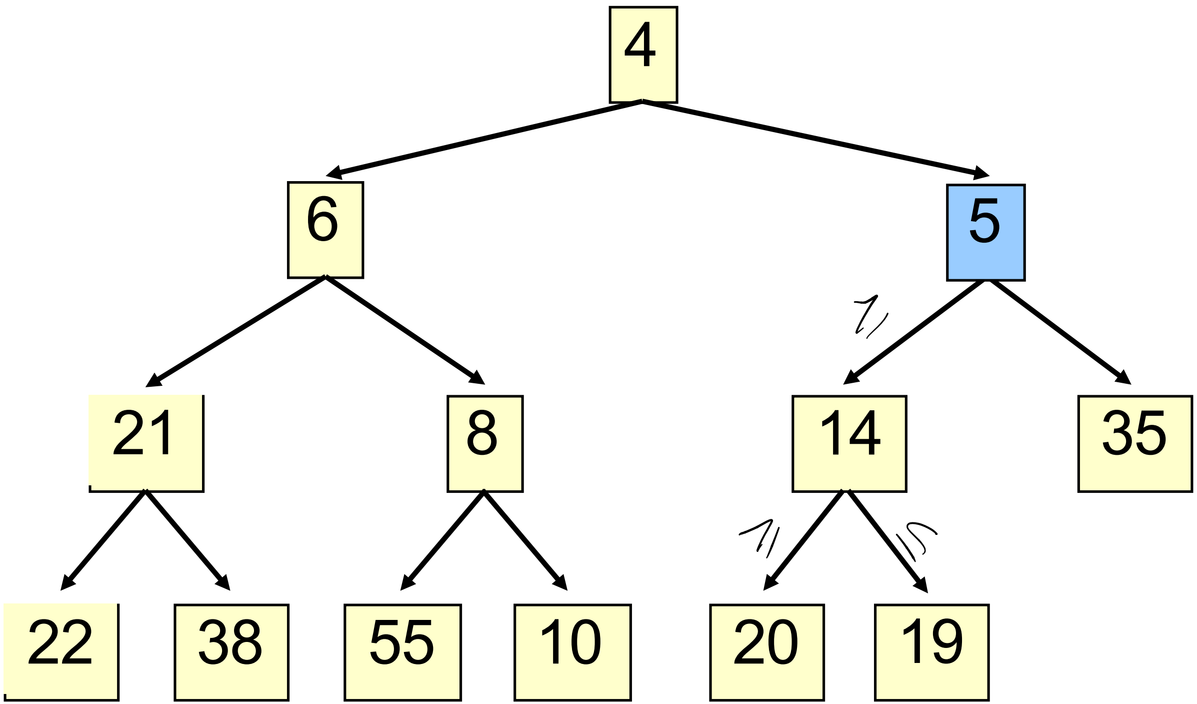

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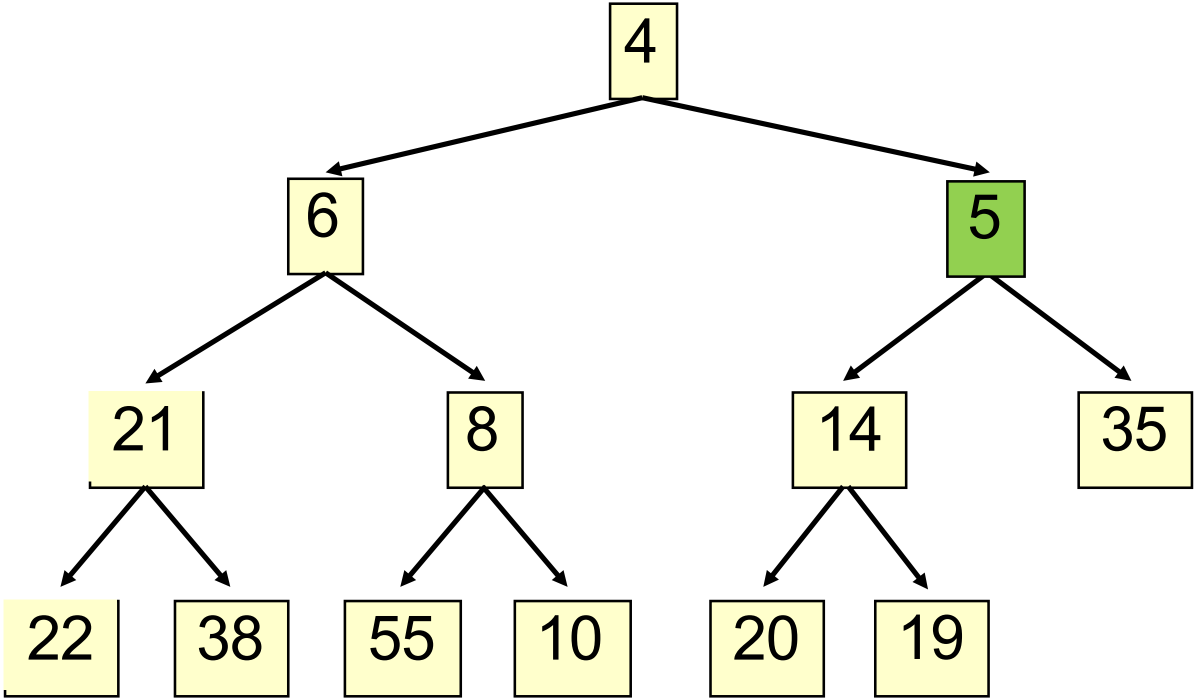

```
void add(E e);
```

Algorithm:

- Add e in the wrong place (the leftmost empty leaf)
- While e is in the wrong place (it is less than its parent)
  - move e towards the right place (swap with parent)

