CSCI 241

Lecture 14a
AVL trees
AVL insertion

Goals

- Know the definition and properties of an AVL tree.
- Know how AVL insertion uses rebalance to correct violations of the AVL property due to an insertion.

AVL Trees

An AVL tree is a Binary Search Tree in which:

$$\boxed{-1 <= \text{balance}(n) <= 1 \text{ for all nodes n.}}$$

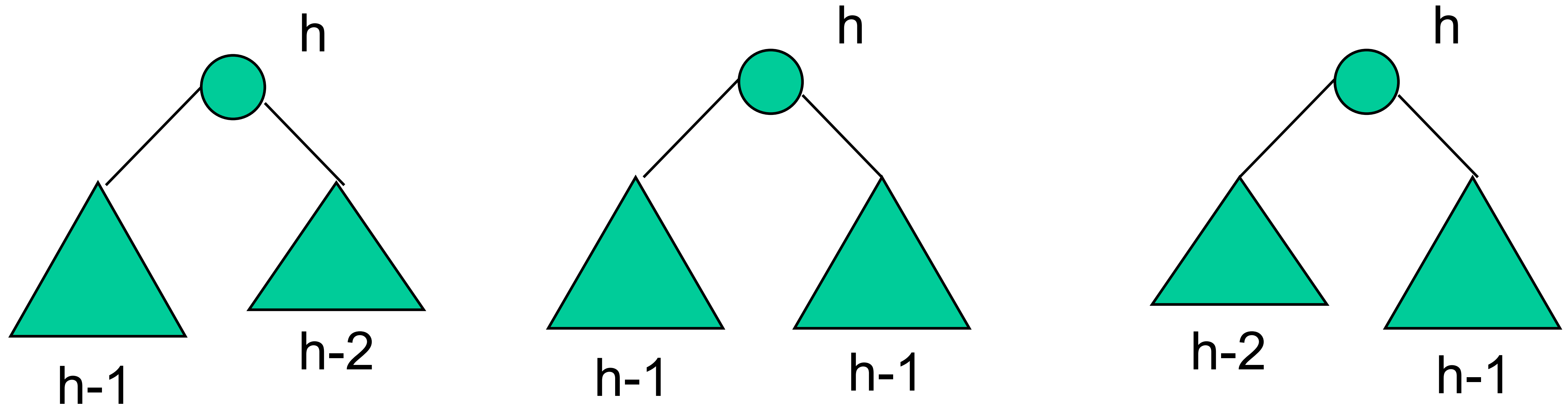
Balance(n): height(n.right) - height(n.left)

Devised by **A**delson-**V**elsky and **L**andis

Balance Factor in AVL Trees

AVL property: -1 <= balance(n) <= 1 for all nodes n.

Every subtree in an AVL tree looks like one of these three trees:



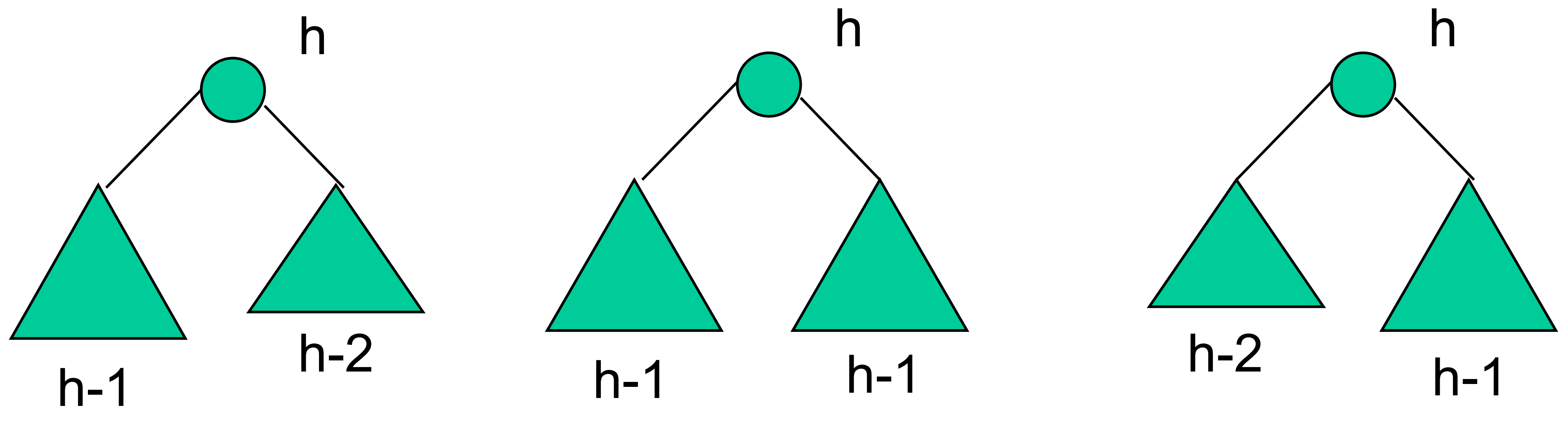
(a) Balance factor:-1 (b) Balance factor: 0 (c) Balance factor: +1

AVL Trees: Insertion

AVL property: -1 <= b(n) <= 1 for all nodes n.

To insert into an AVL tree:

1. Do a normal BST insertion
2. Fix any violations of the AVL property using rotations.



(a) Balance factor:-1 (b) Balance factor: 0 (c) Balance factor: +1

Refresher: BST Insertion

```
/* insert a node with value v into the
 * tree rooted at n. pre: n is not null. */
insert(Node n, int v):
  if n.value == v: return // (duplicate)
  if v < n.value:
    if n has left:
      insert(n.left, v)
    else:
      // attach new node w/ value v to n.left
  else: // v > n.value
    if n has right:
      insert(n.right, v)
    else:
      // attach new node w/ value v to n.right
```

