CSCI 241

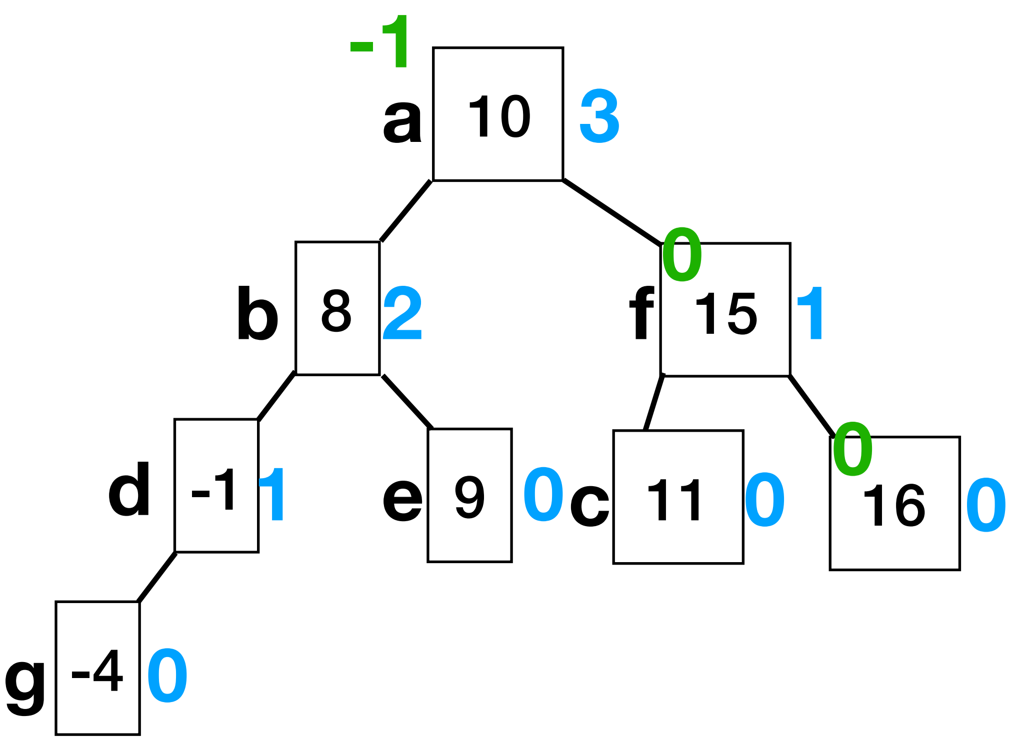
Lecture 14b

AVL rebalancing

Goals

- Understand how rebalance decides to what rotations to perform.
- Be prepared implement rebalance.

AVL Insertion




```
insert(Node n, int v):
  //…(other case, irrelevant here)
  else: // v > n.value
    if n has right:
      insert(n.right, v)
   else:
      // attach new node w/ value
      //   v to n.right
rebalance(n);
```

How did we know what rotation to do?

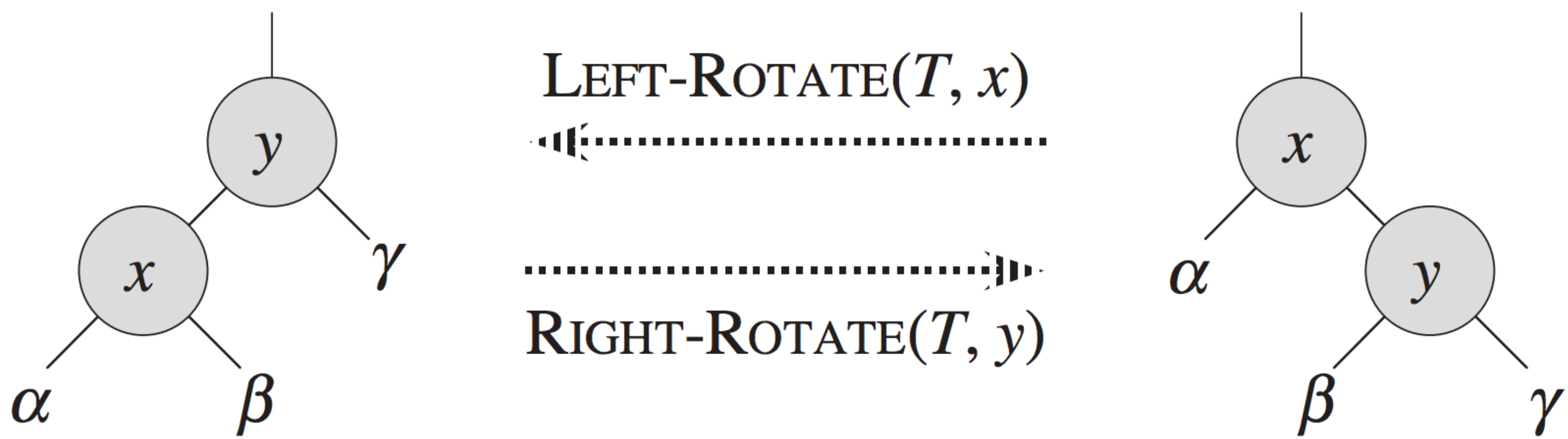
```
insert(a, 16)
=>insert(c, 16)
  =>insert(f, 16)
    =>attach new node
      rebalance(f)
    rebalance(c)
  rebalance(a)
```

rebalance(f) **already balanced**

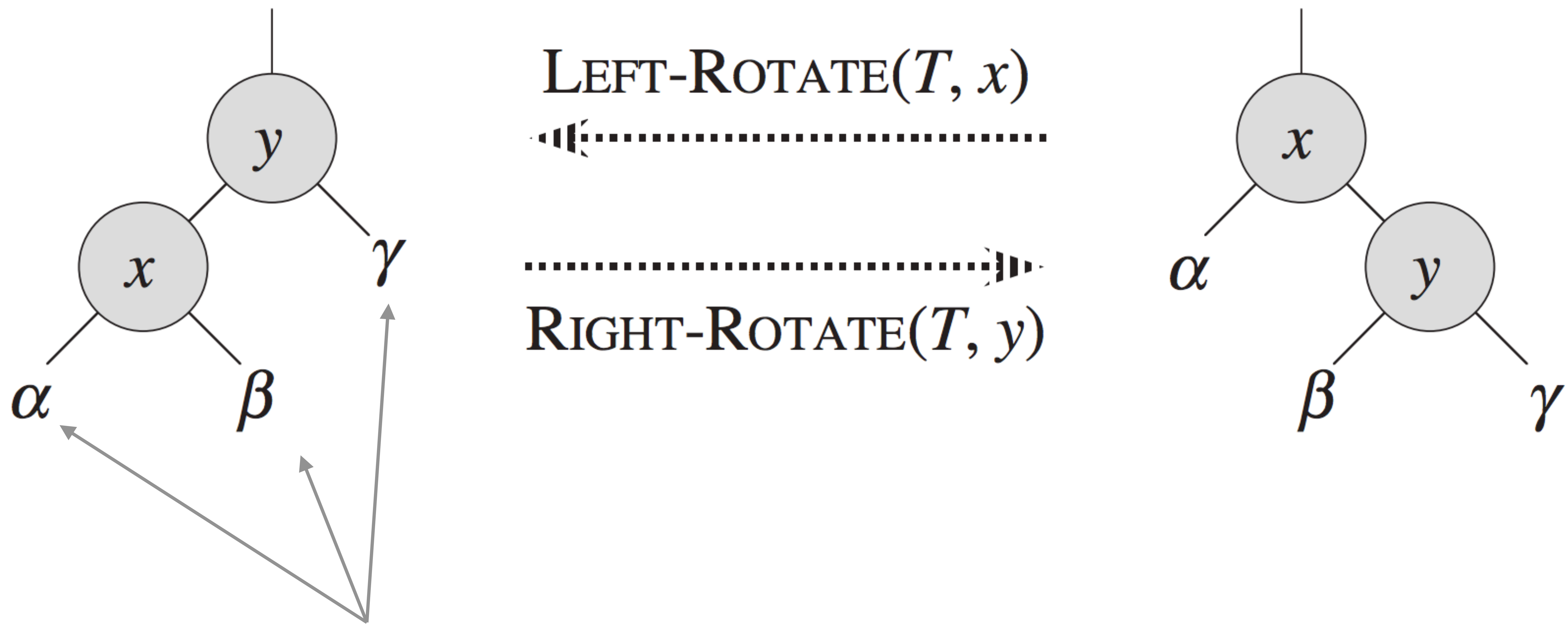
rebalance(c) **perform rotation**

rebalance(a) **already balanced**

Reminder: Tree Rotations



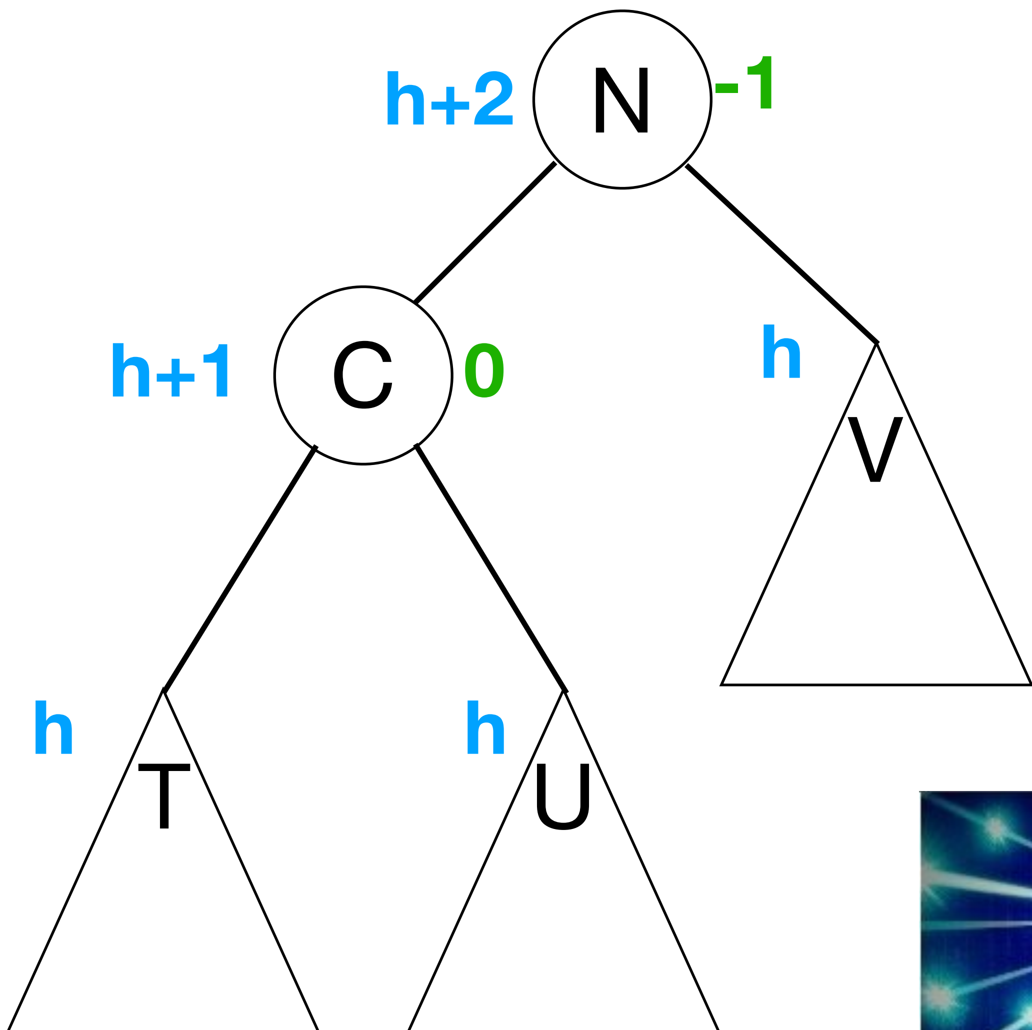
Reminder: Tree Rotations

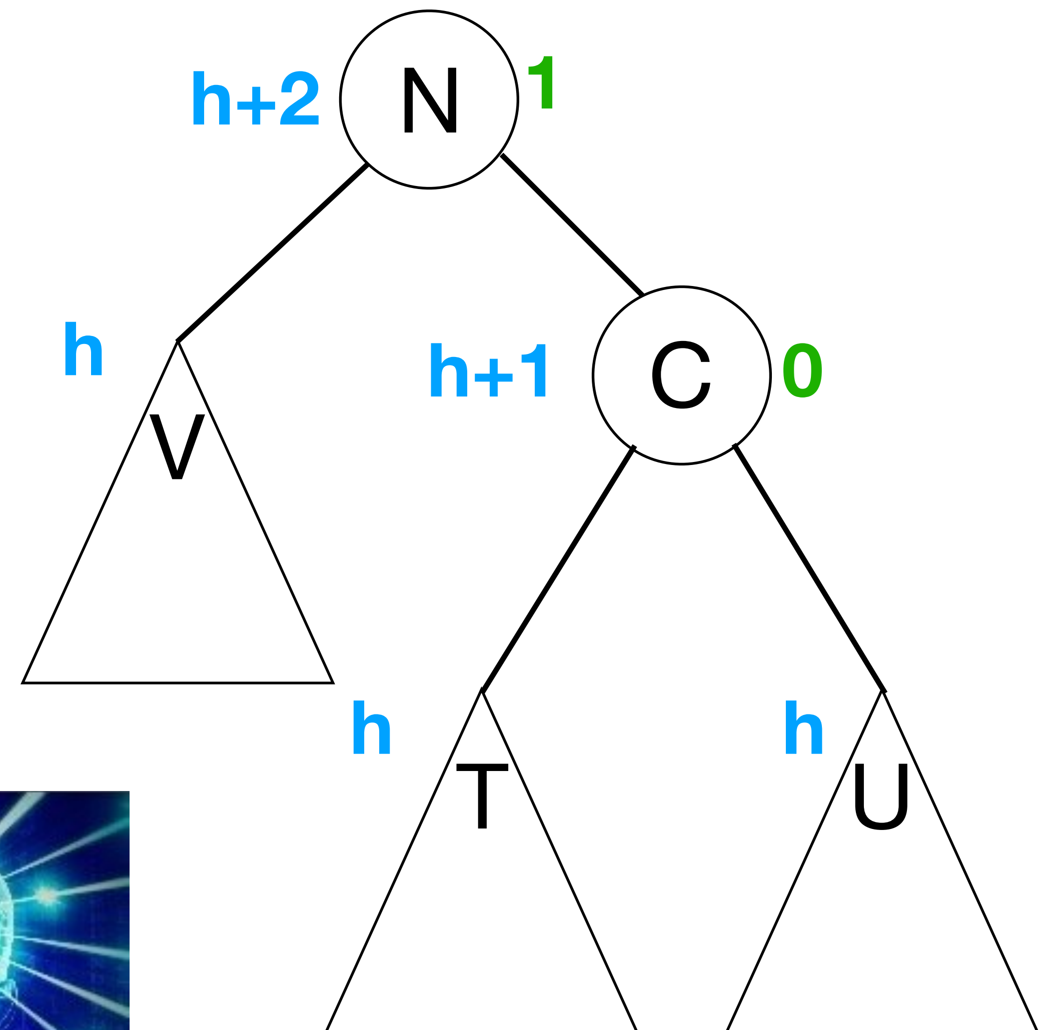


subtrees (could be null, leaf, or tree with many nodes)