The heap invariant is maintained!

Runtime?

If **k** is less than **h**, the height of the tree, how many nodes are at depth **k**?

A. We can't know for sure

B. 2^k

C. 2^{k-1}

D. $2^k - 1$

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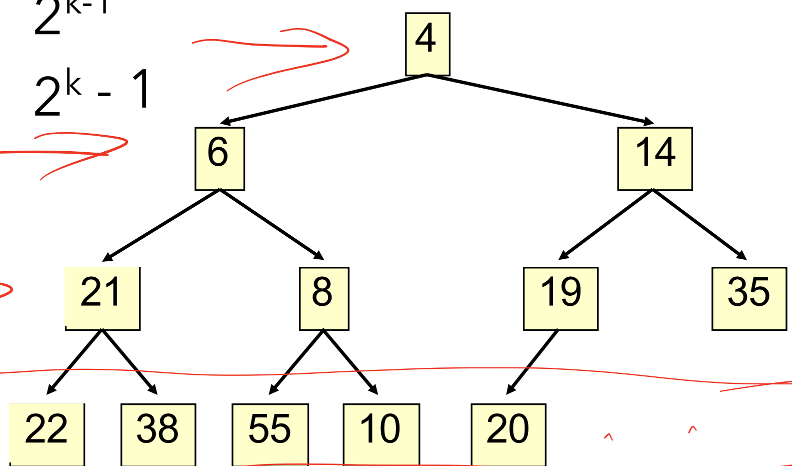