AVL Insertion

```
/* insert a node with value v into the
 * tree rooted at n. pre: n is not null. */
insert(Node n, int v):
  if n.value == v: return // (duplicate)
  if v < n.value:
    if n has left:
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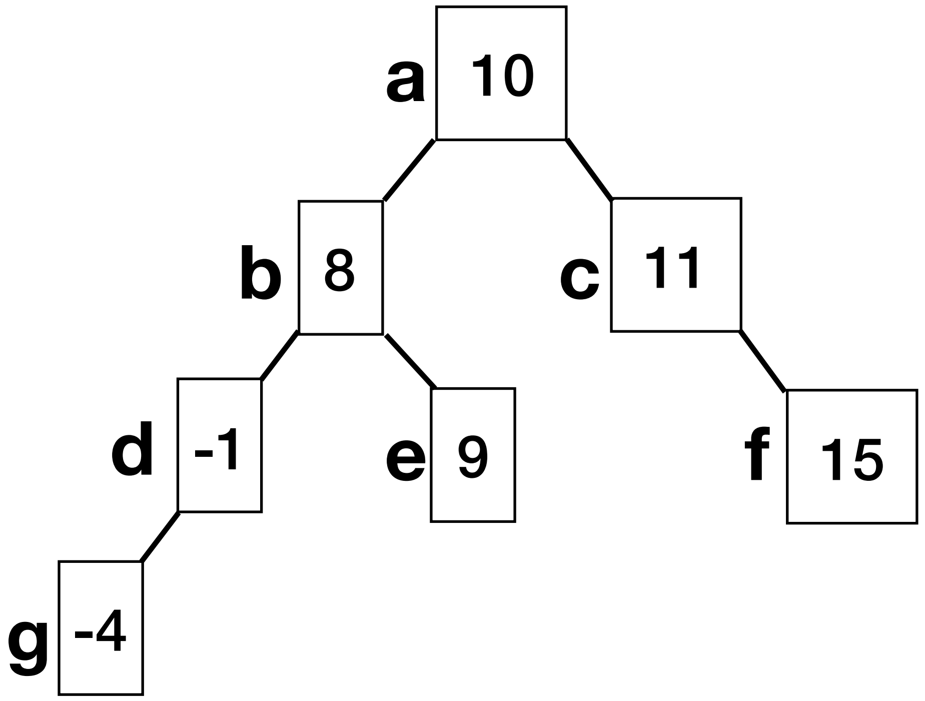
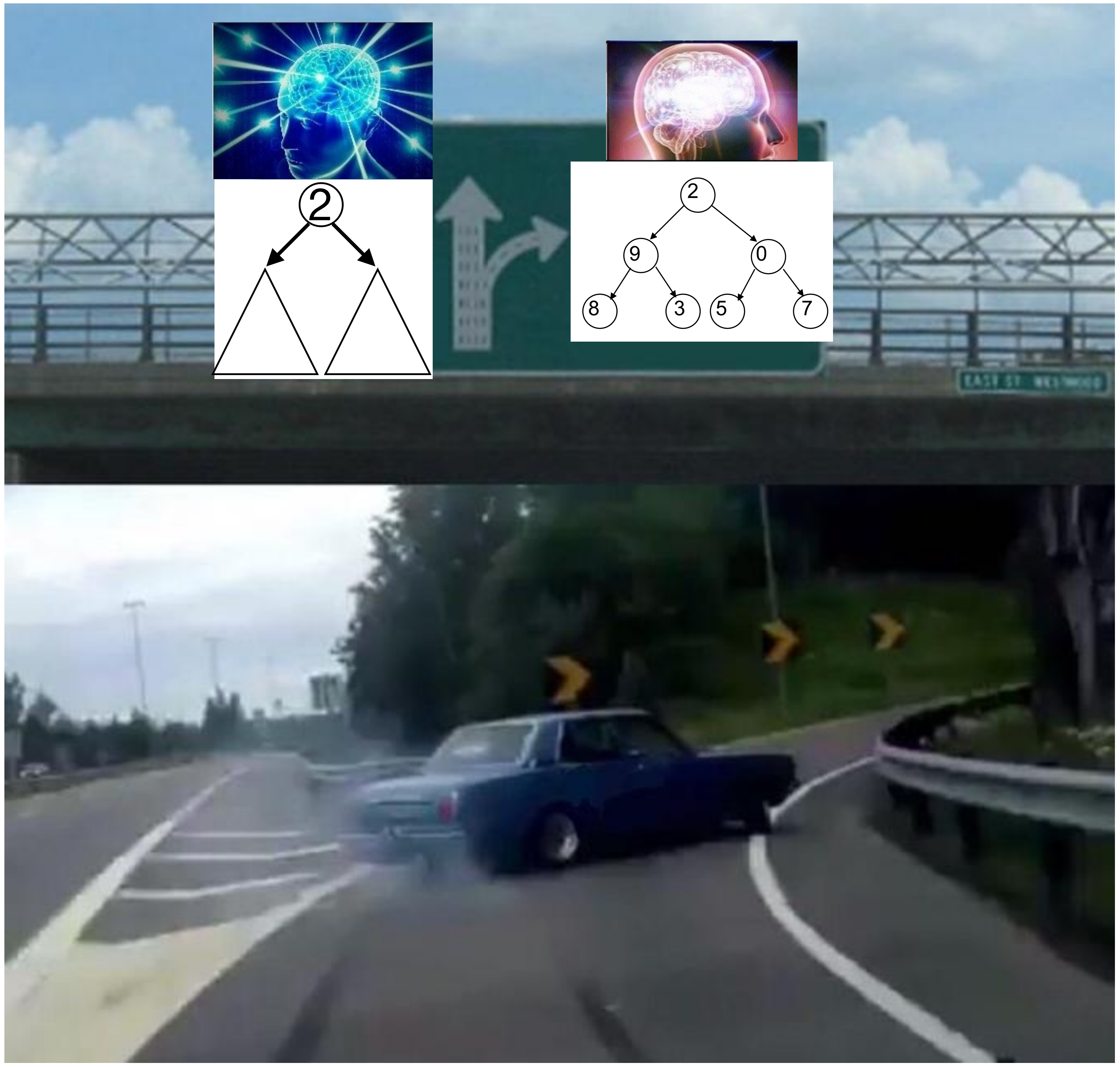
    else:
      // attach new node w/ value v to n.left
  else: // v > n.value
    if n has right:
      insert(n.right, v)
    else:
      // attach new node w/ value v to n.right
  rebalance(n);  ←
```

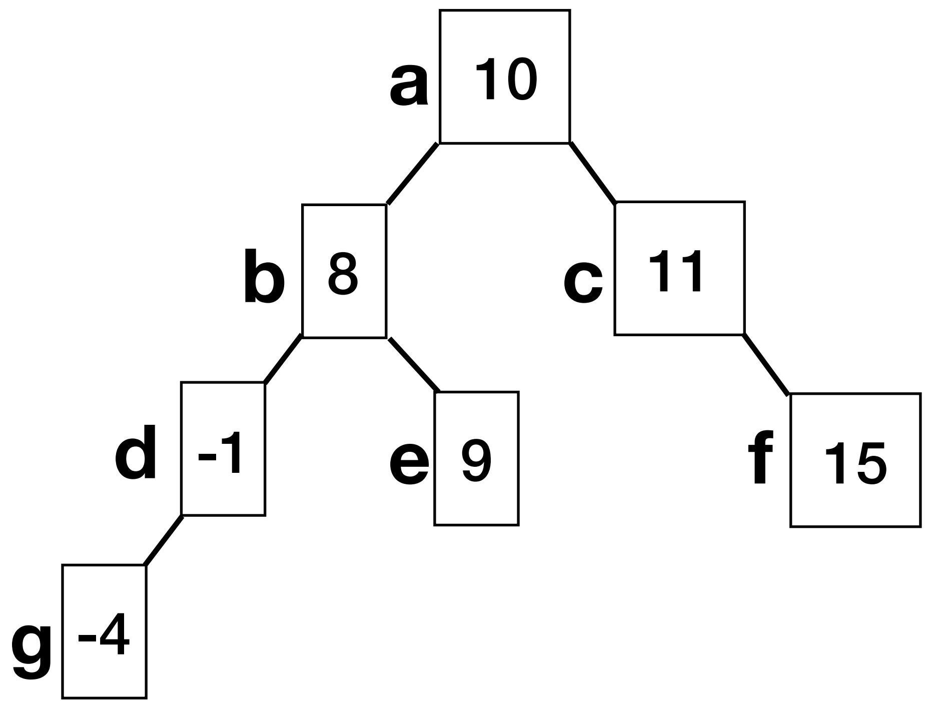
AVL Insertion



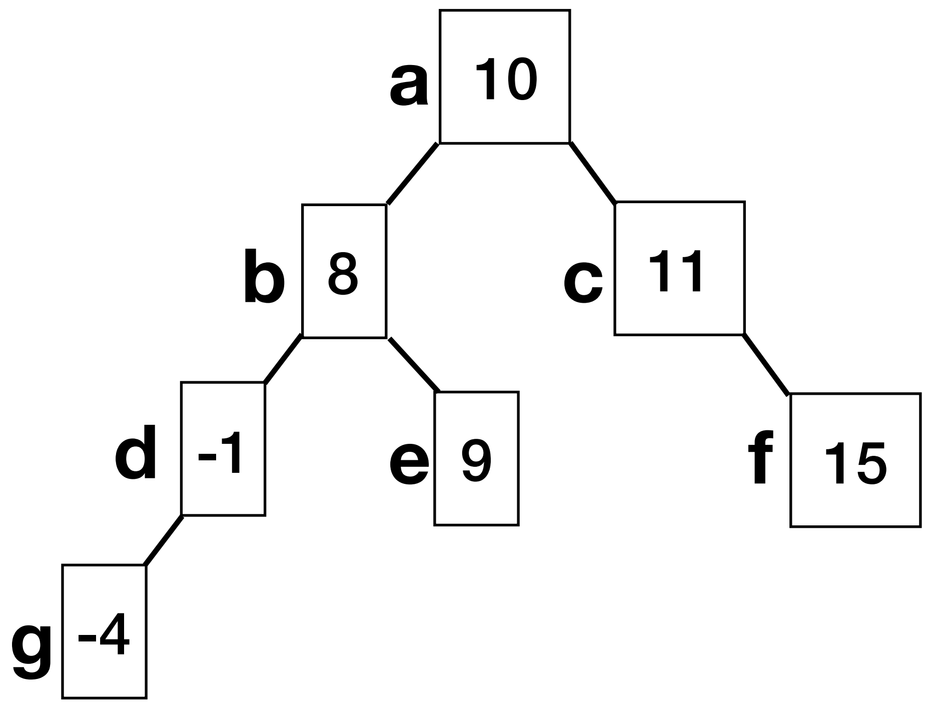


AVL Insertion

First: is this an AVL tree?