AVL Rebalance

Before an insertion that unbalances n,
the tree must look like one of these:





AVL Rebalance

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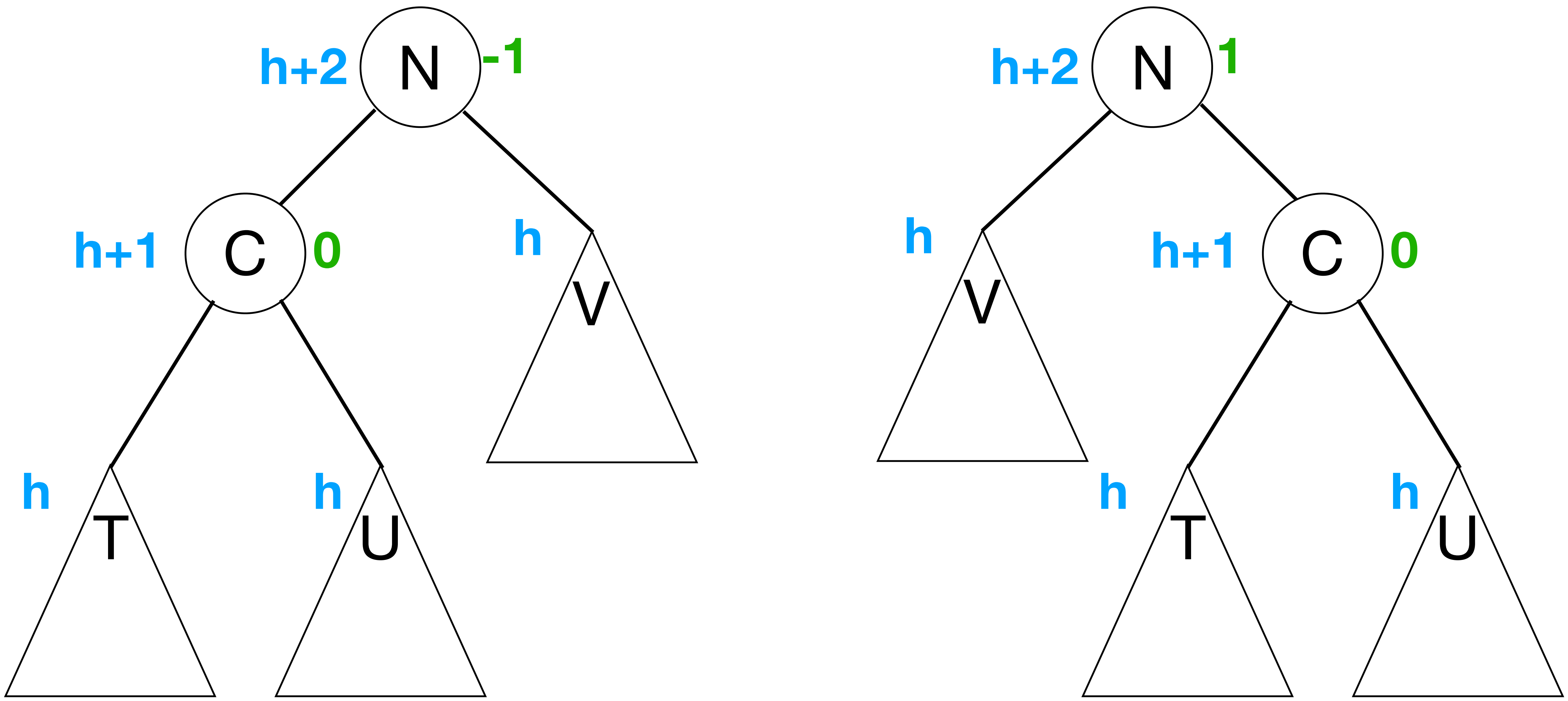
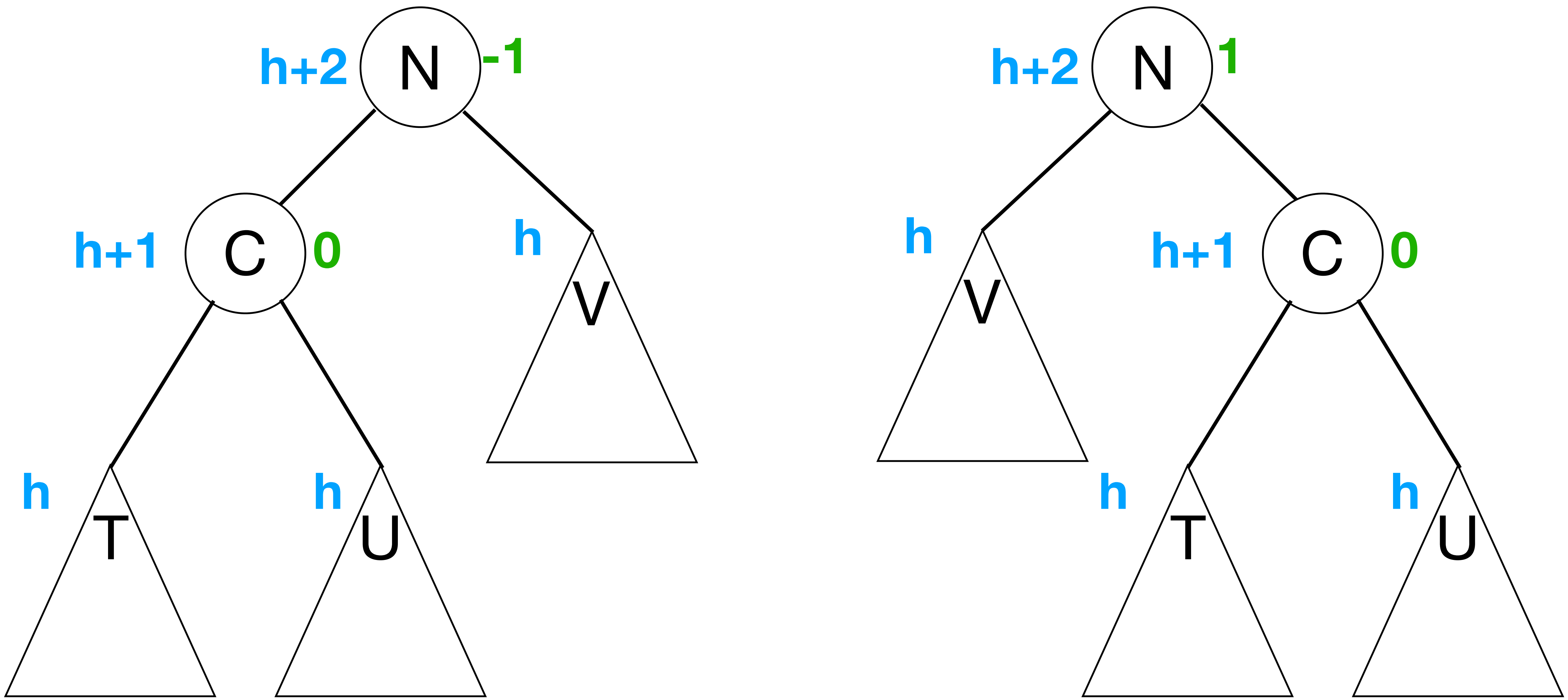


AVL Rebalance

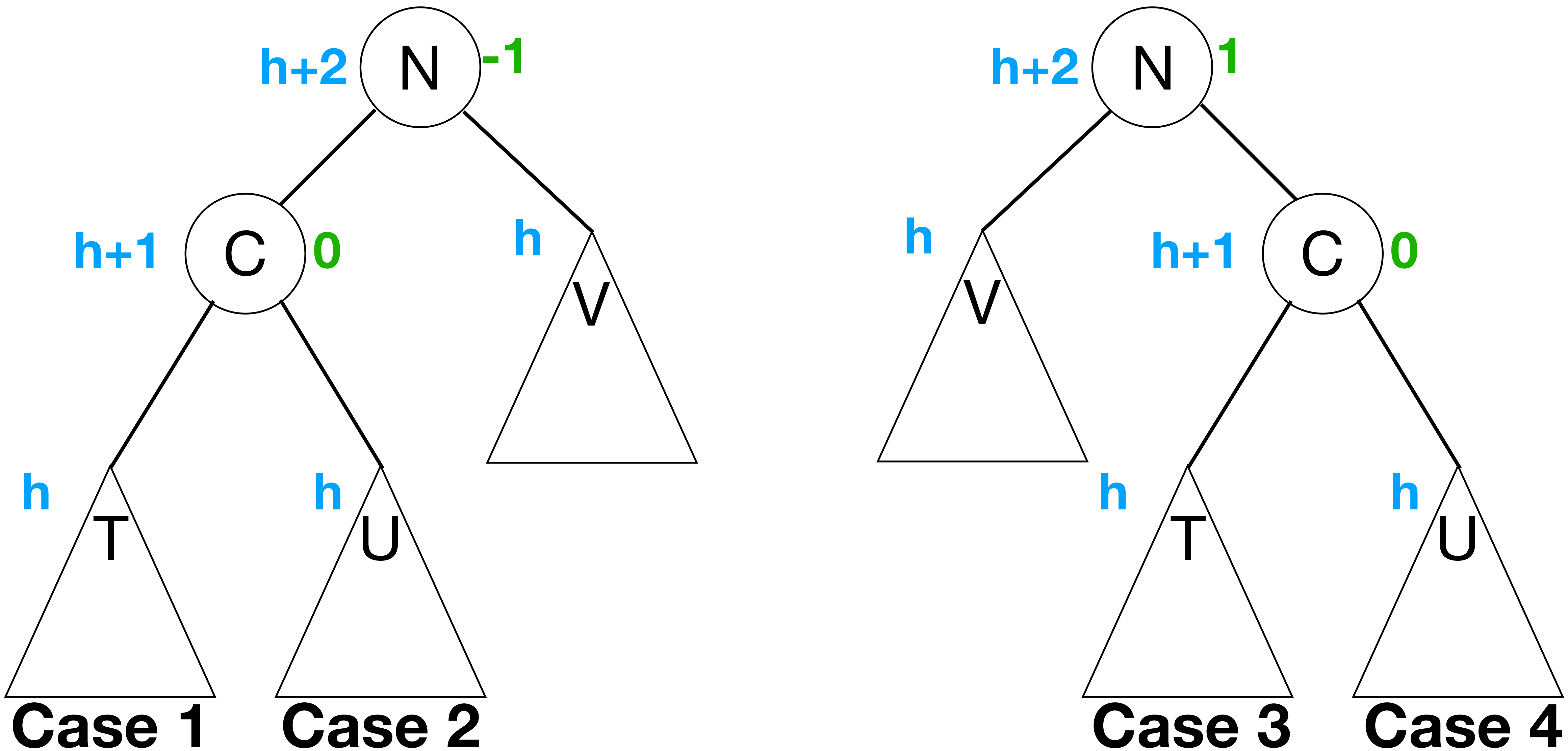
Before an insertion that unbalances N,
the tree must look like one of these:



An insertion that *unbalances* N could go one of four places.

AVL Rebalance

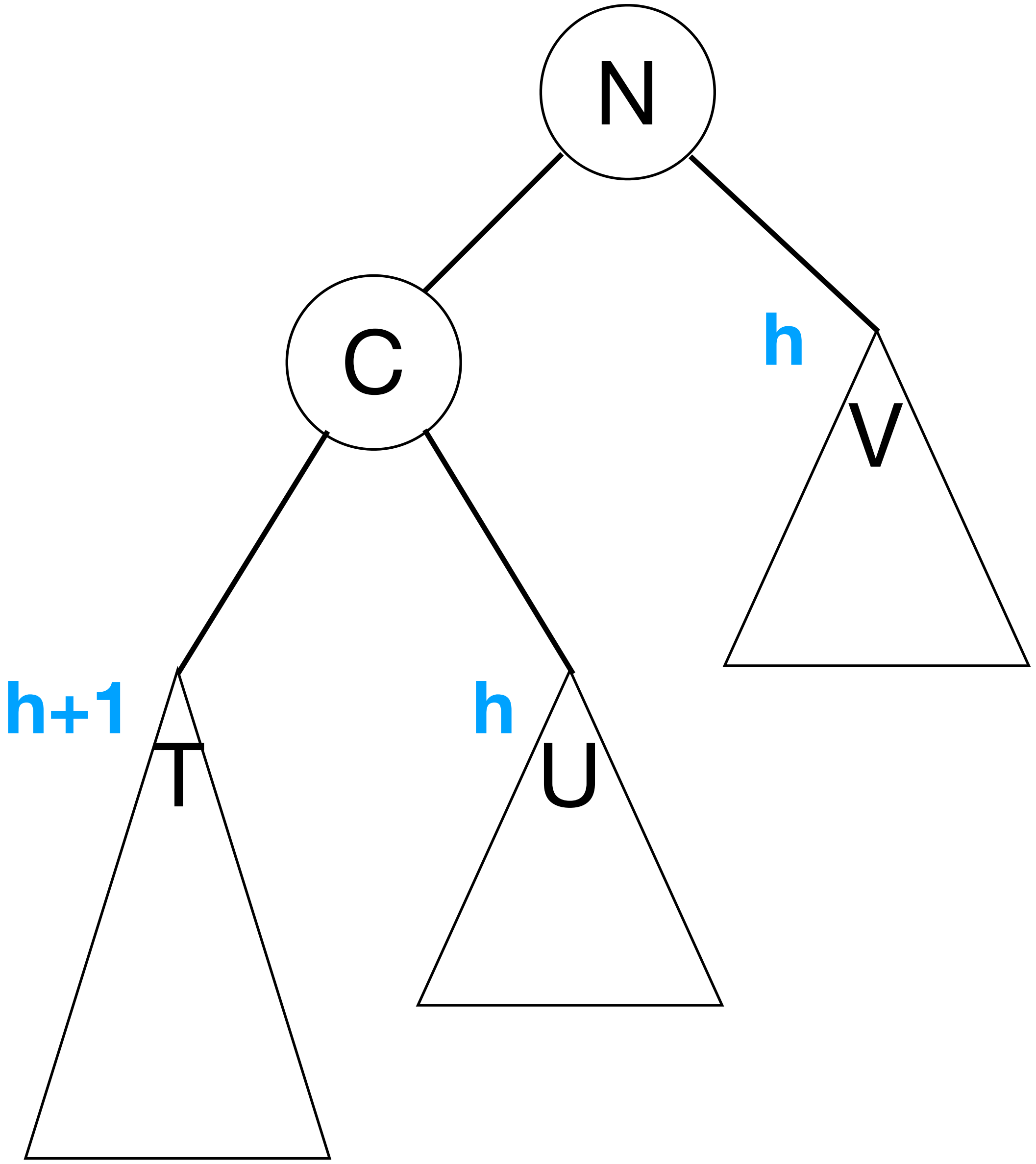
Before an insertion that unbalances n,
the tree must look like one of these:



An insertion that unbalances n could go one of four places.