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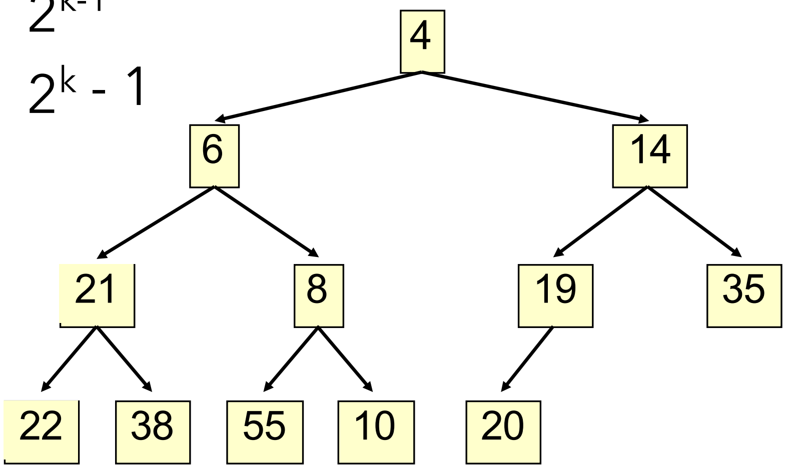

Depth	Nodes
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
…	
k	

So... runtime?

$O(\text{swaps}) \cdot \text{runtime of swap}$

$O(h) \quad O(1)$

↓

$O(h)$

h is $O(\log n)$

add(e) is $O(\log n)$!

Runtime.

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- O(number of swap/bubble operations) = O(height)

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- Complete => balanced => h is **O(log n)**

Runtime.

- O(number of swap/bubble operations) = O(height)
- Complete => balanced => h is **O(log n)**
- Maximum number of swaps is O(log n)

add(e)

Algorithm:

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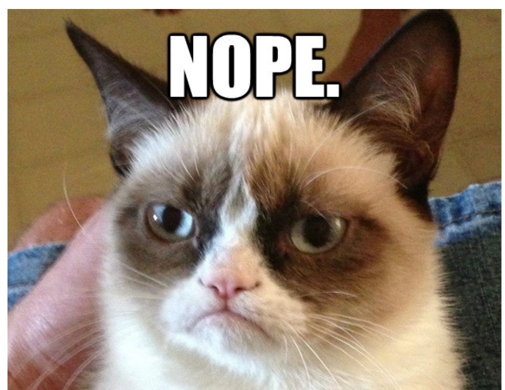
Implementing Heaps

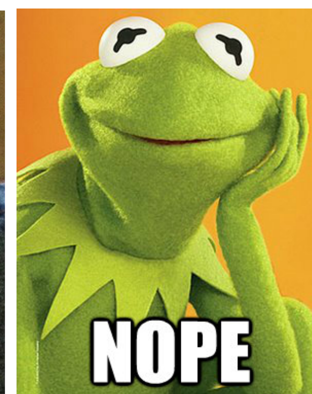
Implementing Heaps

```
public class HeapNode {
    private int value;
    private HeapNode left;
    private HeapNode right;
    ...
}
public class Heap {
    HeapNode root;
    ...
```

Implementing Heaps

```
public class HeapNope {
   private int value;
   private HeapNope left;
   private HeapNope right;
   ...
}
```