AVL Insertion



```
insert(Node n, int v):
  //…(other case, irrelevant here)
  else: // v > n.value
    if n has right:
      insert(n.right, v)
   else:
      // attach new node w/ value
      //   v to n.right
  rebalance(n);
```

AVL Insertion

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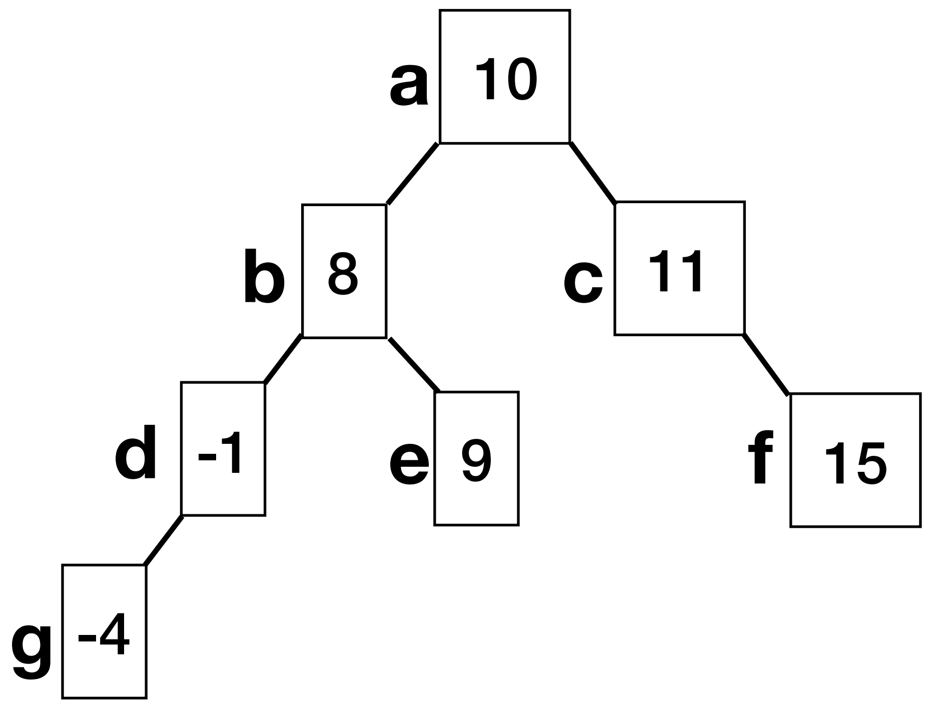

```
insert(a, 16)
```

AVL Insertion

```
insert(Node n, int v):
  //…(other case, irrelevant here)
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    if n has right:
      insert(n.right, v)
   else:
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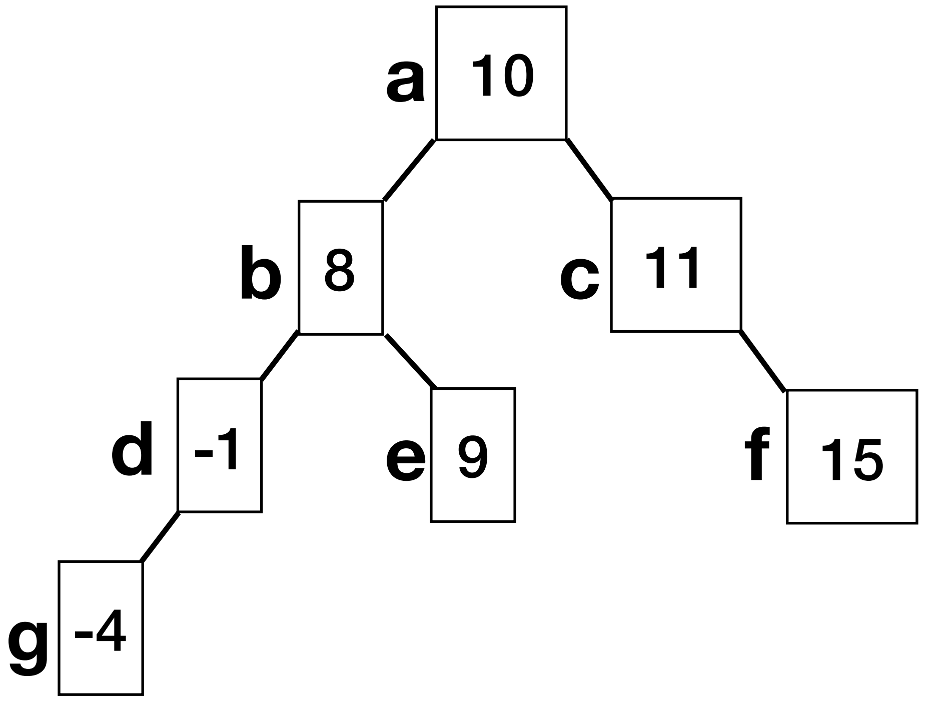
      //   v to n.right
  rebalance(n);
```



```
insert(a, 16)
=>insert(c, 16)




  rebalance(a)
```

AVL Insertion

```
insert(Node n, int v):
  //…(other case, irrelevant here)
  else: // v > n.value
    if n has right:
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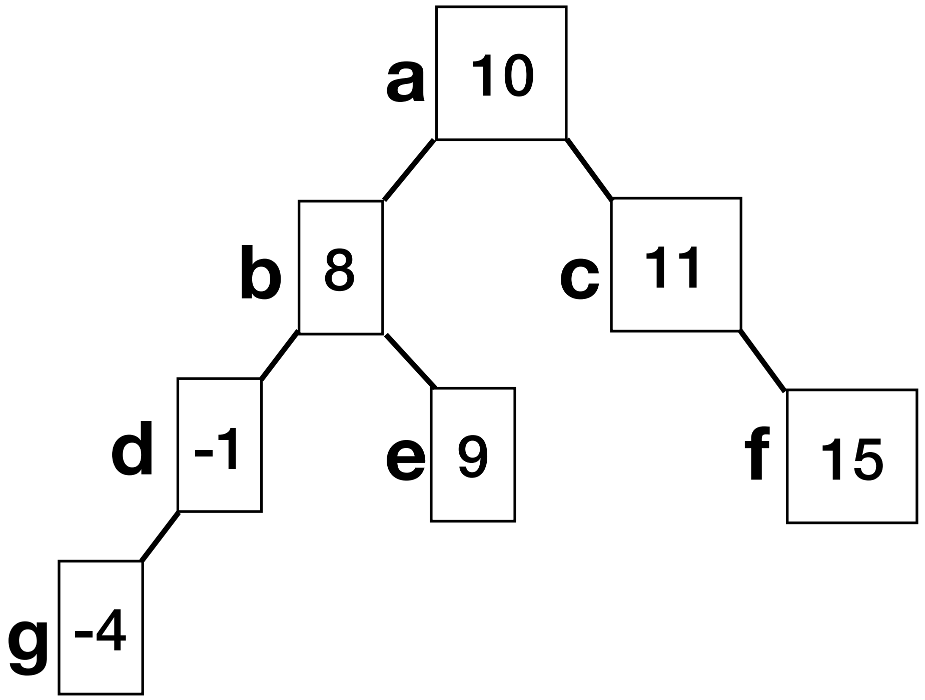
   else:
      // attach new node w/ value
      //   v to n.right
  rebalance(n);
```



```
insert(a, 16)
=>insert(c, 16)
  =>insert(f, 16)



    rebalance(c)
  rebalance(a)
```