AVL Rebalance

Case 1: After BST insertion step, the tree looks like this.



AVL Rebalance

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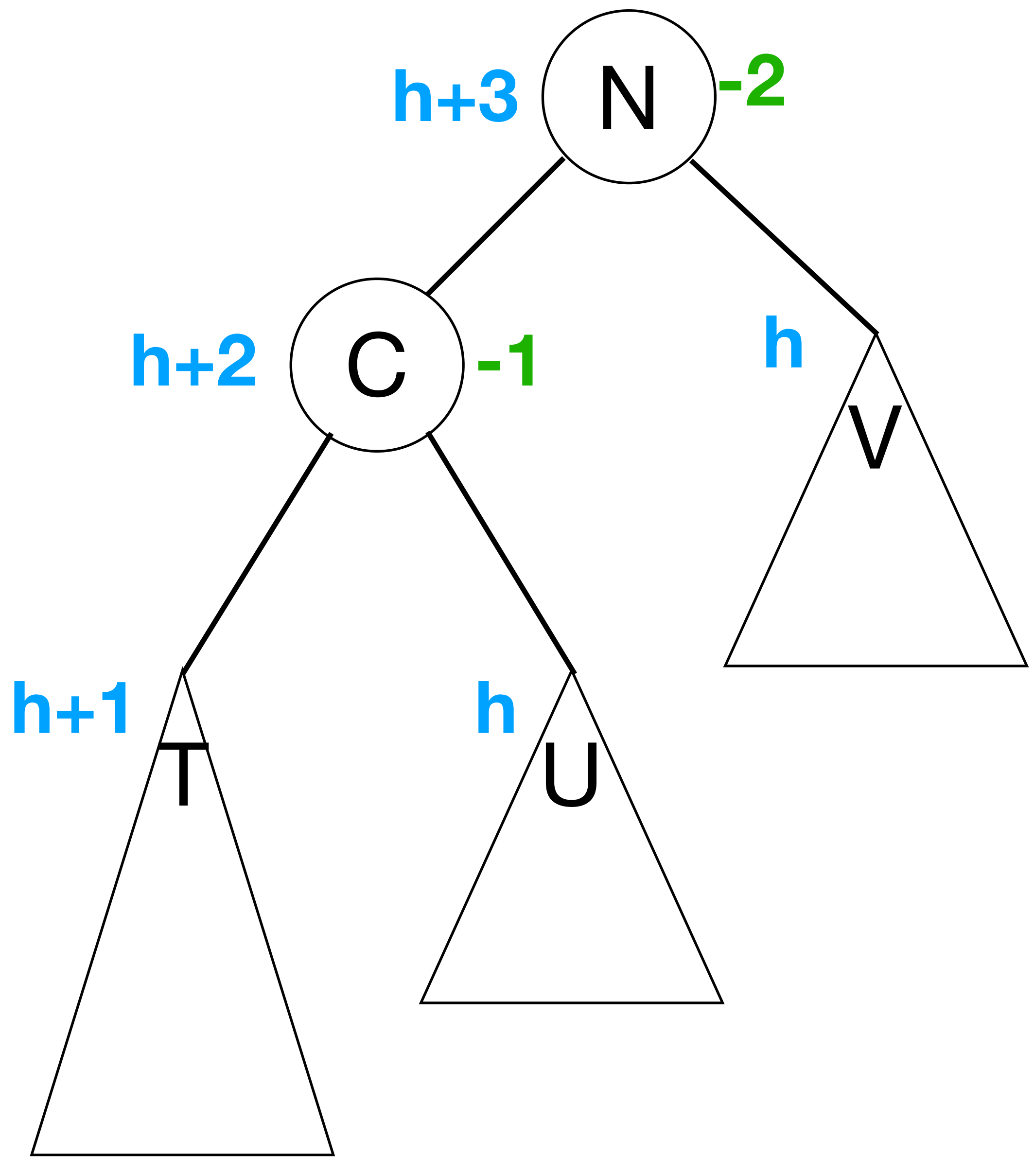
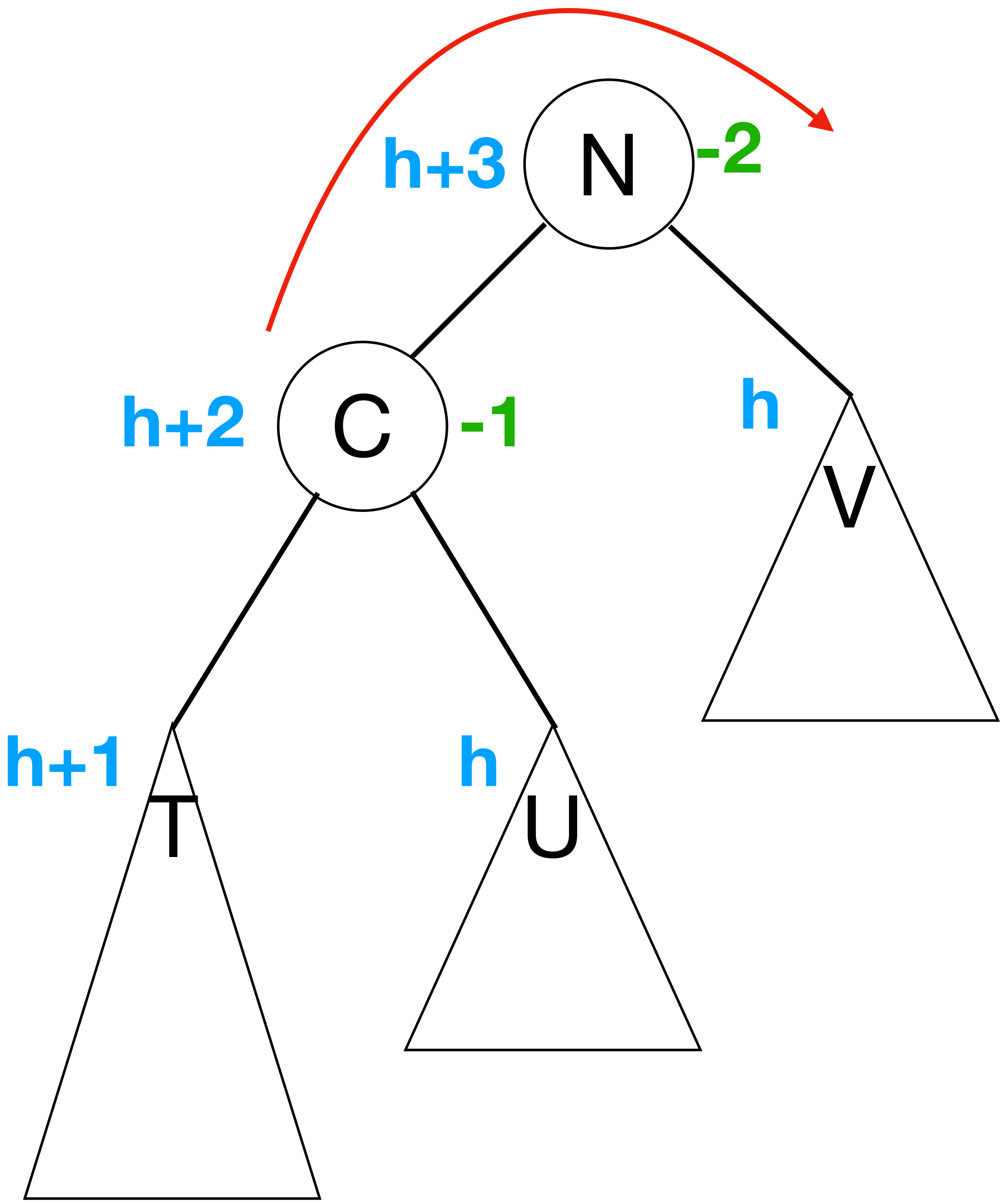


AVL Rebalance

Case 1: After BST insertion step, the tree looks like this.

Solution: right rotate on N.



AVL Rebalance

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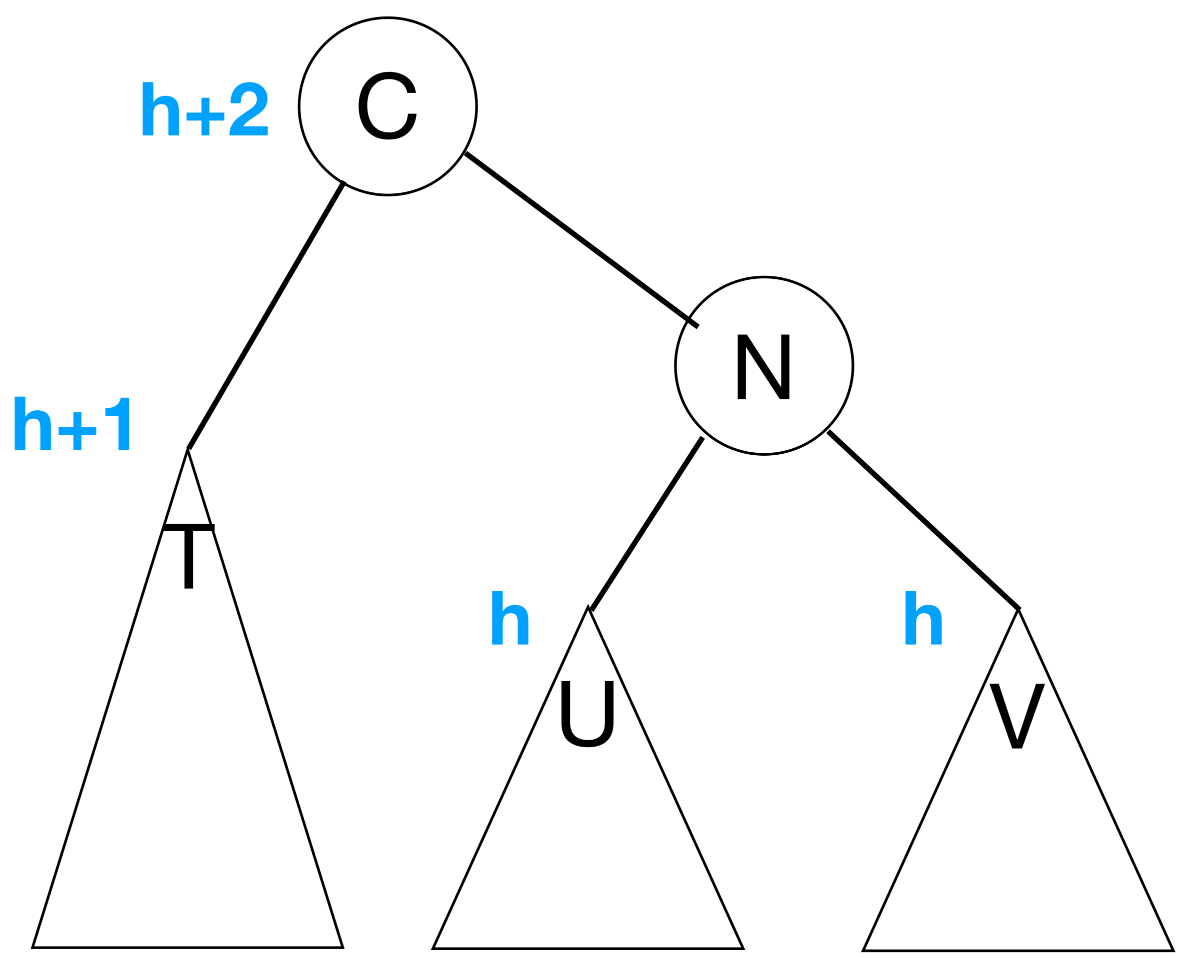
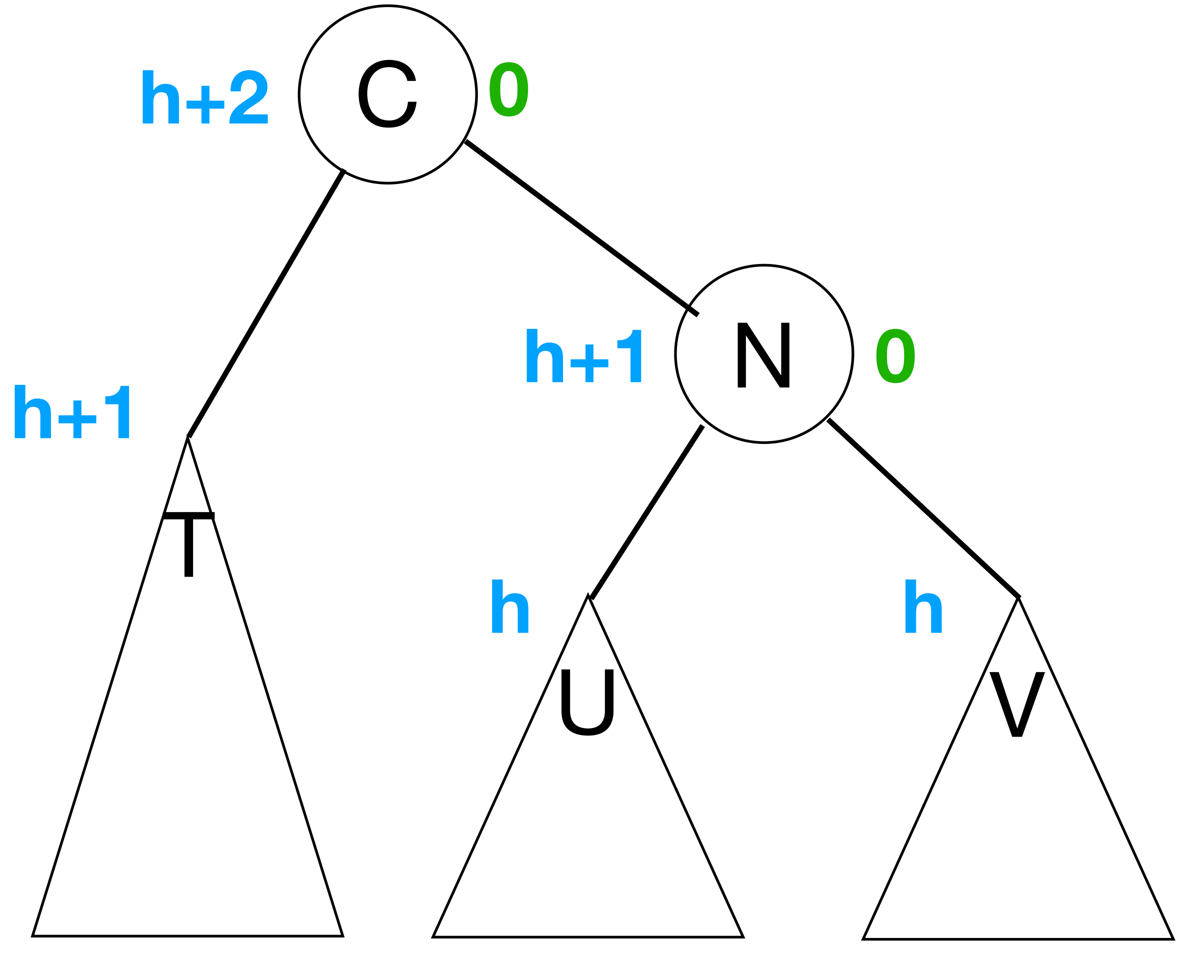


AVL Rebalance

Case 1: After BST insertion step, the tree looks like this.