Implementing Heaps

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   private HeapNope right;
   ...
}
```

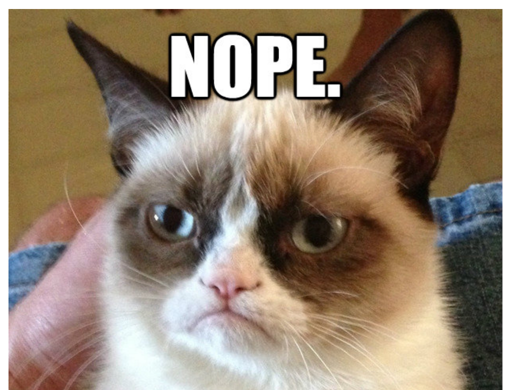

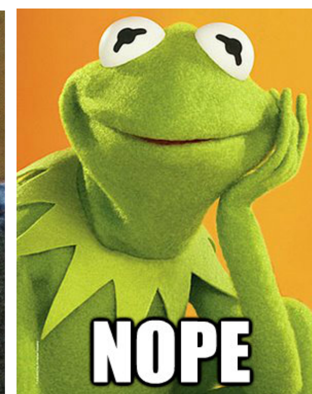

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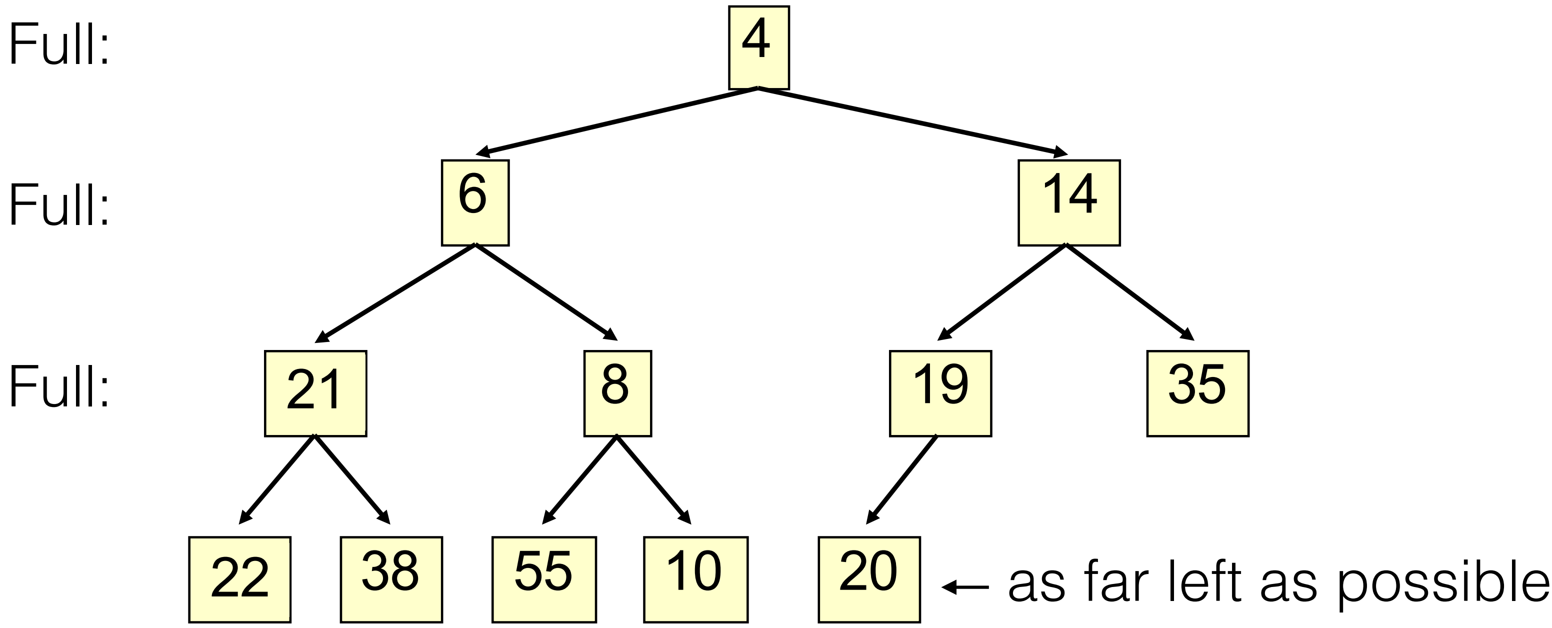
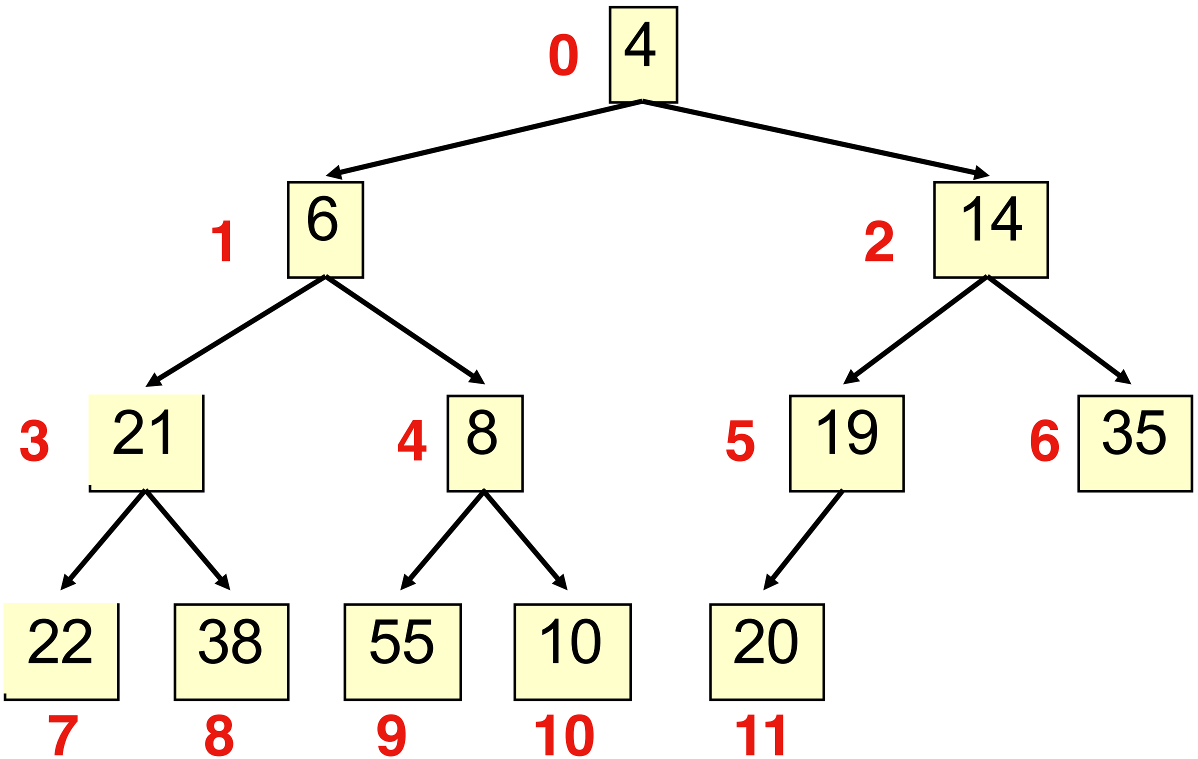