AVL Insertion

```
insert(Node n, int v):
  //…(other case, irrelevant here)
  else: // v > n.value
    if n has right:
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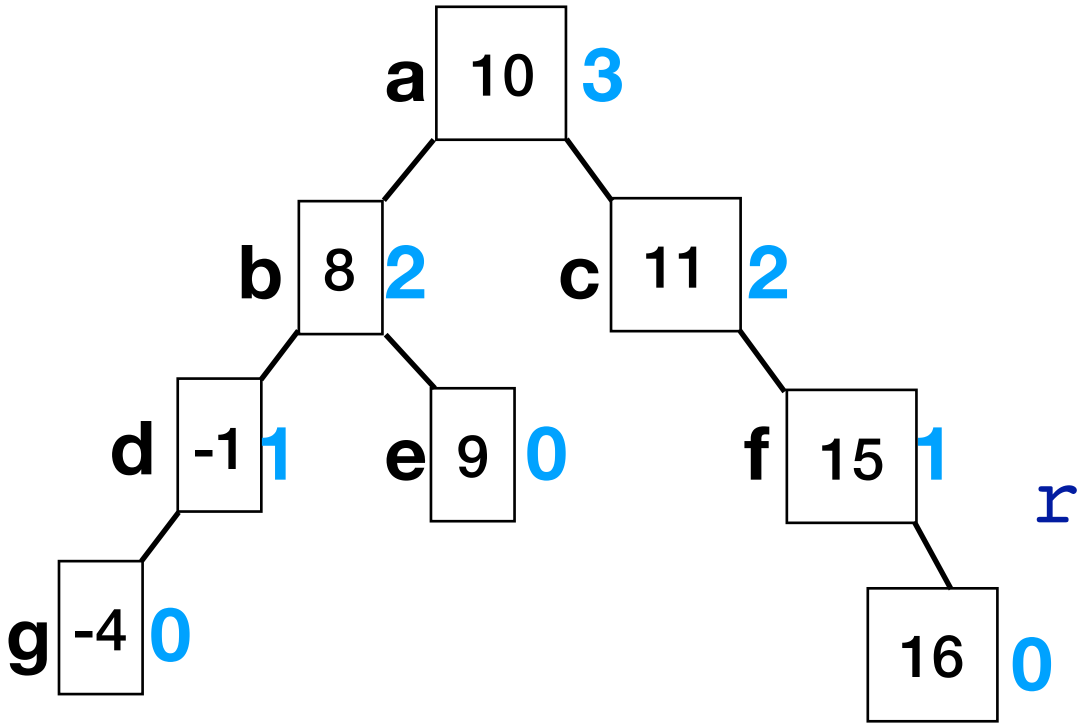
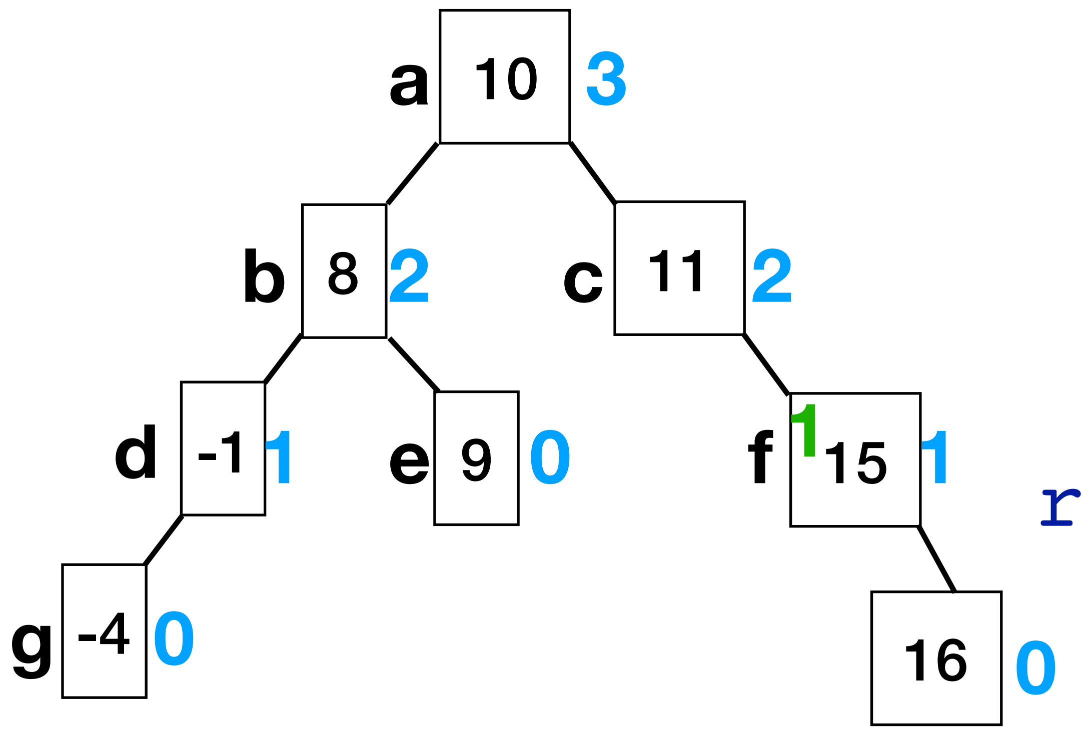
   else:
      // attach new node w/ value
      //   v to n.right
  rebalance(n);
```



```
insert(a, 16)
=>insert(c, 16)
  =>insert(f, 16)
    =>attach new node
      rebalance(f)
    rebalance(c)
  rebalance(a)
```

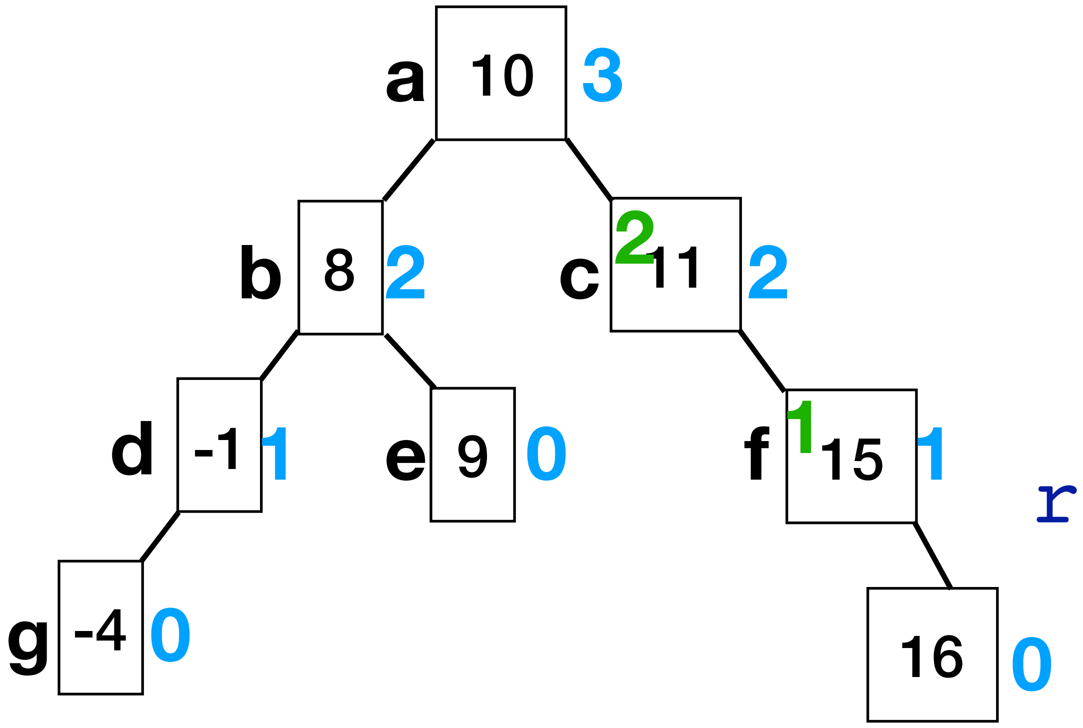
AVL Insertion

```
insert(Node n, int v):
  //…(other case, irrelevant here)
  else: // v > n.value
    if n has right:
      insert(n.right, v)
   else:
      // attach new node w/ value
      //   v to n.right
  rebalance(n);
```



```
insert(a, 16)
=>insert(c, 16)
  =>insert(f, 16)
    =>attach new node
      rebalance(f)
    rebalance(c)
  rebalance(a)
```
already balanced