Solution: right rotate on N.

N is now AVL balanced.



AVL Rebalance

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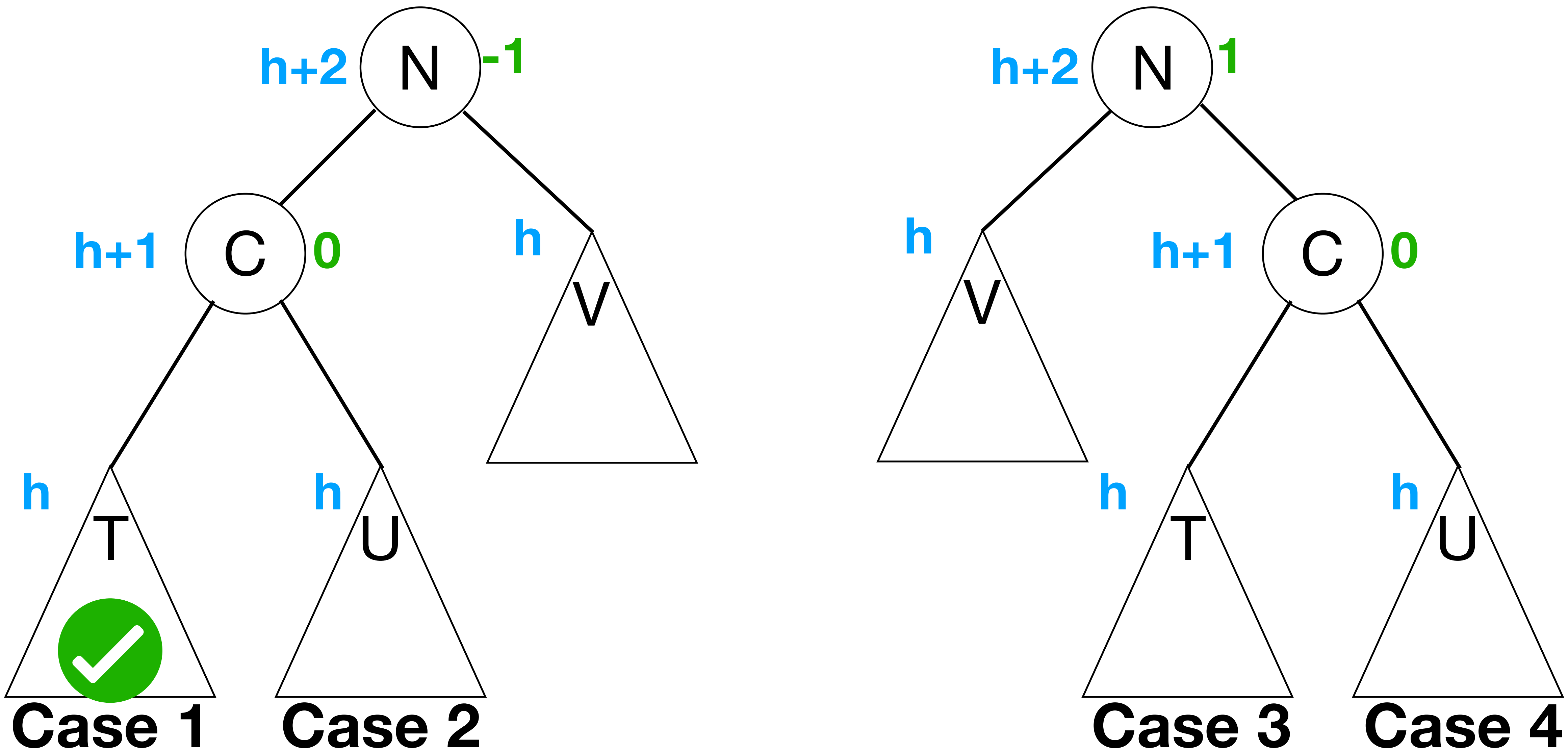
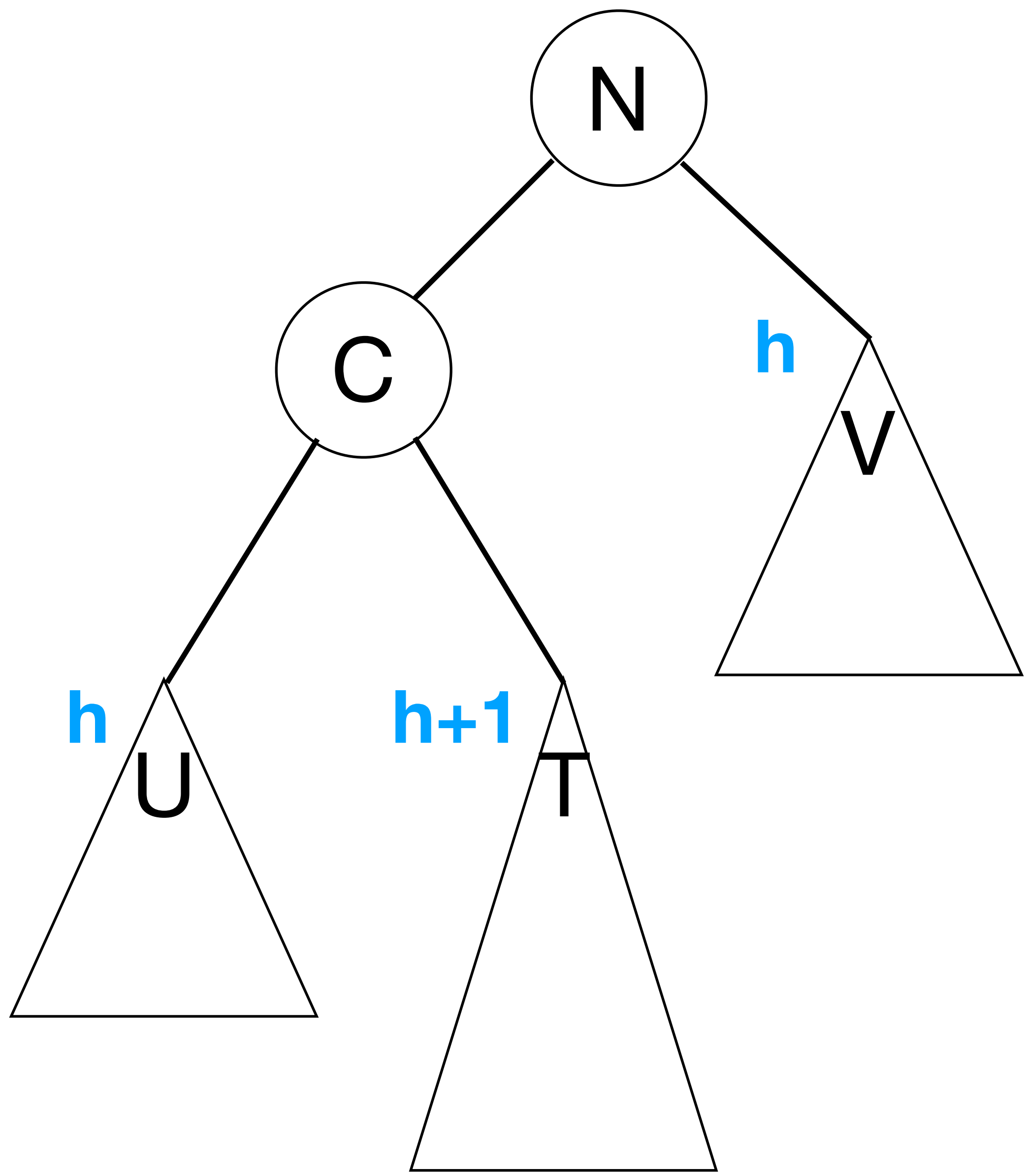


An insertion that unbalances n could gone one of four places.

AVL Rebalance

Case 2: After BST insertion step, the tree looks like this.



AVL Rebalance

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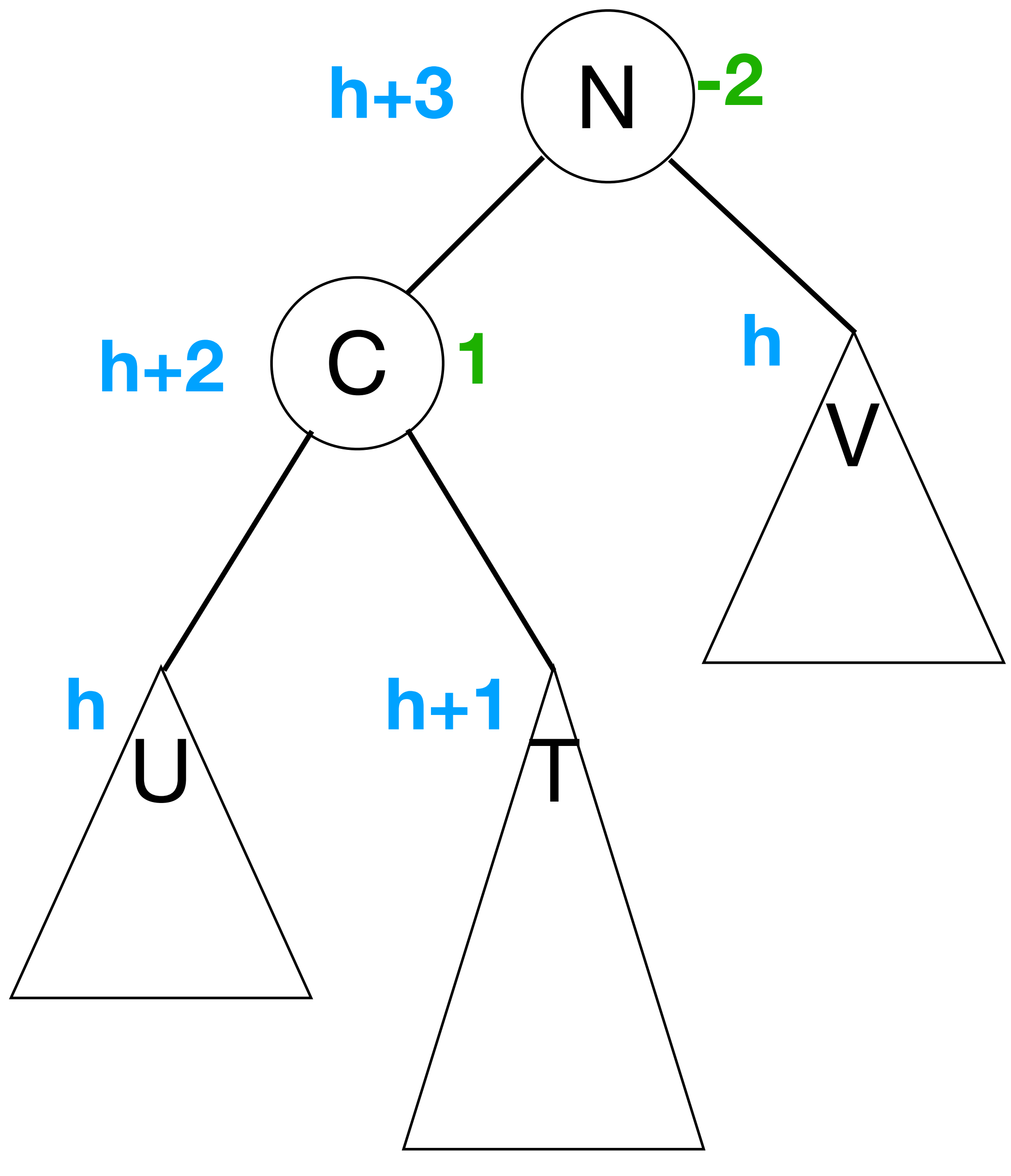