2. **Complete:** no holes!



Numbering Nodes

Level-order traversal:



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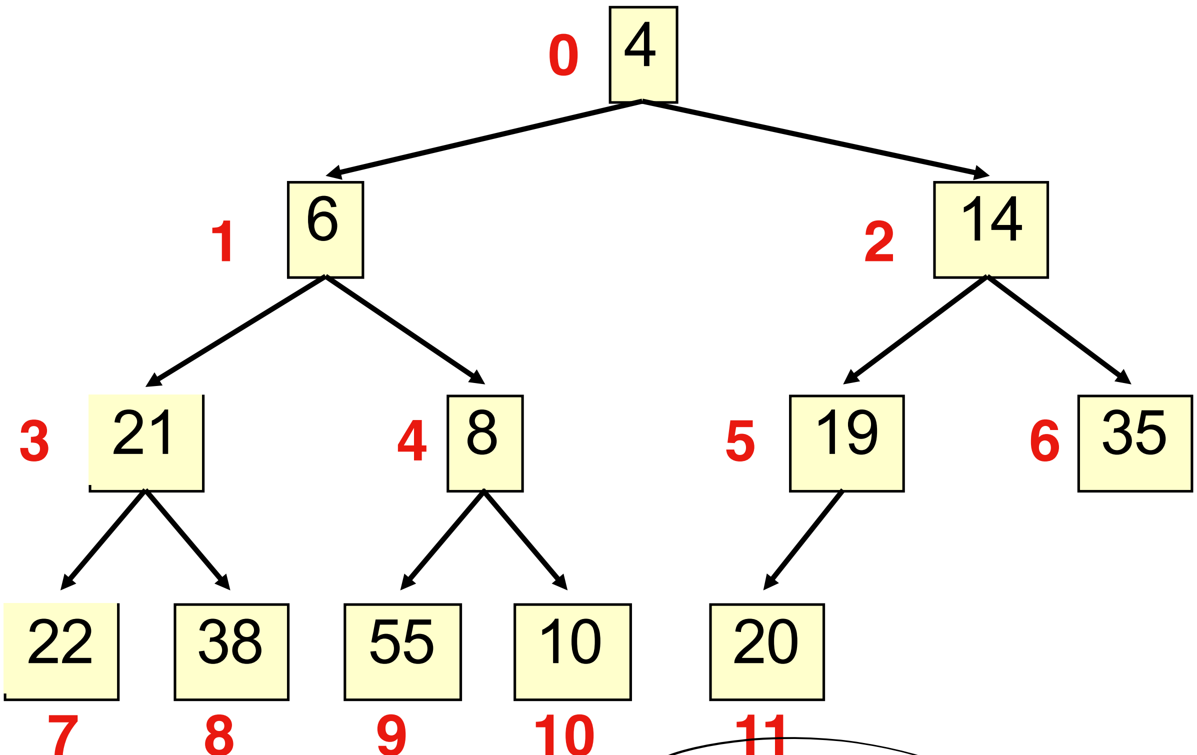
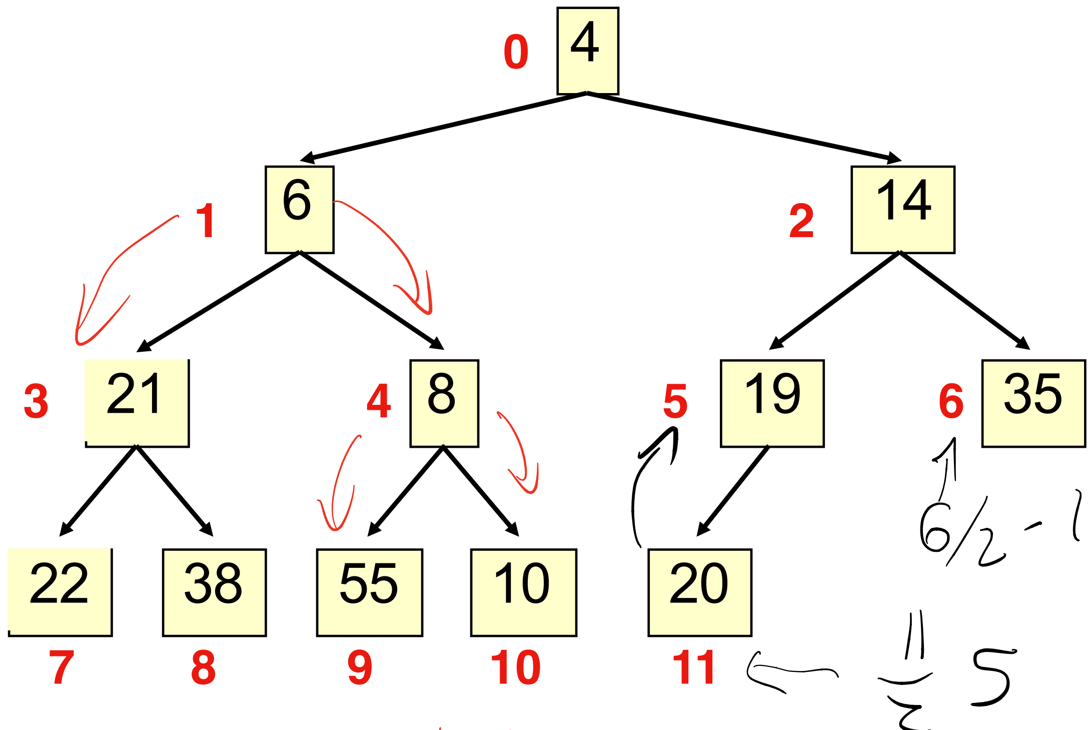


2. Complete: **no holes!**

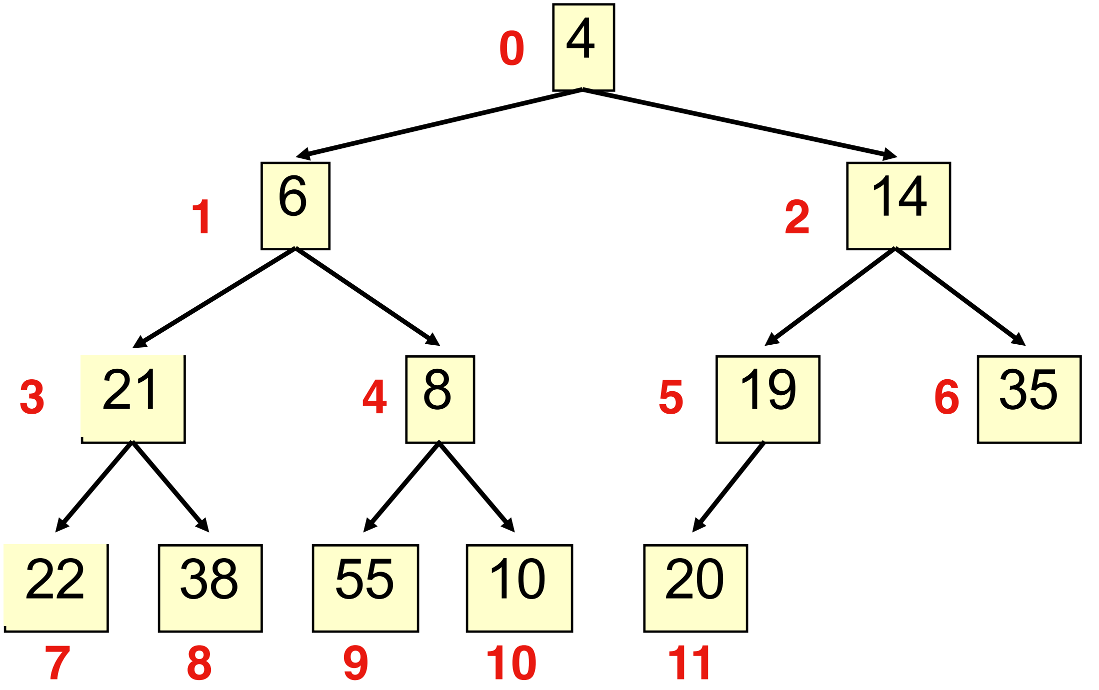
Numbering Nodes