AVL Insertion



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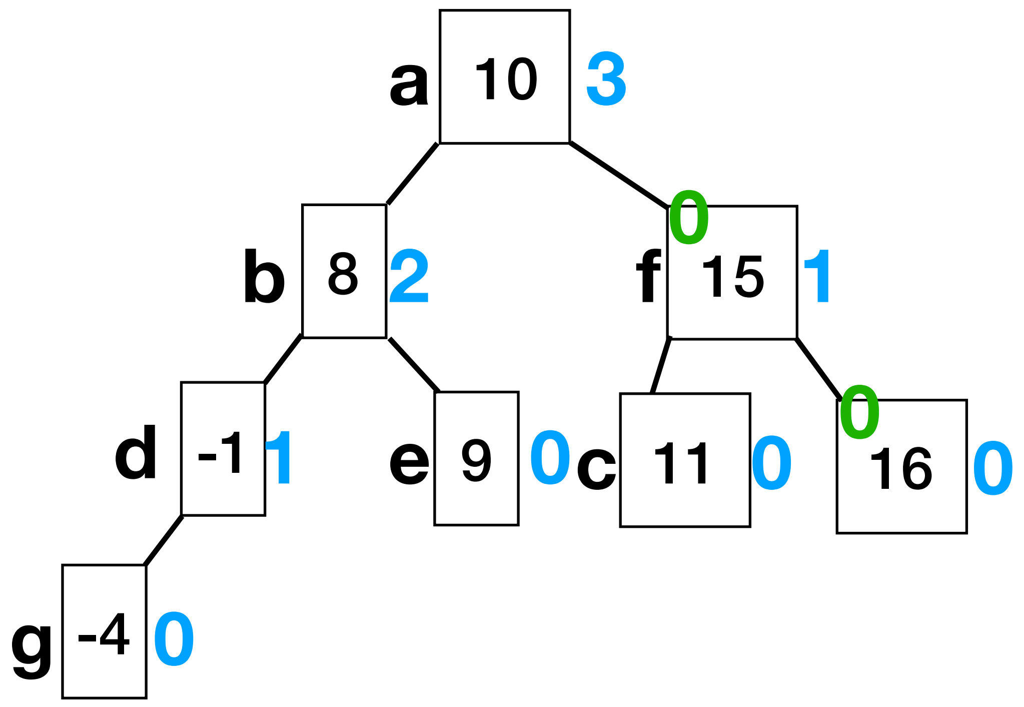
    if n has right:
      insert(n.right, v)
   else:
      // attach new node w/ value
      //   v to n.right
  rebalance(n);
```

```
insert(a, 16)
=>insert(c, 16)
  =>insert(f, 16)
    =>attach new node
      rebalance(f) already balanced
    rebalance(c) perform rotation
  rebalance(a)
```

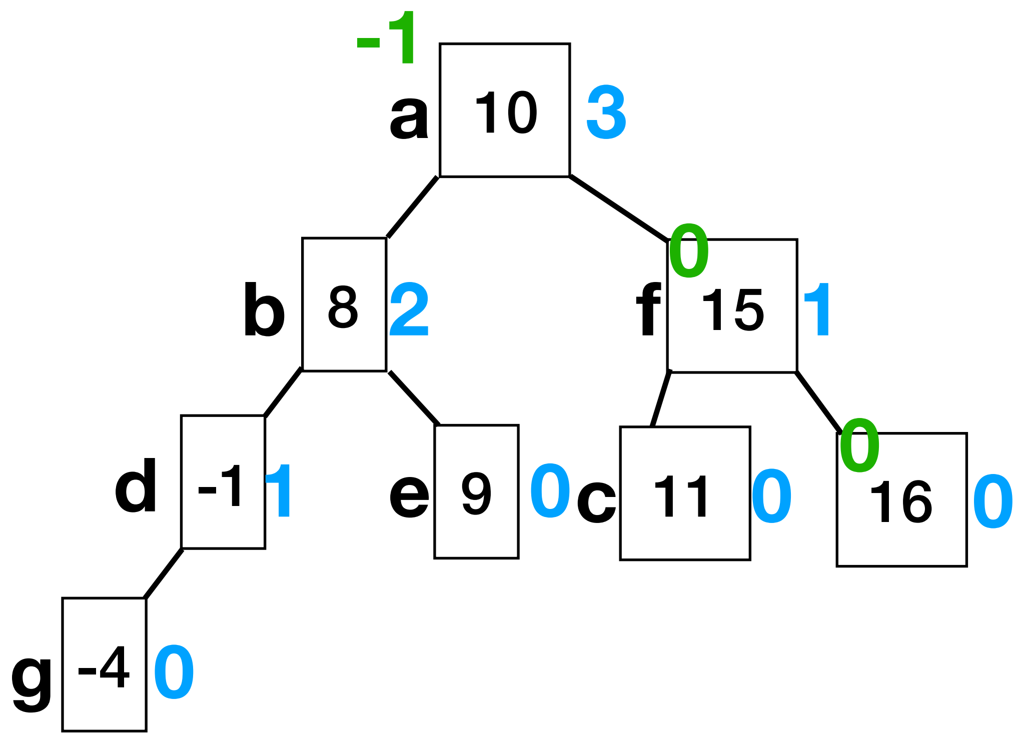
AVL Insertion

```
insert(Node n, int v):
  //…(other case, irrelevant here)
  else: // v > n.value
    if n has right:
      insert(n.right, v)
   else:
      // attach new node w/ value
      //    v to n.right
  rebalance(n);
```



```
insert(a, 16)
=>insert(c, 16)
  =>insert(f, 16)
    =>attach new node
      rebalance(f) already balanced
    rebalance(c) perform rotation
  rebalance(a)
```

AVL Insertion



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      //   v to n.right
rebalance(n);
```