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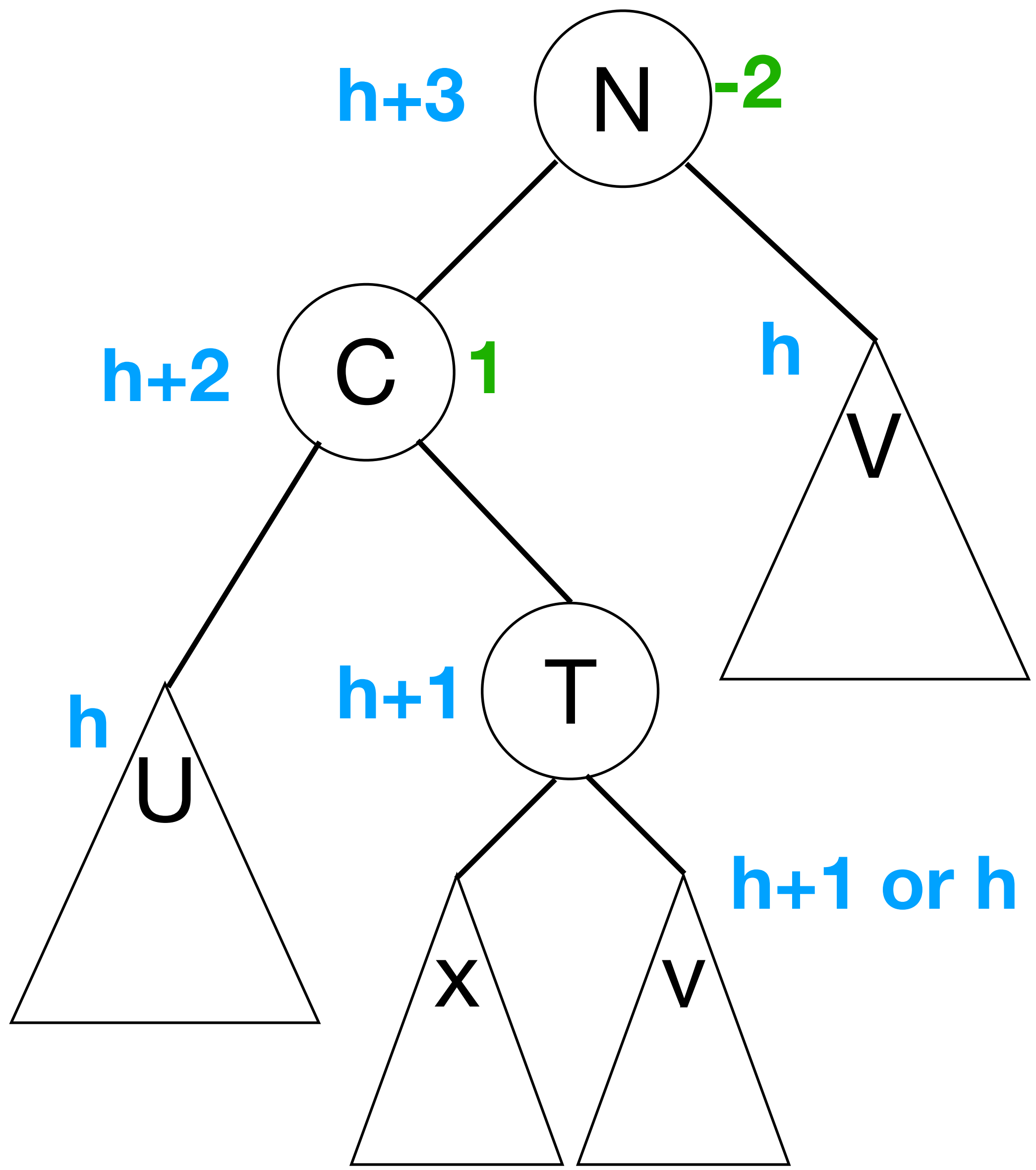
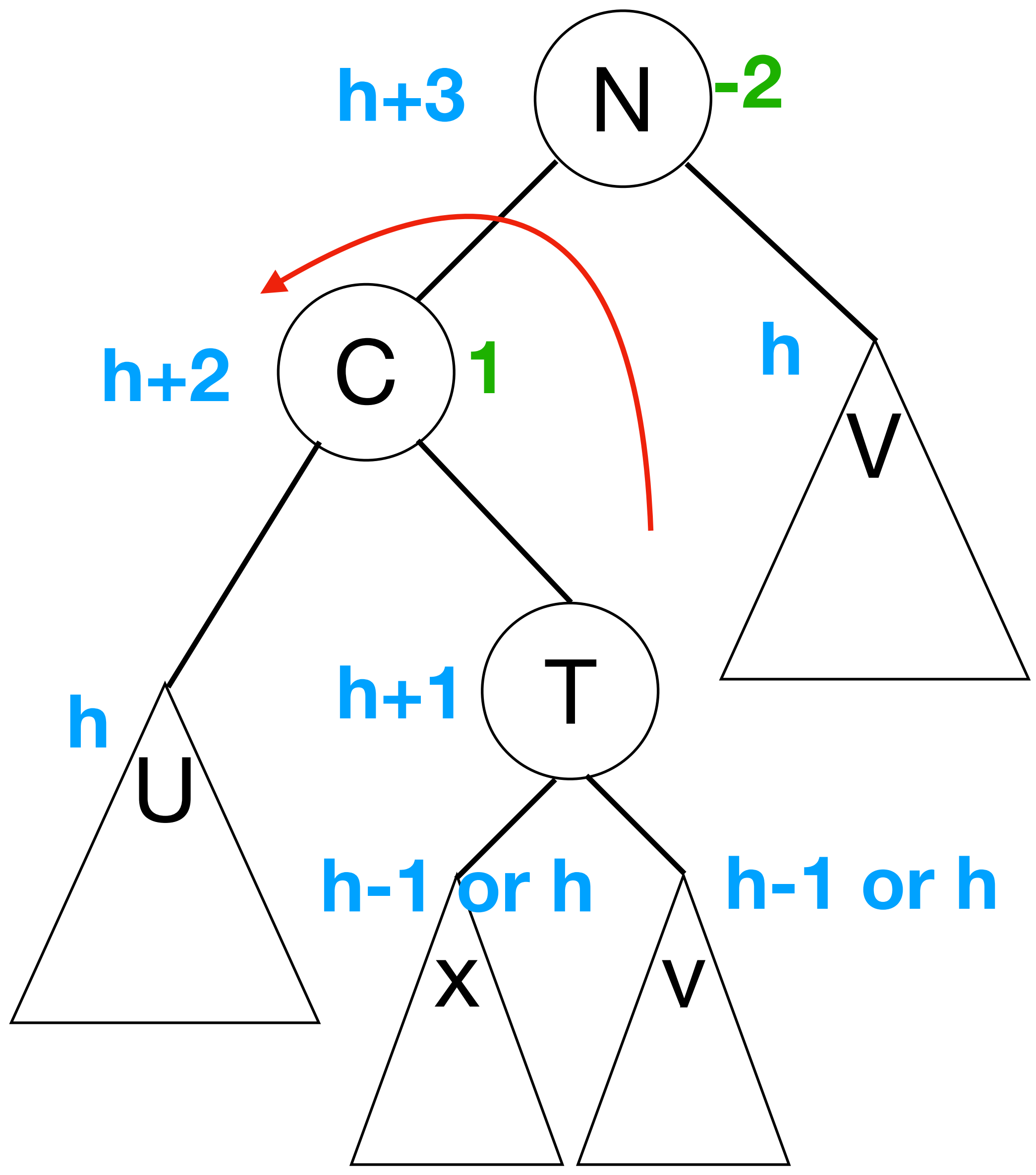


Solution - two rotations:

1. Left rotate C
2. Right rotate N

AVL Rebalance

Case 2: After BST insertion step, the tree looks like this.

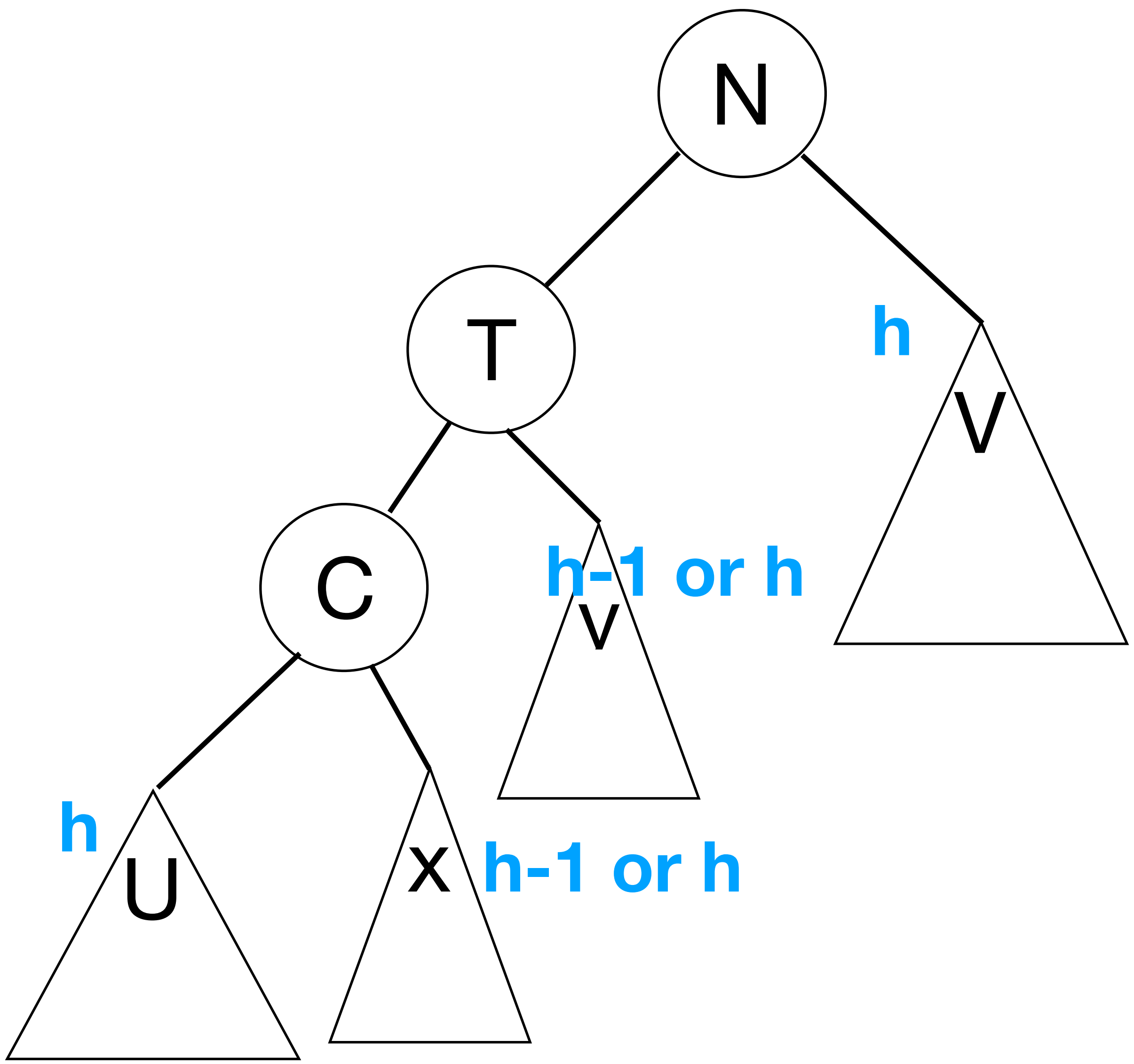


Solution - two rotations:

1. **Left rotate C**
2. Right rotate N

AVL Rebalance

Case 2: After BST insertion step, the tree looks like this.



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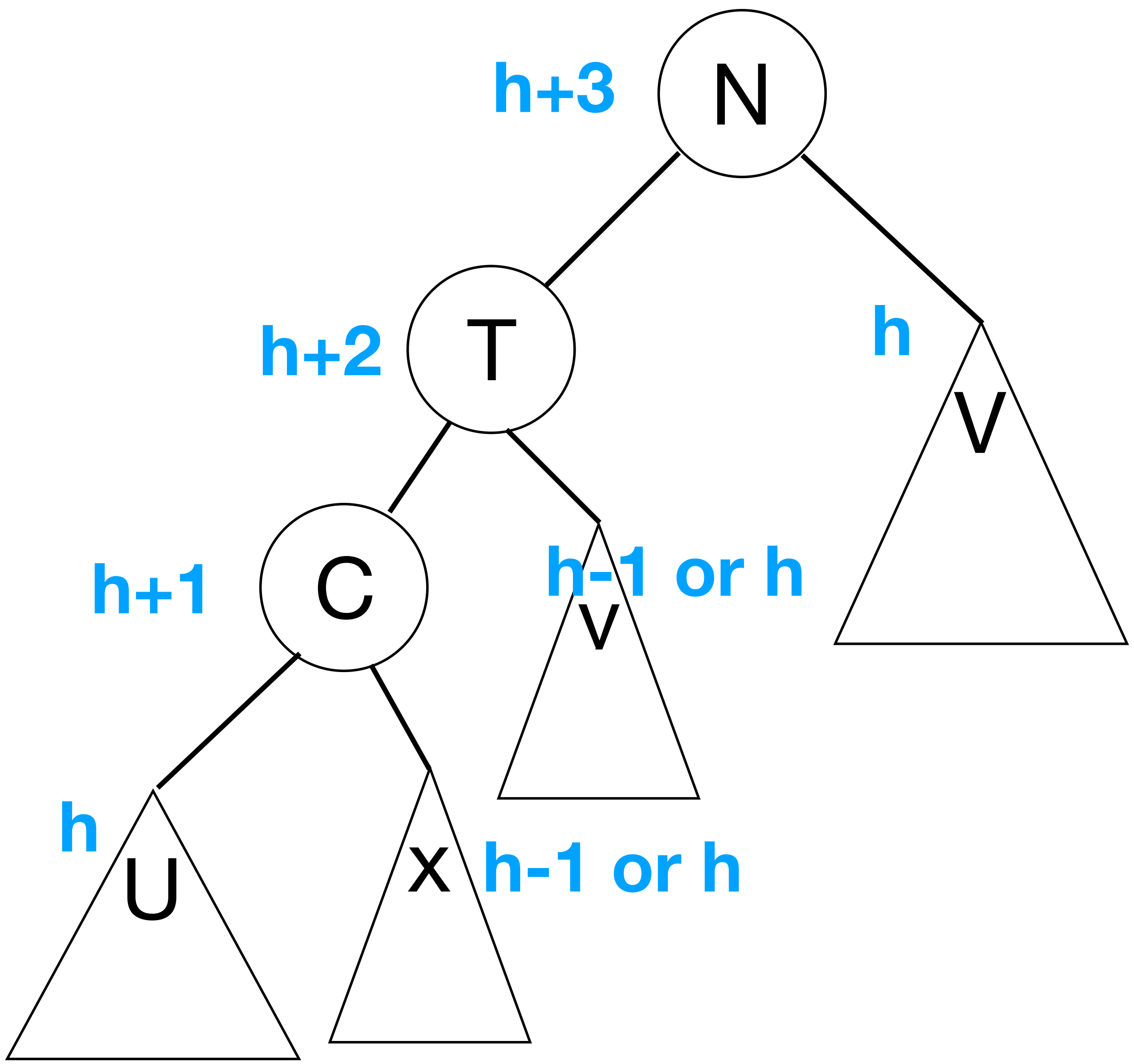