node **k**'s parent is (k-1)/2

node **k**'s children are nodes 2k+1 and 2k+2

Numbering Nodes



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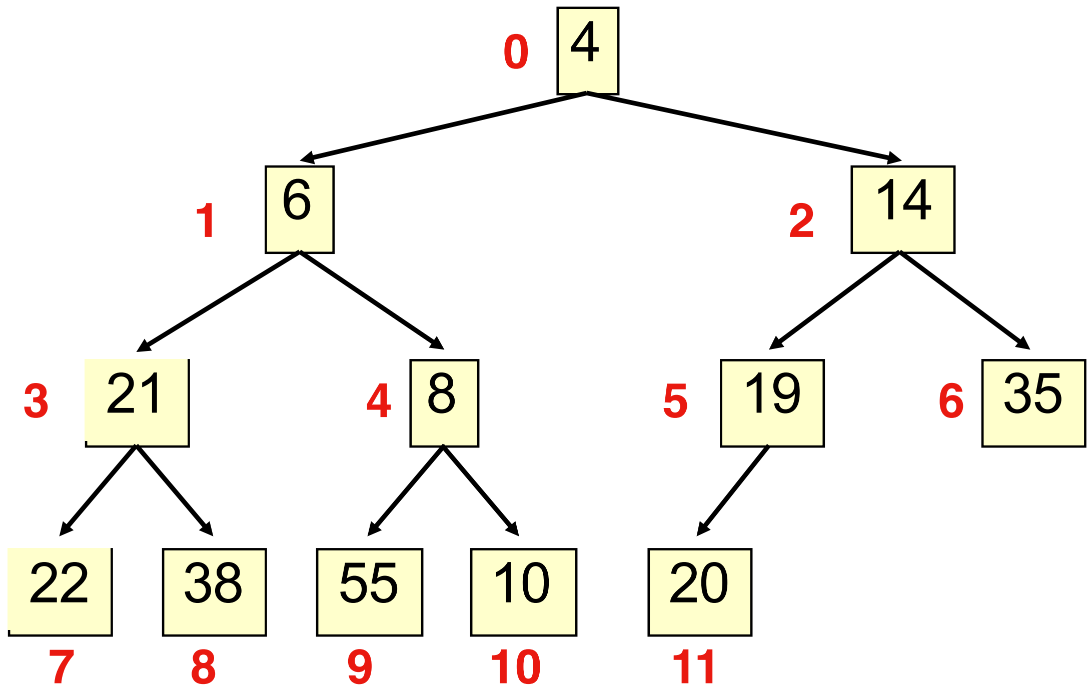
Numbering Nodes



node **k**’s parent is **(k – 1)/2**

node **k**’s children are nodes **2k + 1** and **2k + 2**