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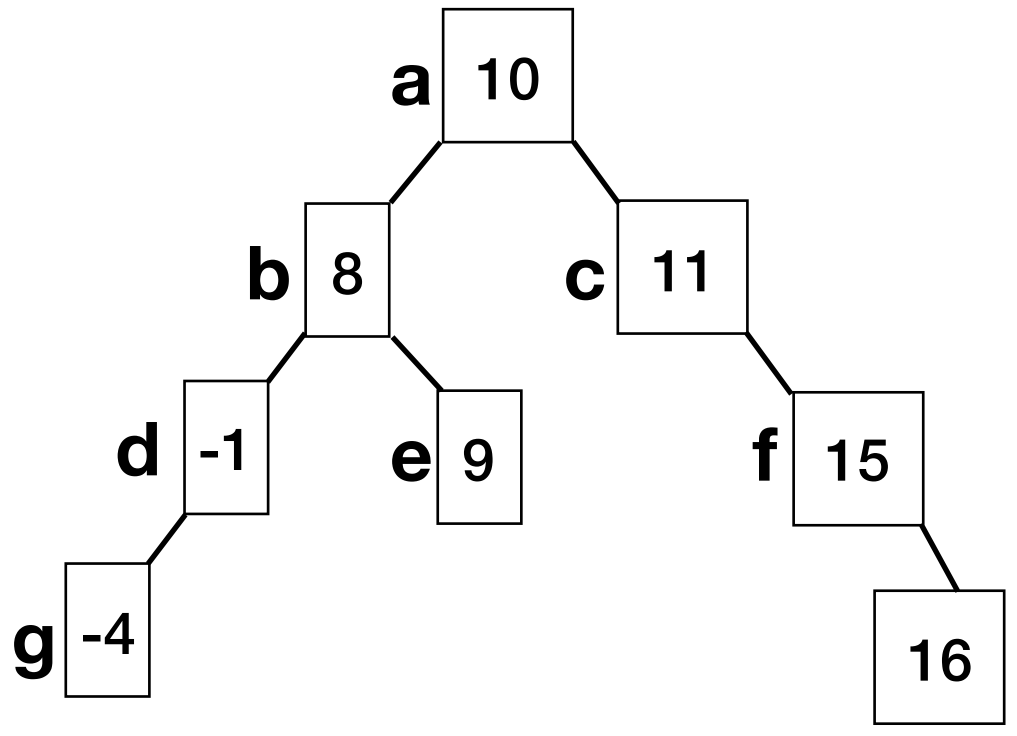
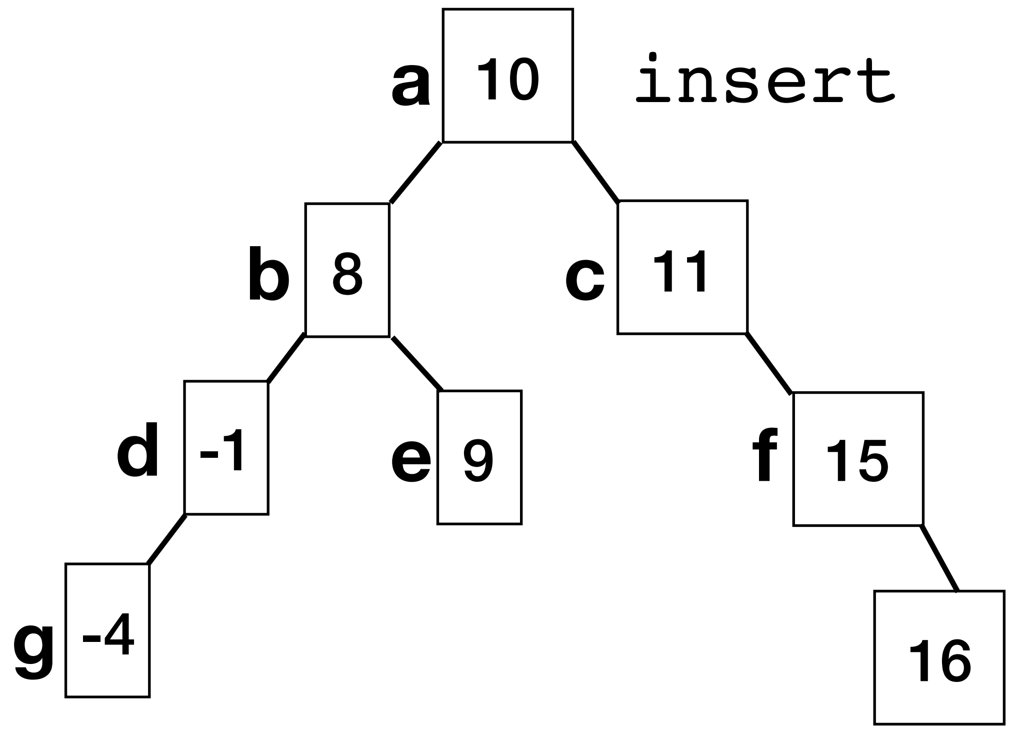
insert(a, 16)
=>insert(c, 16)
  =>insert(f, 16)
    =>attach new node
      rebalance(f) already balanced
    rebalance(c) perform rotation
  rebalance(a) already balanced
```