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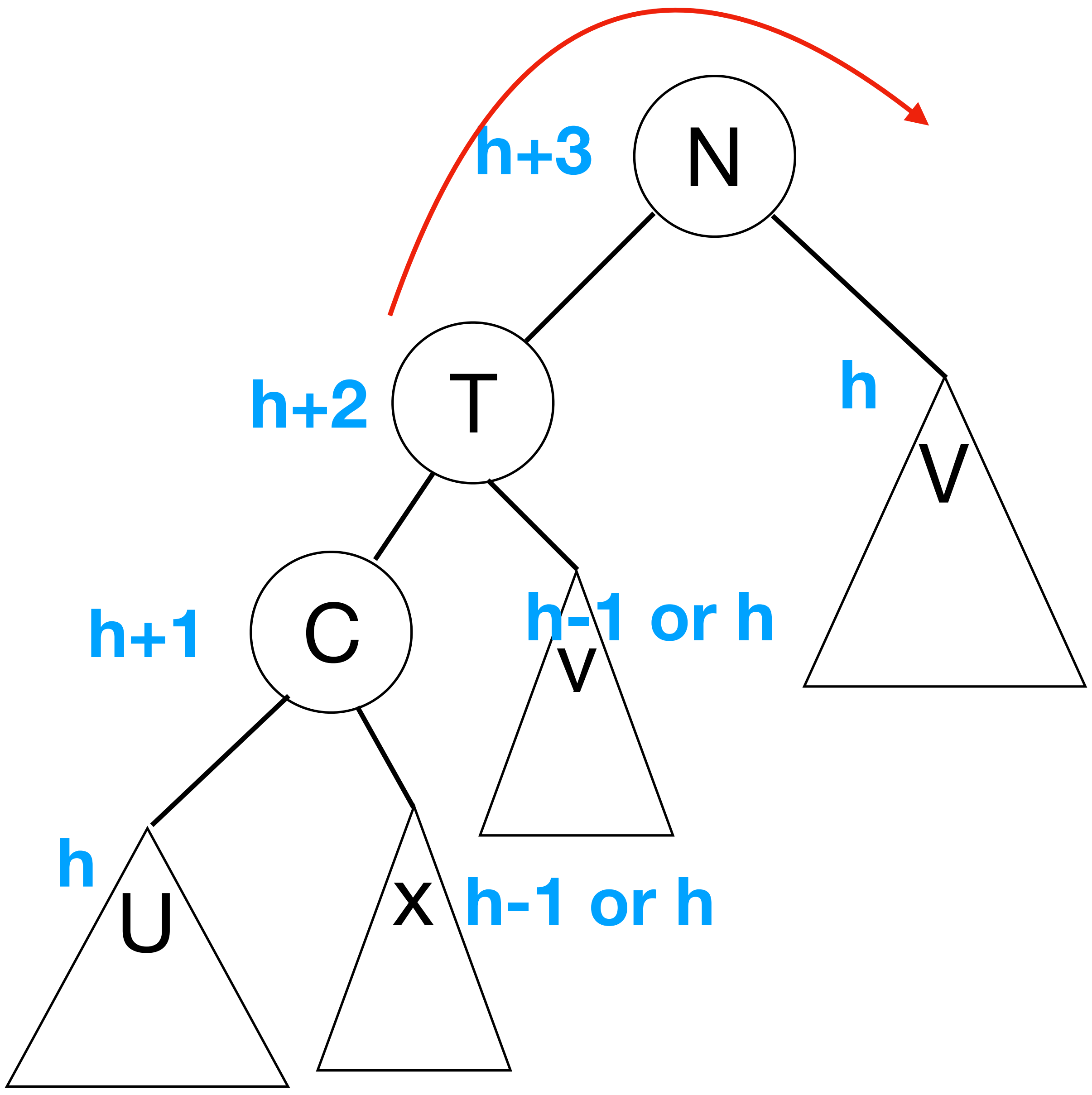

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AVL Rebalance

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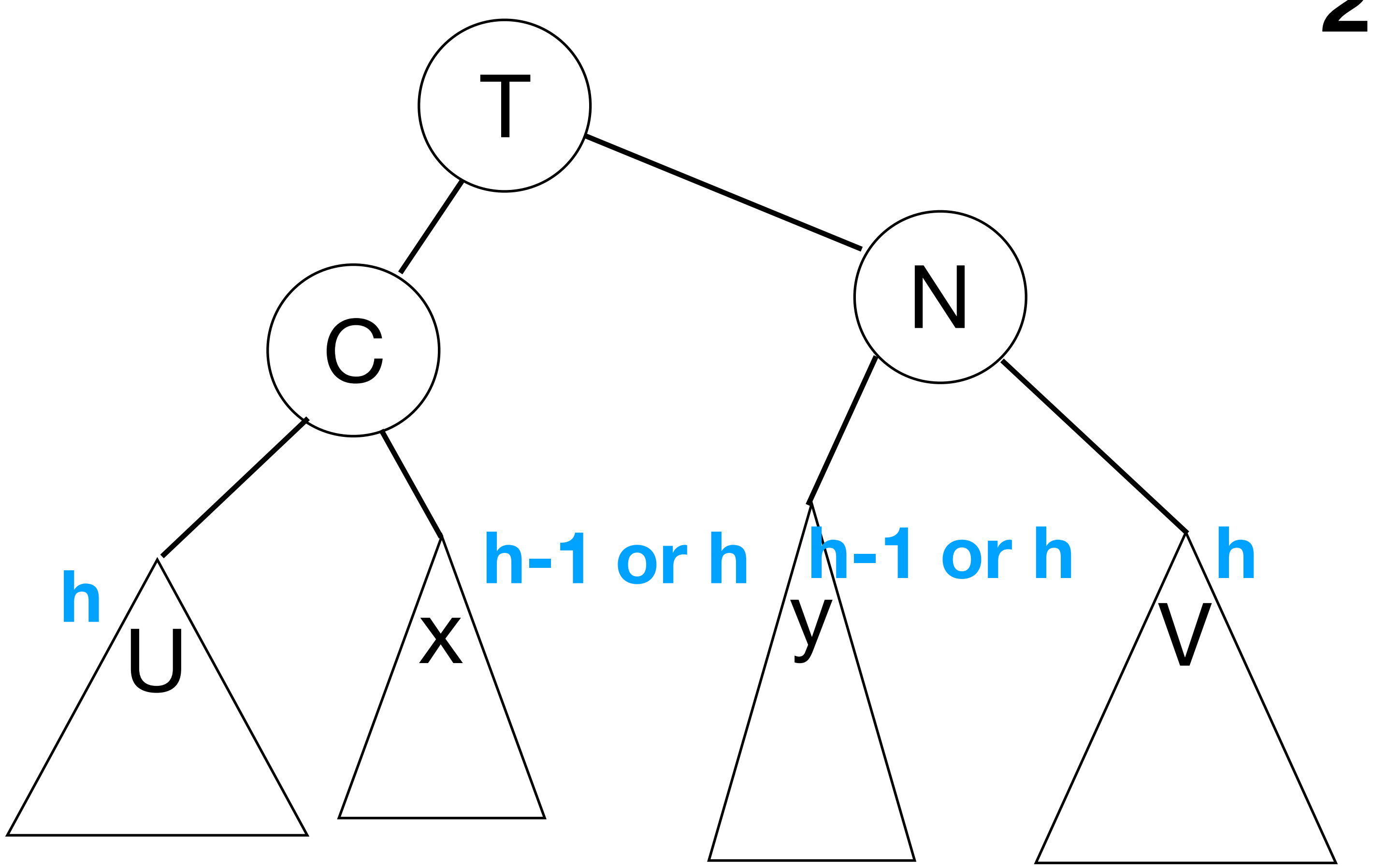

AVL Rebalance

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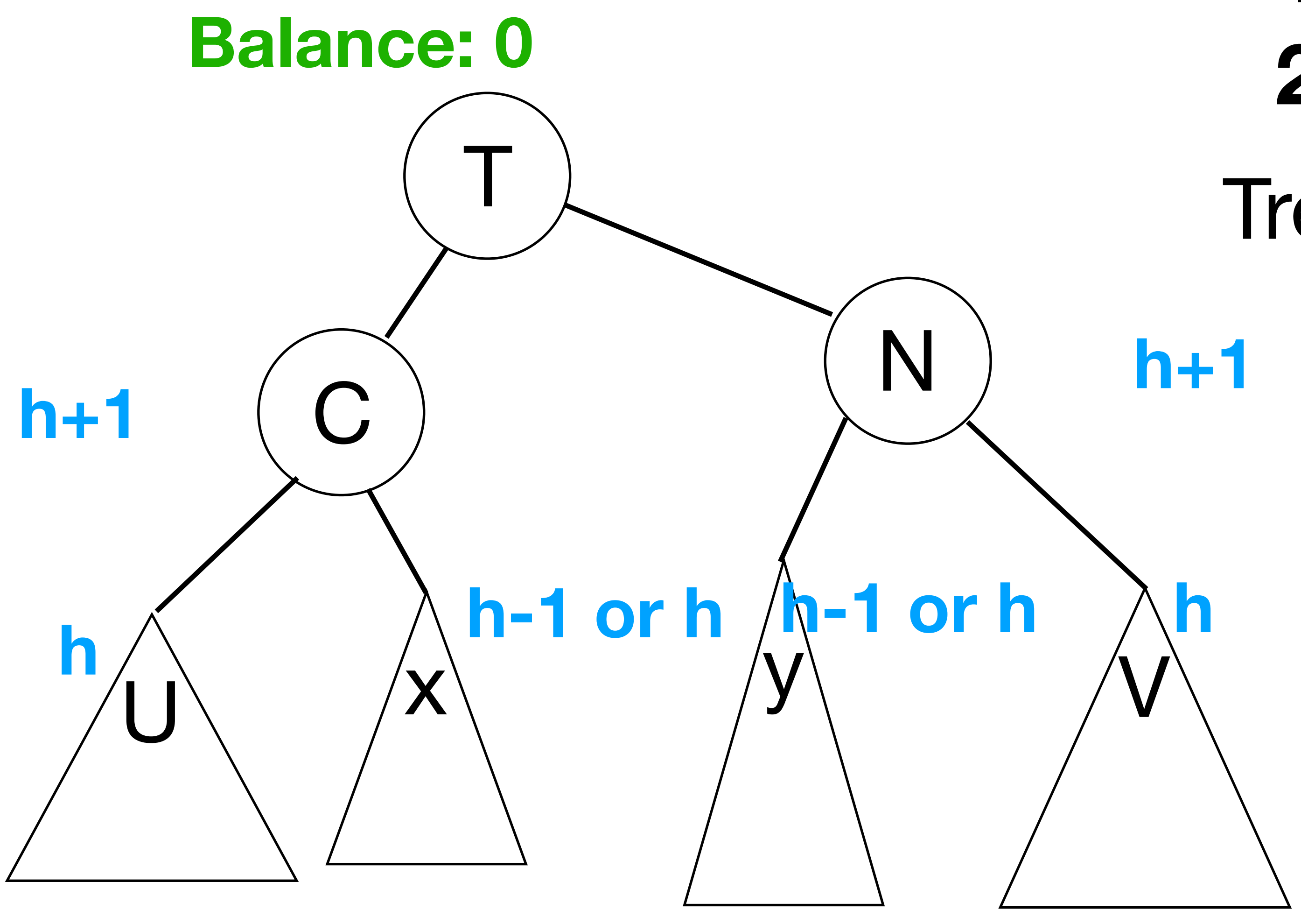


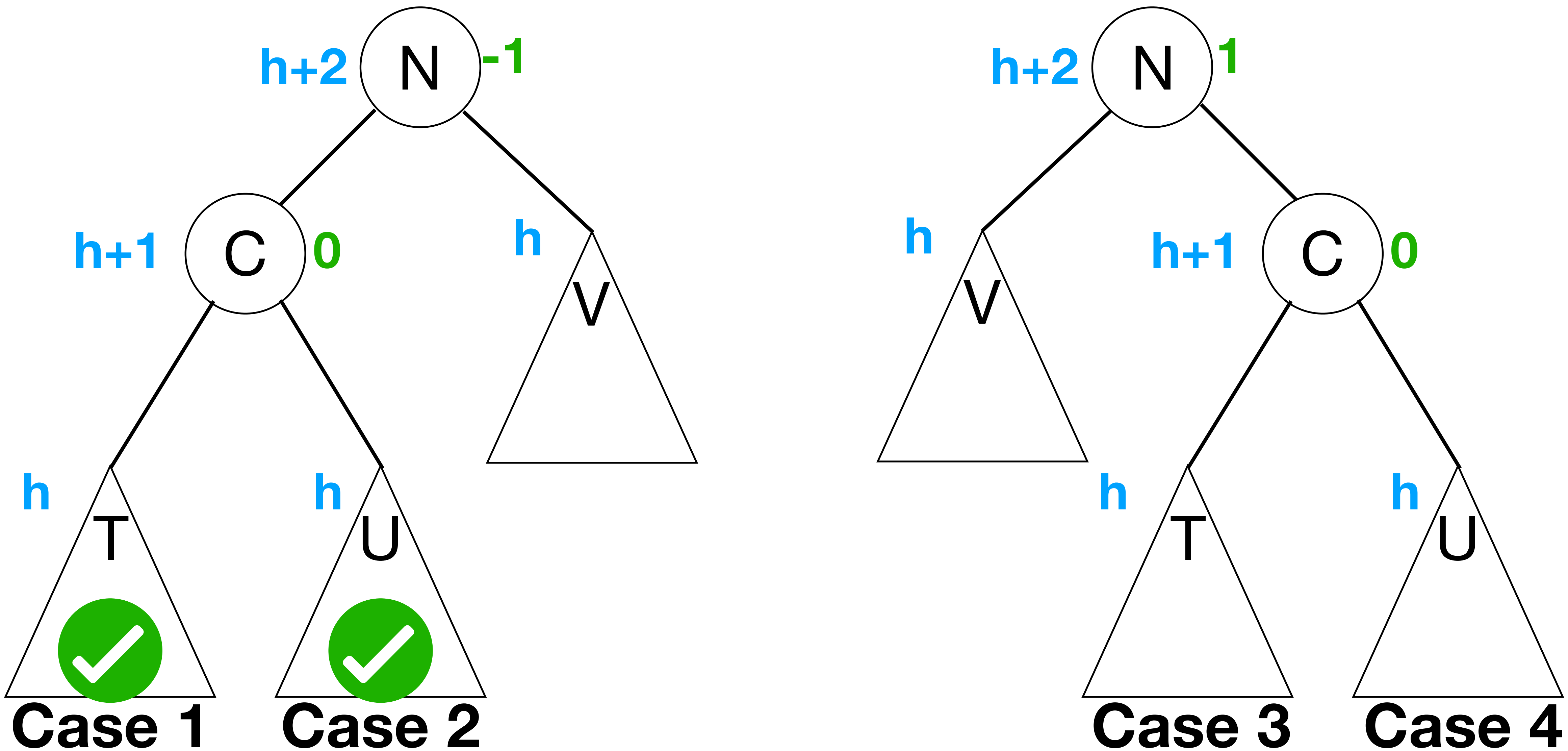
Solution - two rotations:

1. Left rotate C
2. **Right rotate N**

Tree is now AVL balanced.

AVL Rebalance

Before an insertion that unbalances n,
the tree must look like one of these:



An insertion that unbalances n could gone one of four places.