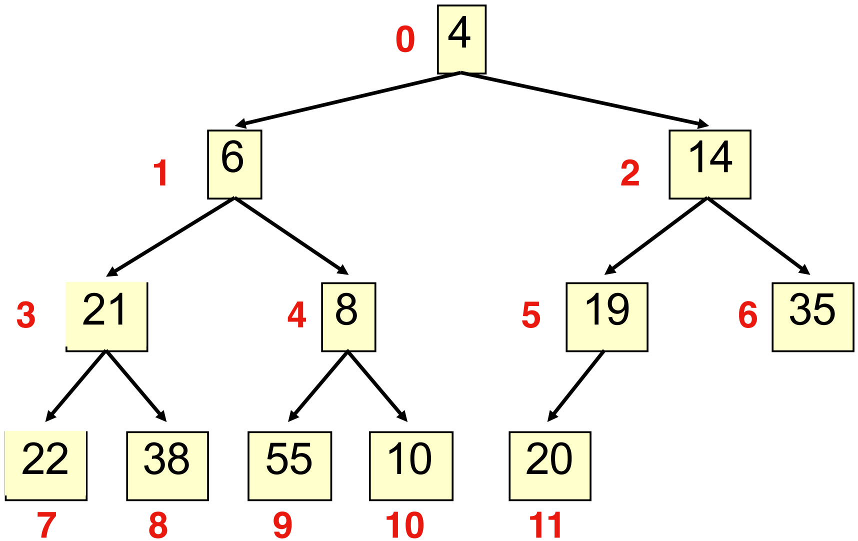
Implementing Heaps

```
public class Heap<E> {
    private E[] heap;
    private int size;
    ...
}
```

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
4	6	14	21	8	19	35	22	38	55	10	20				

Implicit Tree Structure

2. Complete: **no holes!**



0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
4	6	14	21	8	19	35	22	38	55	10	20				

Heap it real, part 2.

Here's a heap, stored in an array:

[1 5 7 6 7 10]

Write the array after execution of **add(4)**.

Assume the array is large enough to store the additional element.

A. [1 5 7 6 7 10 4]
B. [1 4 5 6 7 10 7]
C. [1 5 4 6 7 10 7]
D. [1 4 56 7 6 7 10]