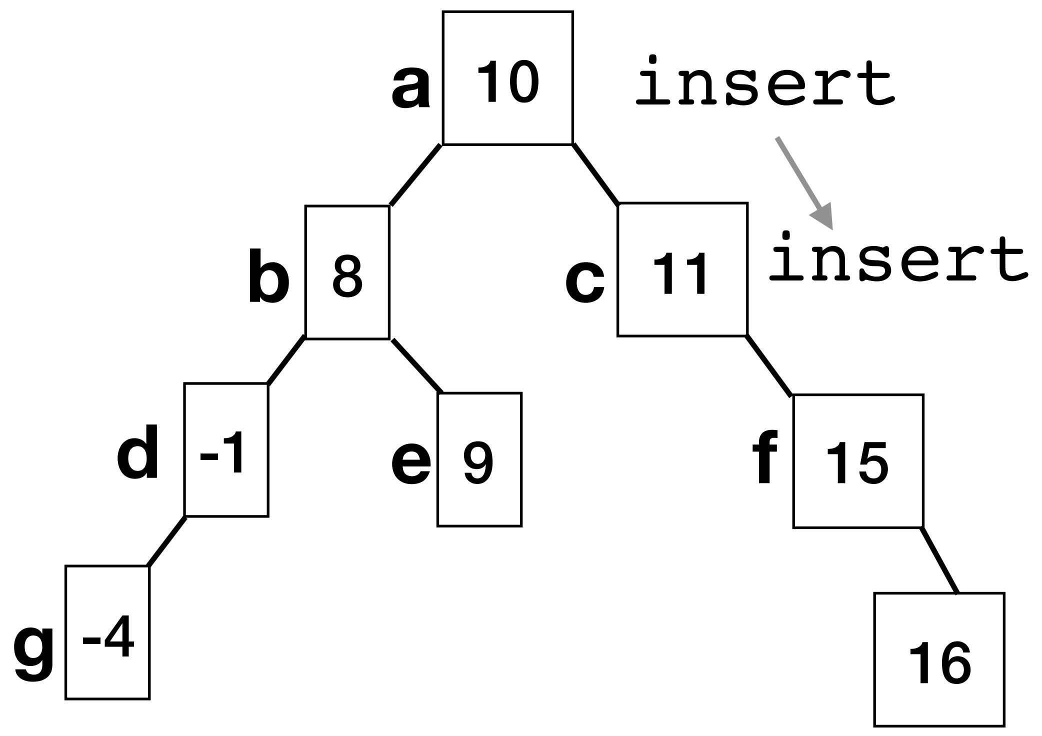
Order of actual execution



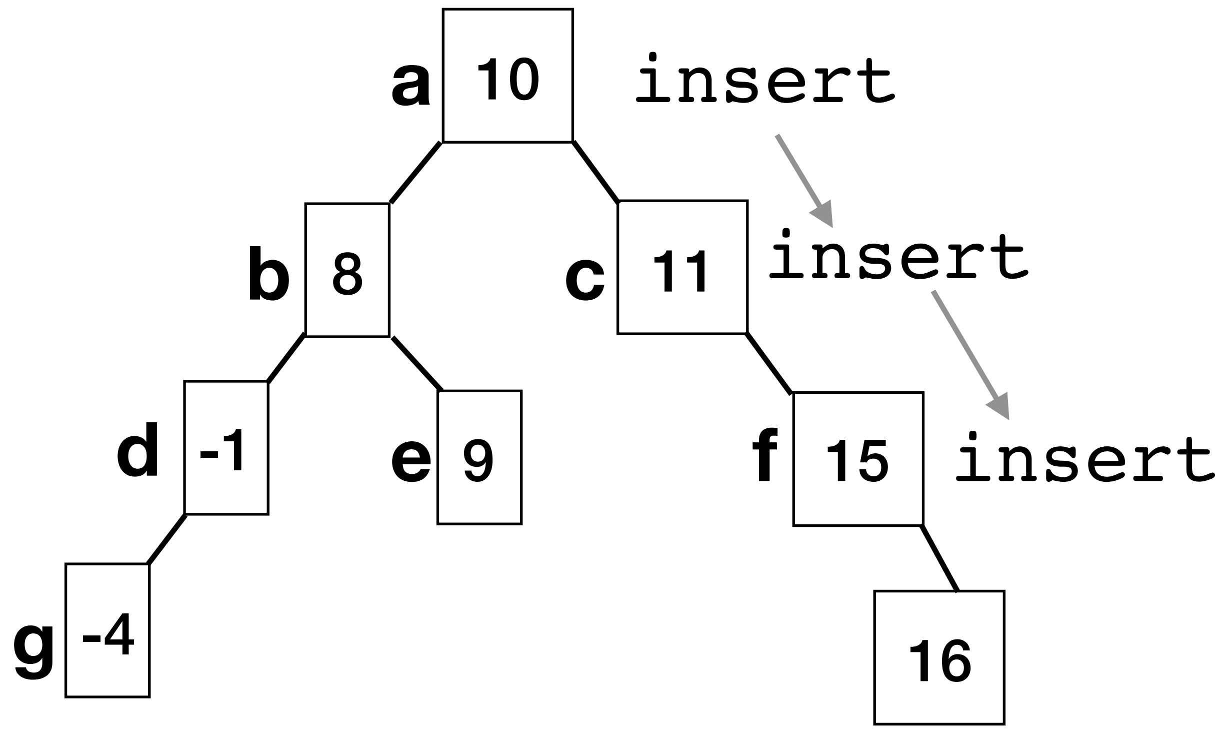
Order of actual execution



Order of actual execution



Order of actual execution



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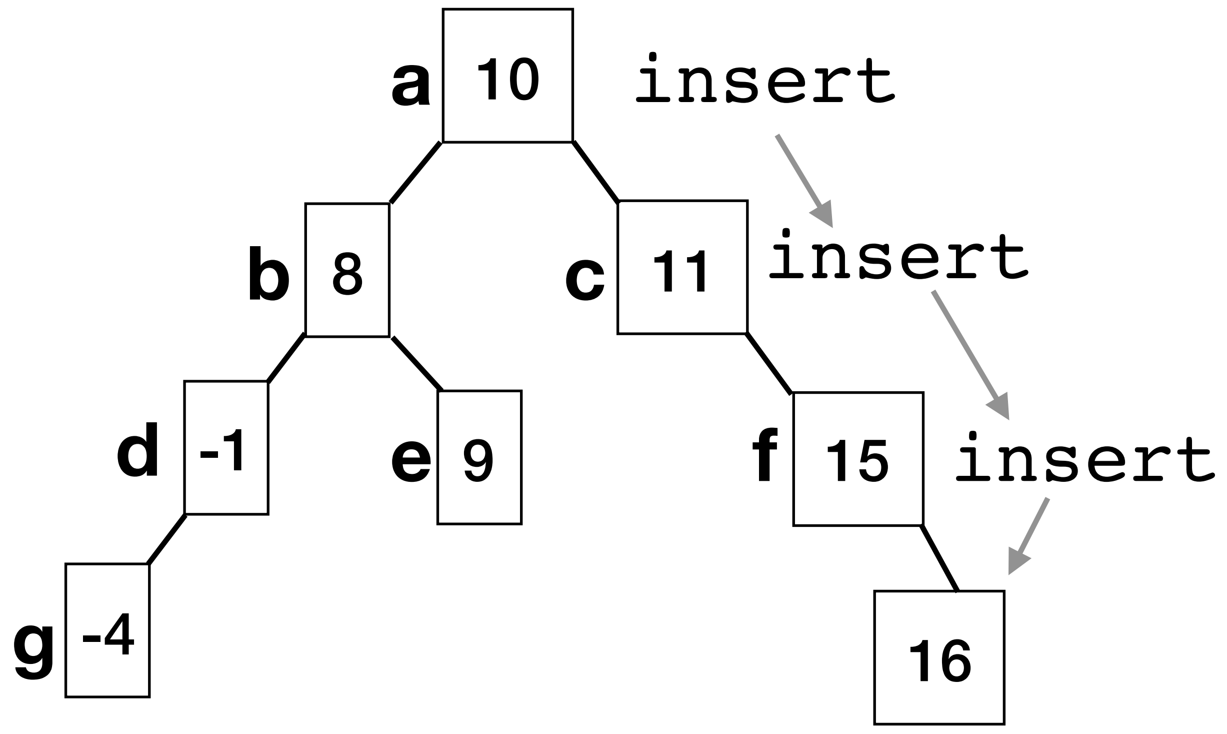