AVL Rebalance

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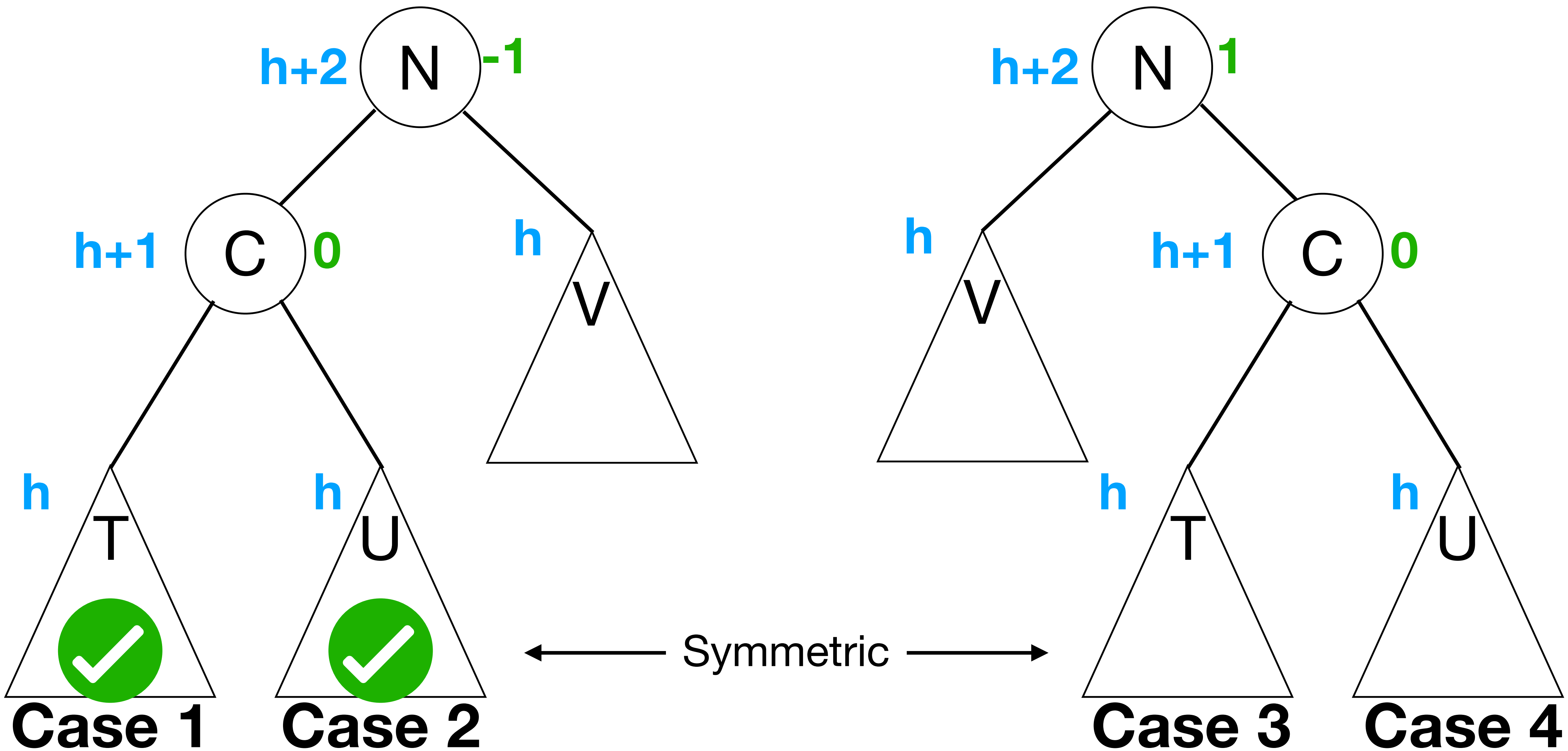

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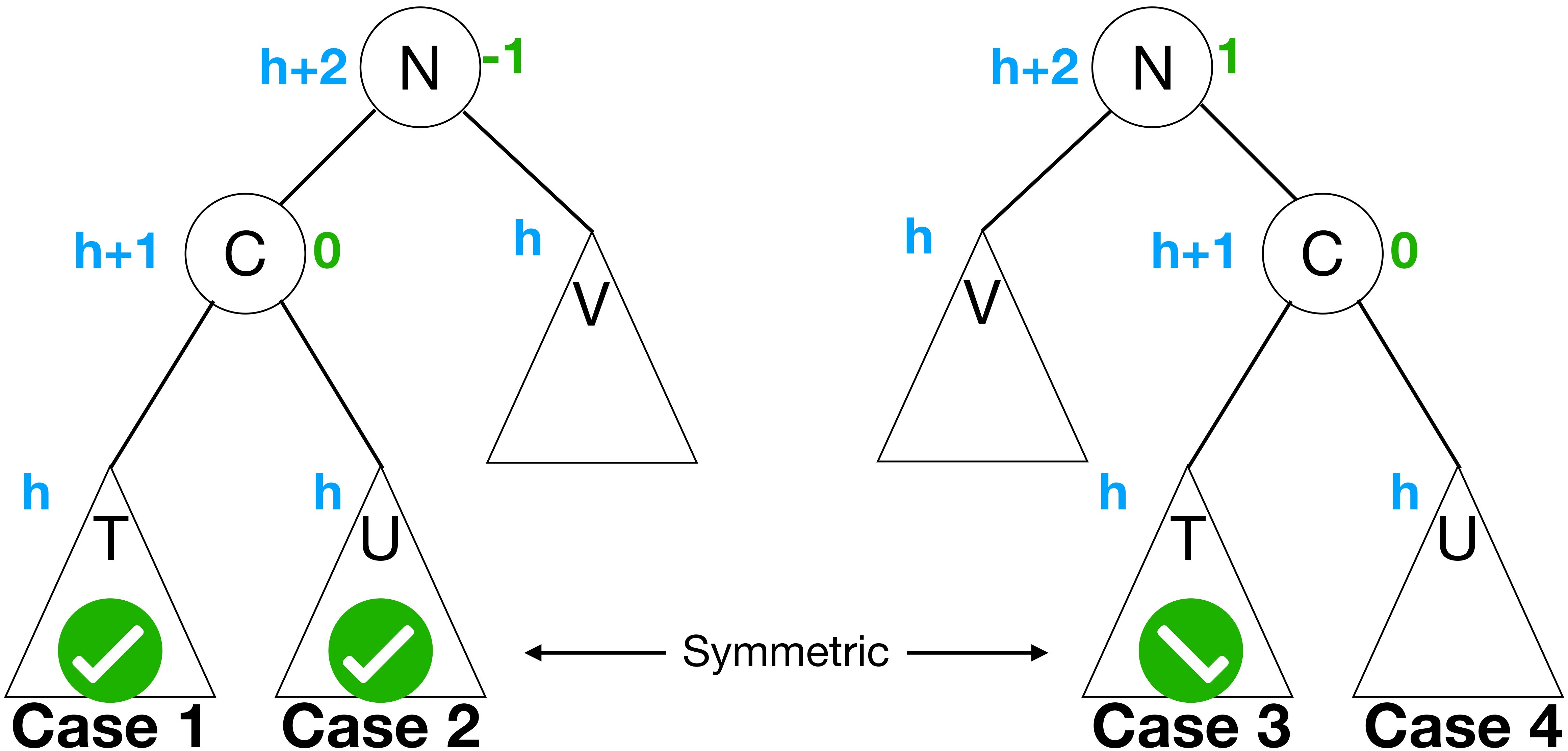

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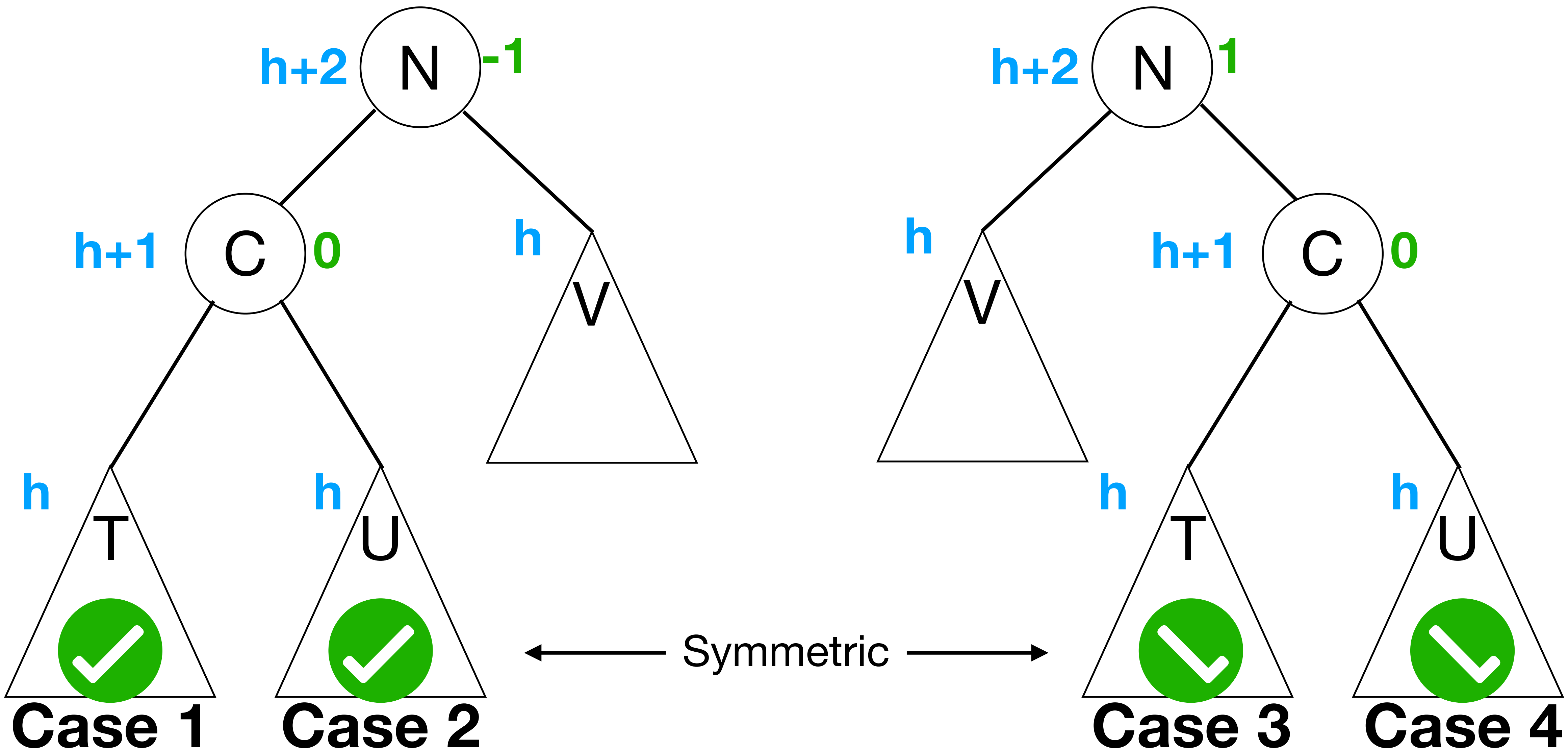
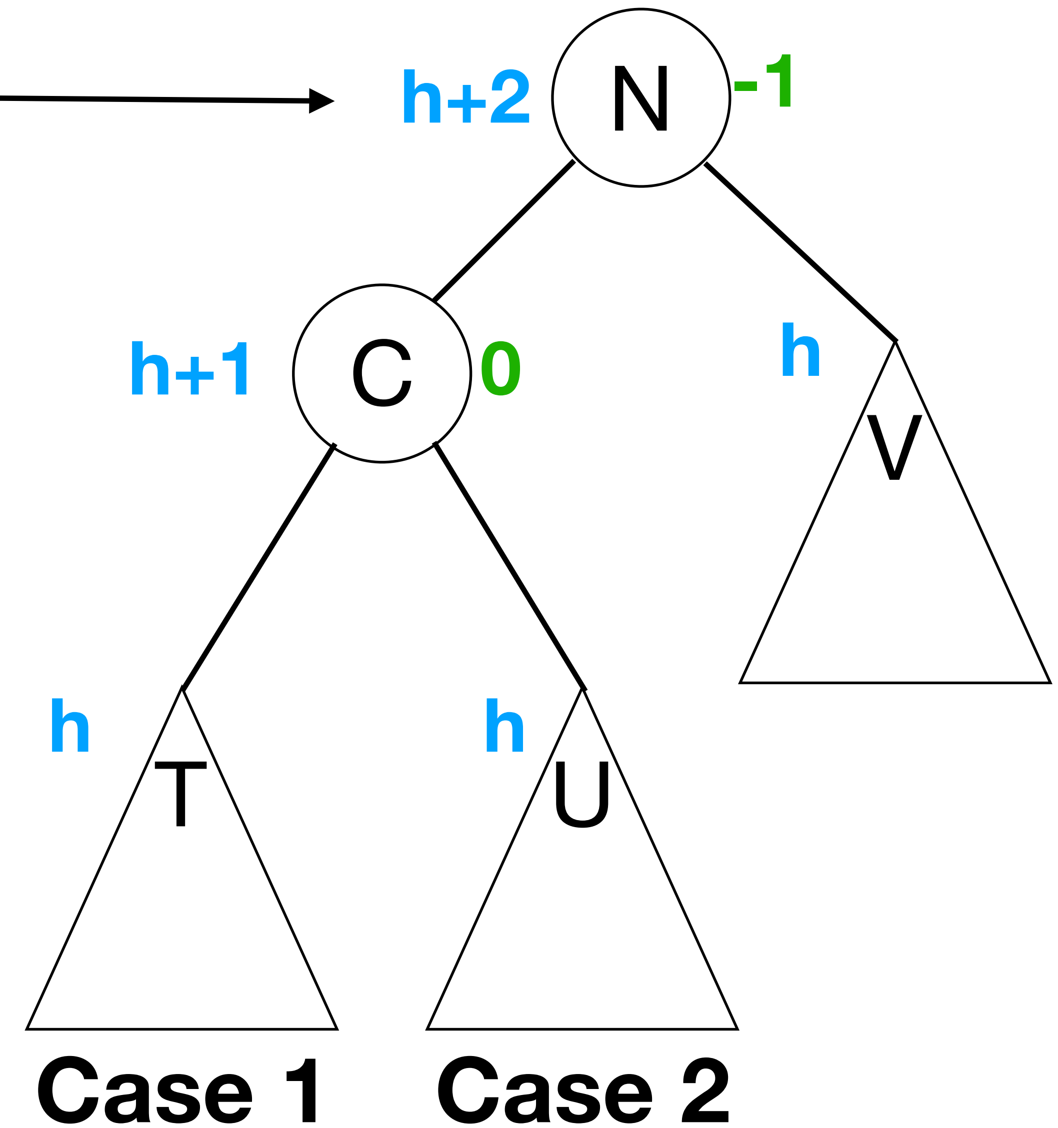


An insertion that unbalances n could gone one of four places.

Implementation

```
void rebalance(n):
  if bal(n) < -1:
    if bal(n.left) < 0
      // case 1:
      // rightRot(n)
    else:
      // case 2:
      // leftRot(n.L);
      // rightRot(n)
  else if bal(n) > 1:
    if bal(n.right) < 0:
      // case 3:
      // rightRot(n.R);
      // leftRot(n)
    else:
      // case 4:
      // leftRot(n)
```



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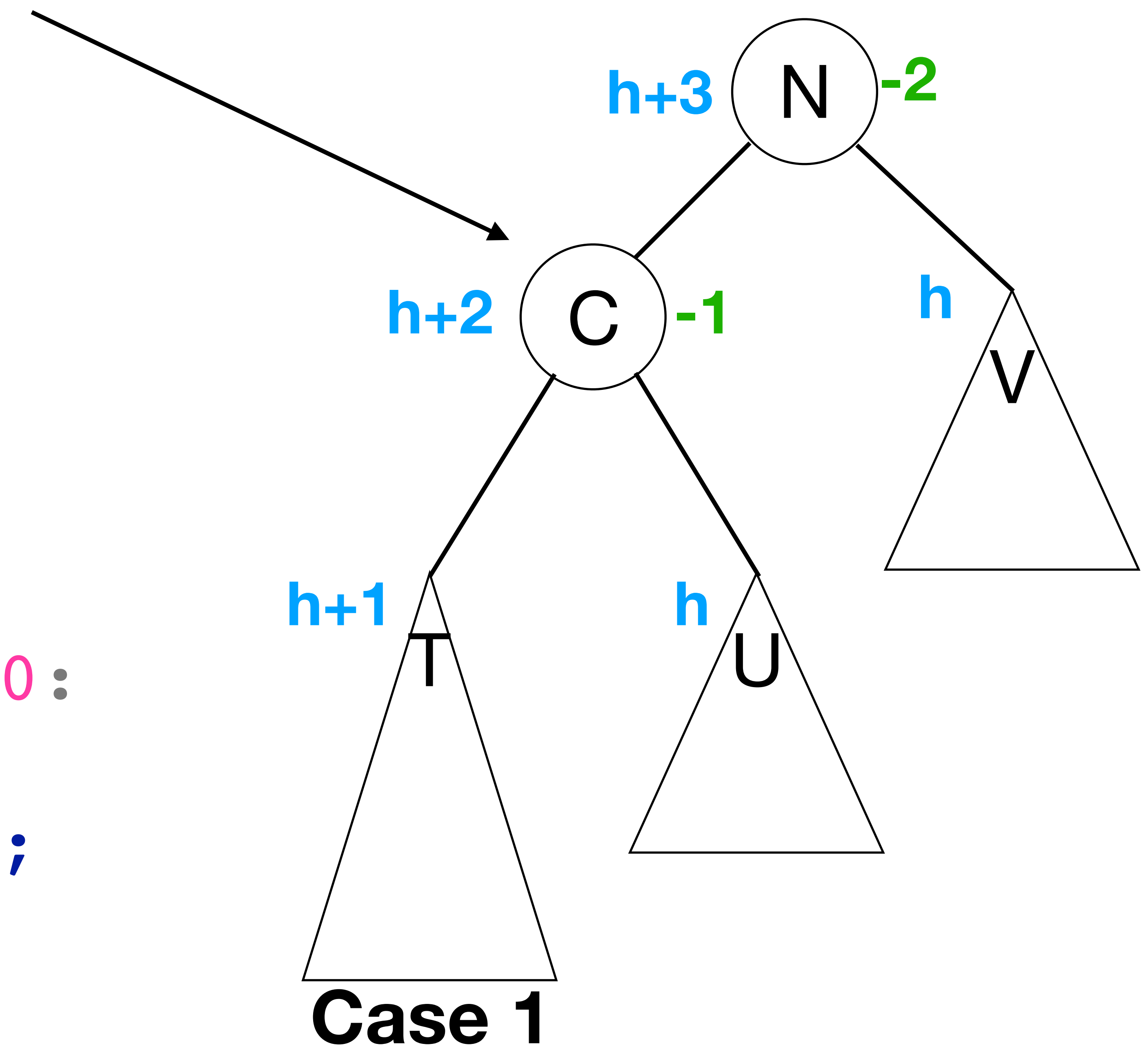

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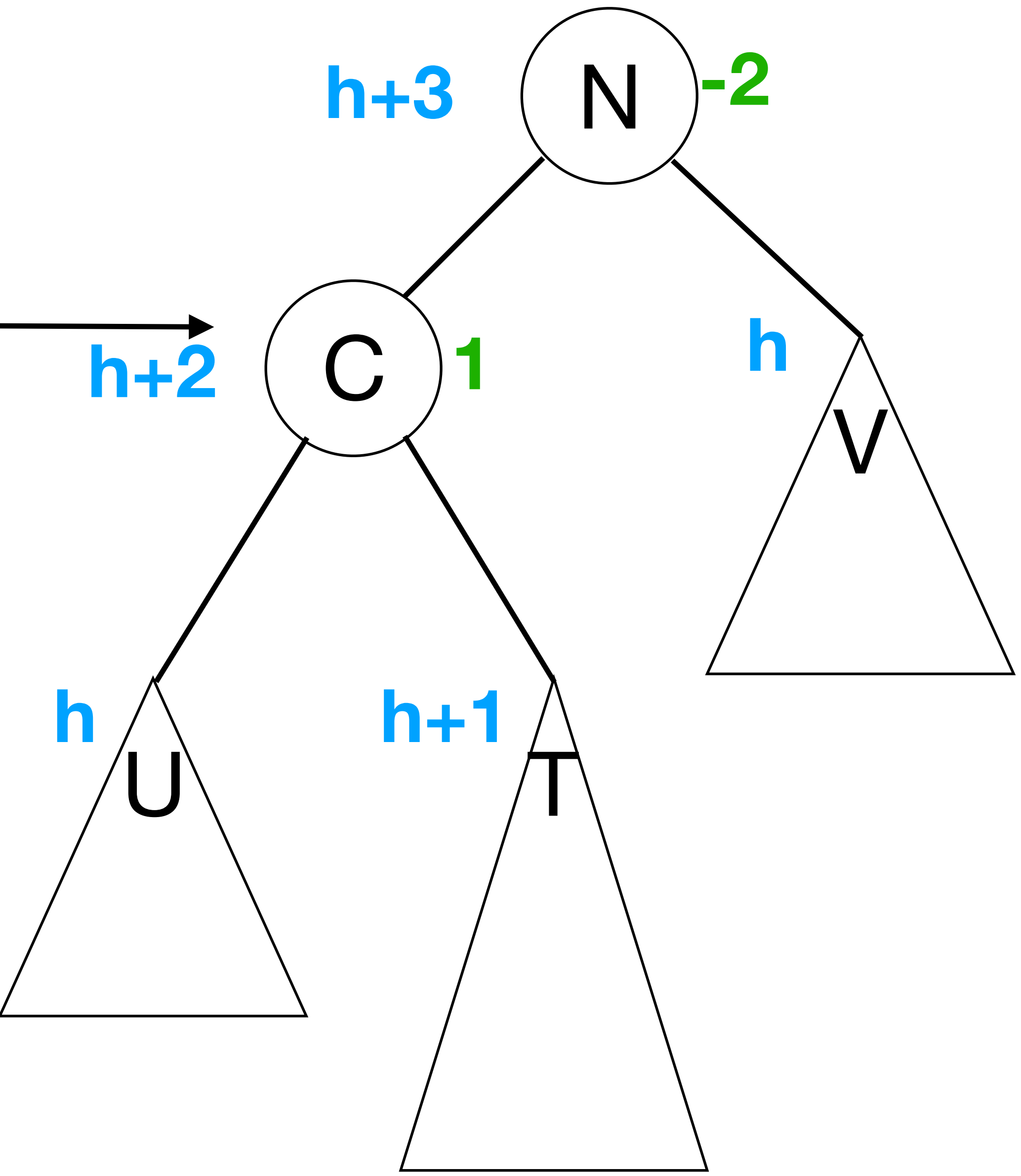

Case 2

Implementation

```
void rebalance(n):
  if bal(n) < -1:
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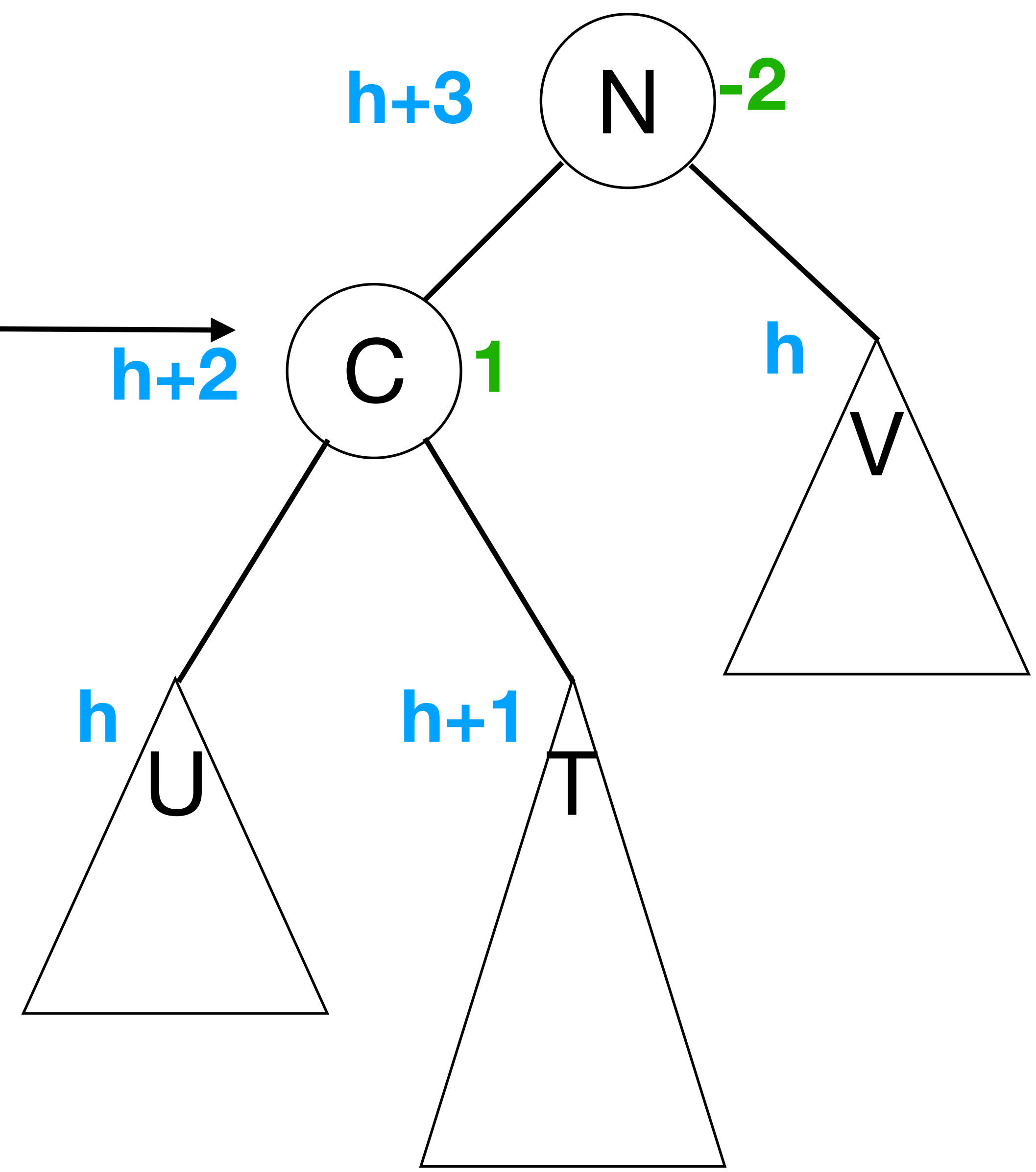
      // case 1:
      // rightRot(n)
    else:
      // case 2:
      // leftRot(n.L);
      // rightRot(n)
  else if bal(n) > 1:
    if bal(n.right) < 0:
      // case 3:
      // rightRot(n.R);
      // leftRot(n)
    else:
      // case 4:
      // leftRot(n)
```



Case 2

Cases 3 and 4 are symmetric with 2 and 1

Implementation

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```

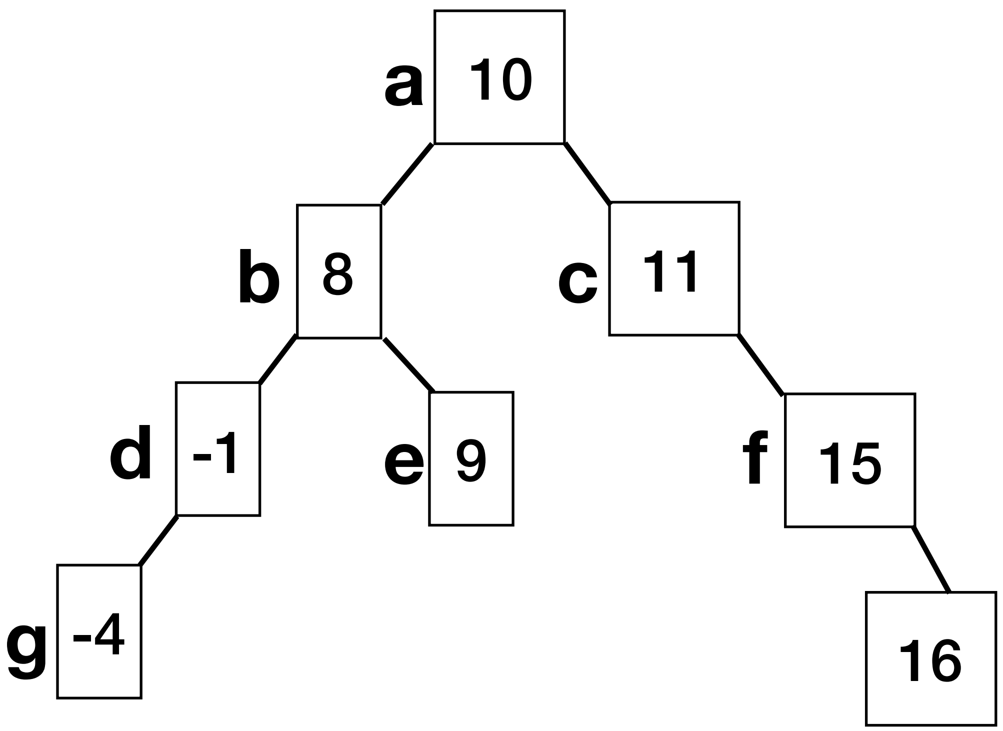
Cases 3 and 4 are symmetric with 2 and 1.

Details

- Implementing `bal`:
  - calculating height as in lab 4 is O(n)! Not good enough.
  - Nodes track their height and update when the tree changes
  - Update each node's height **on the way up the tree**, calculating height using only its children's heights.

Insertion with Rebalance

```
insert(Node n, int v):
  //…(other case, irrelevant here)
  else: // v > n.value
    if n has right:
      insert(n.right, v)
   else:
      // attach new node w/ value
      //   v to n.right
  rebalance(n);
```




```
insert(a, 16)
=>insert(c, 16)
  =>insert(f, 16)
    =>attach new node
      rebalance(f) already balanced
    rebalance(c) perform rotation
  rebalance(a) already balanced
```

How did we know what rotation to do?