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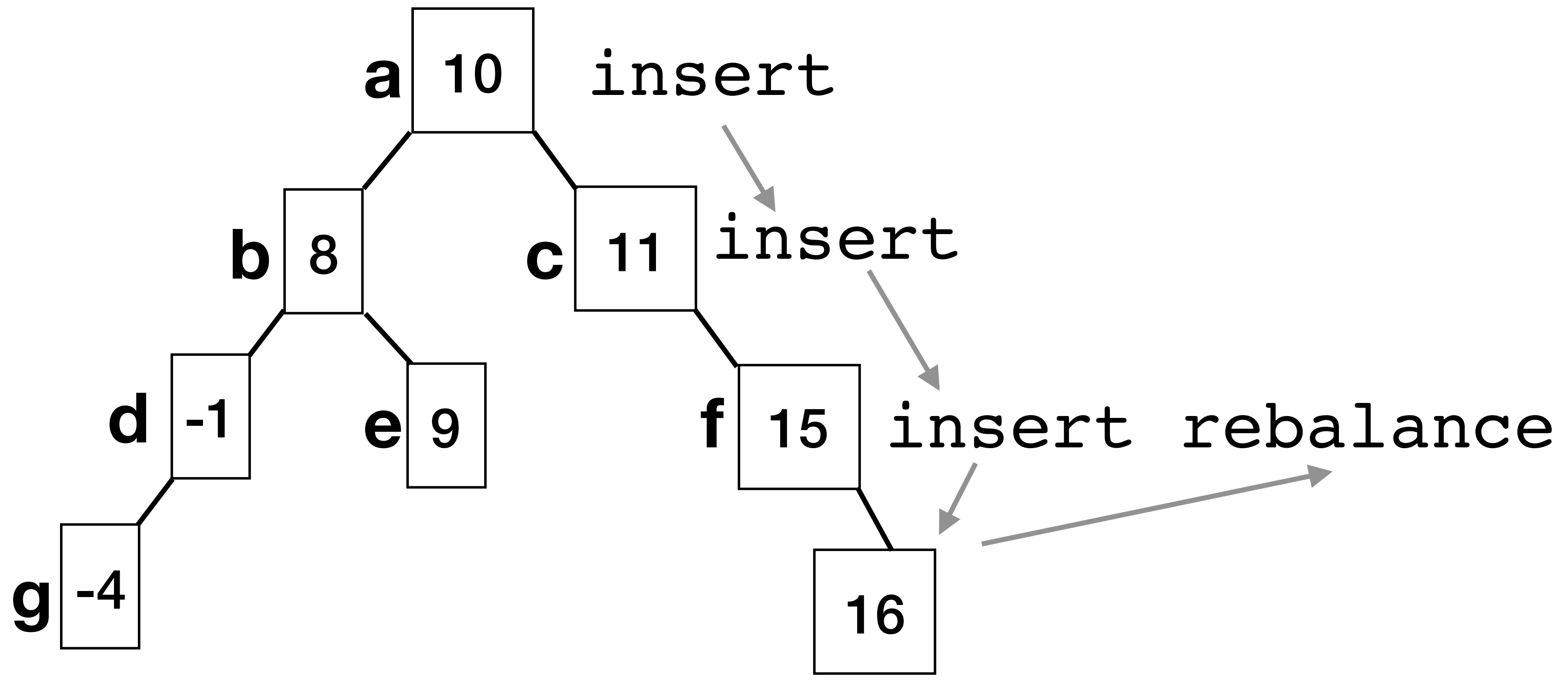

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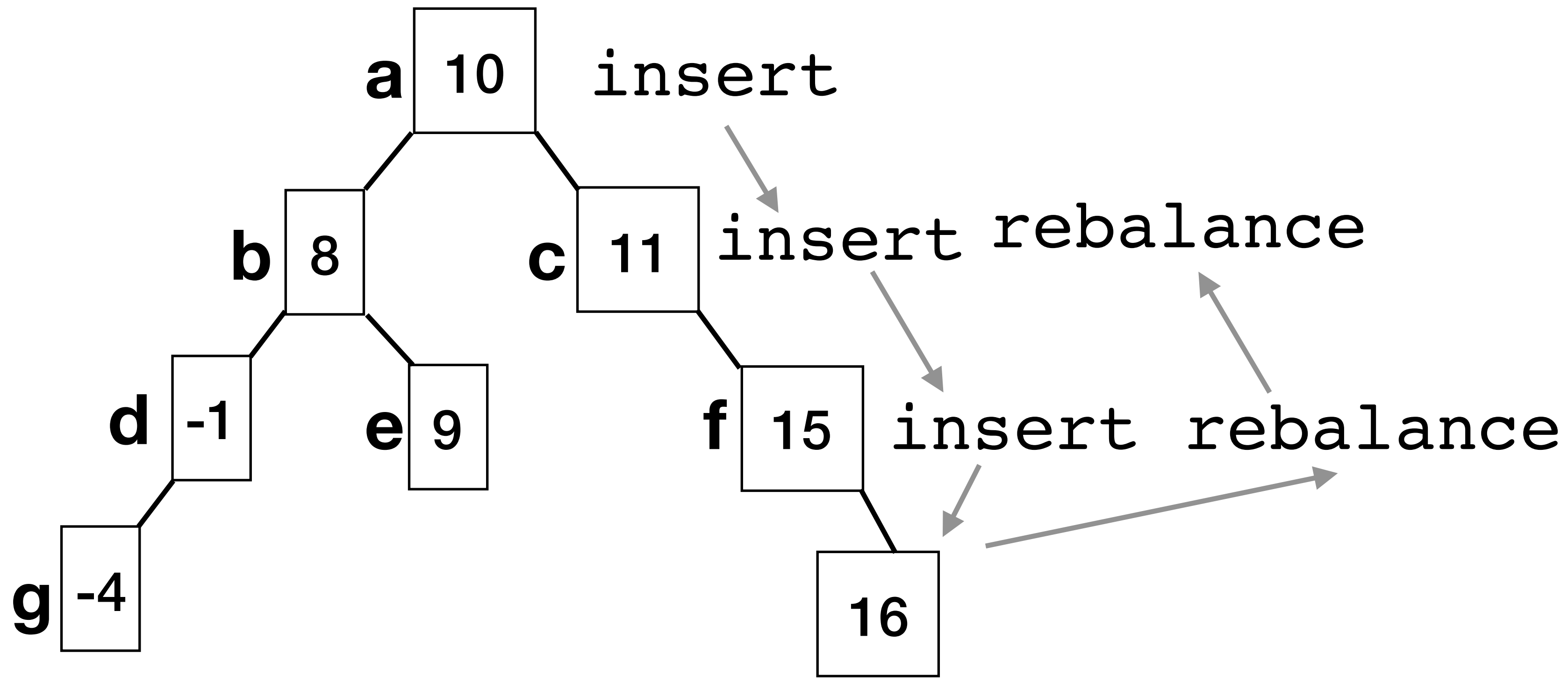

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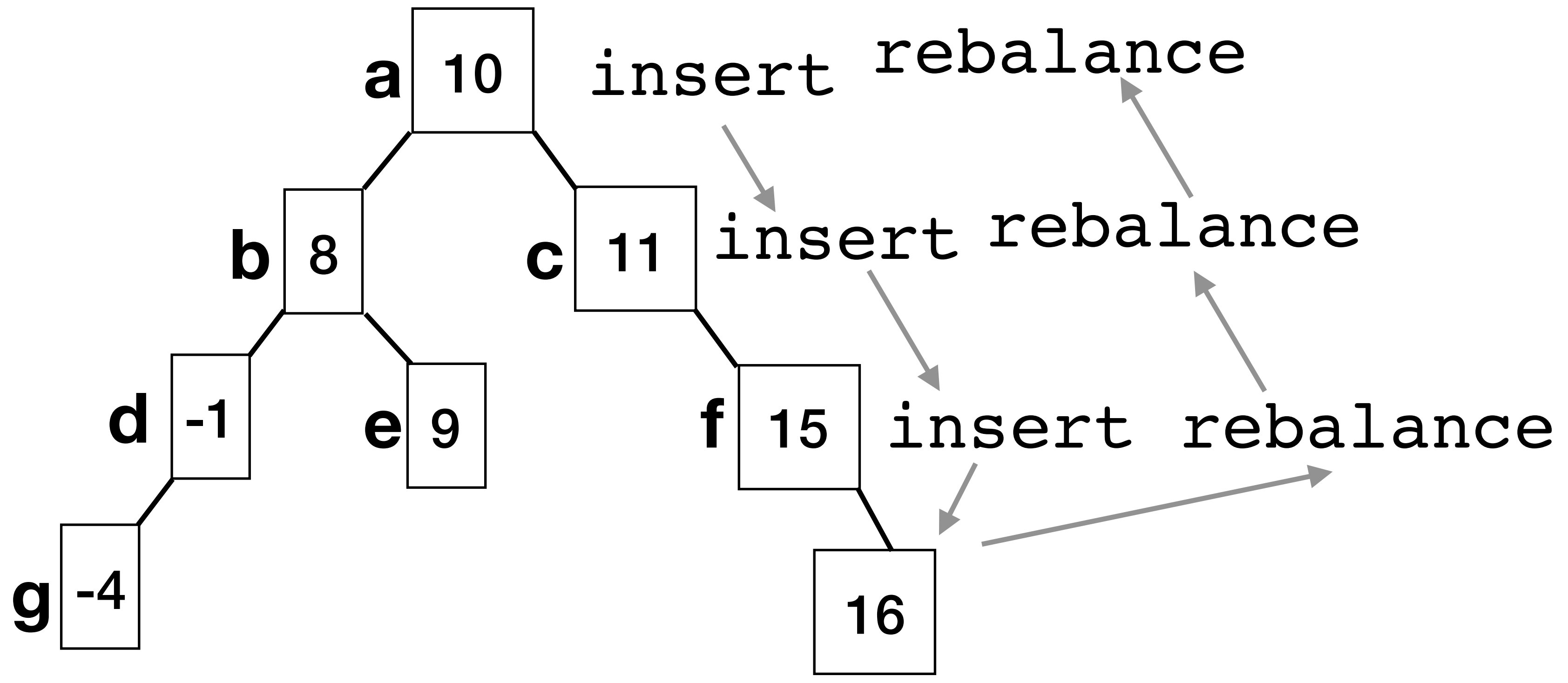
a
10
insert
rebalance
b
8
c
11
insert
rebalance
d
-1
e
9
f
15
insert
rebalance
g
-4
16