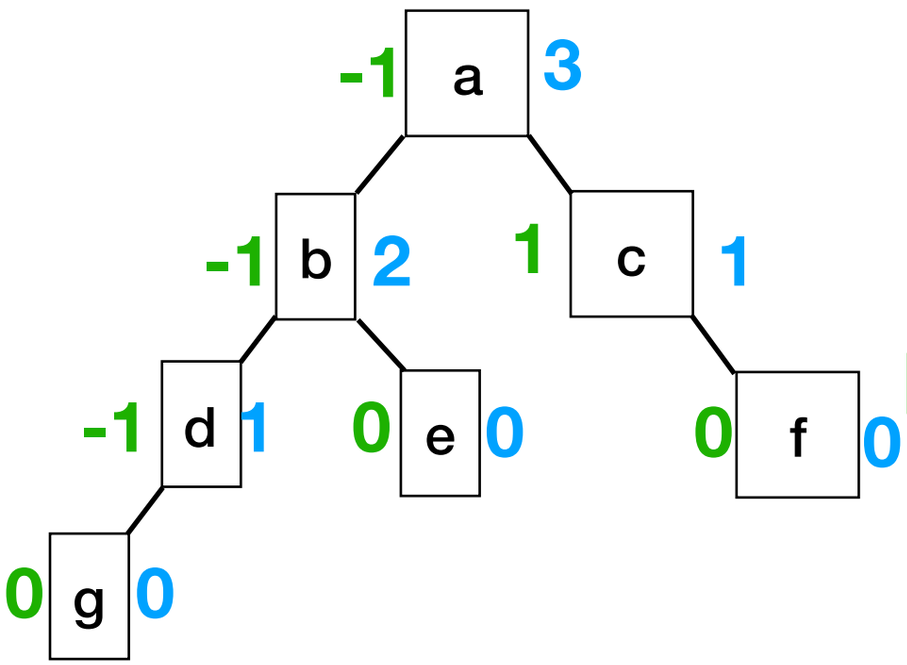
CSCI 241

Lecture 13 Exercises:
Balance Factor
Rotations

Height, Balance Factor



Height(t): path length from t's deepest descendant (leaf) to t's root.

Height(n): height of the subtree rooted at n

Balance(n): height(n.right) - height(n.left)

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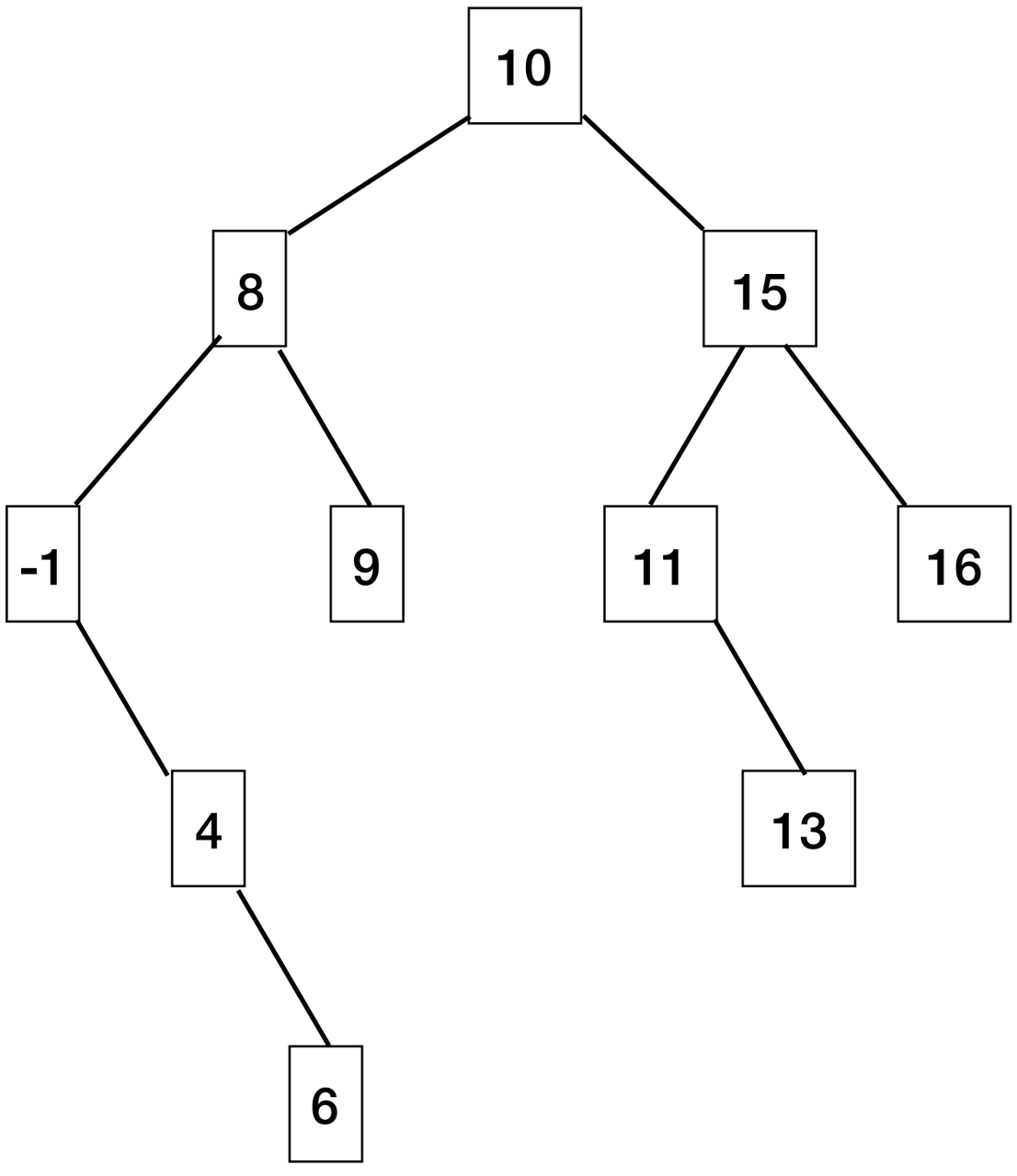
height(null) = -1
height(n) = 1 + max(height(n.left), height(n.right)
```

Height, Balance Factor



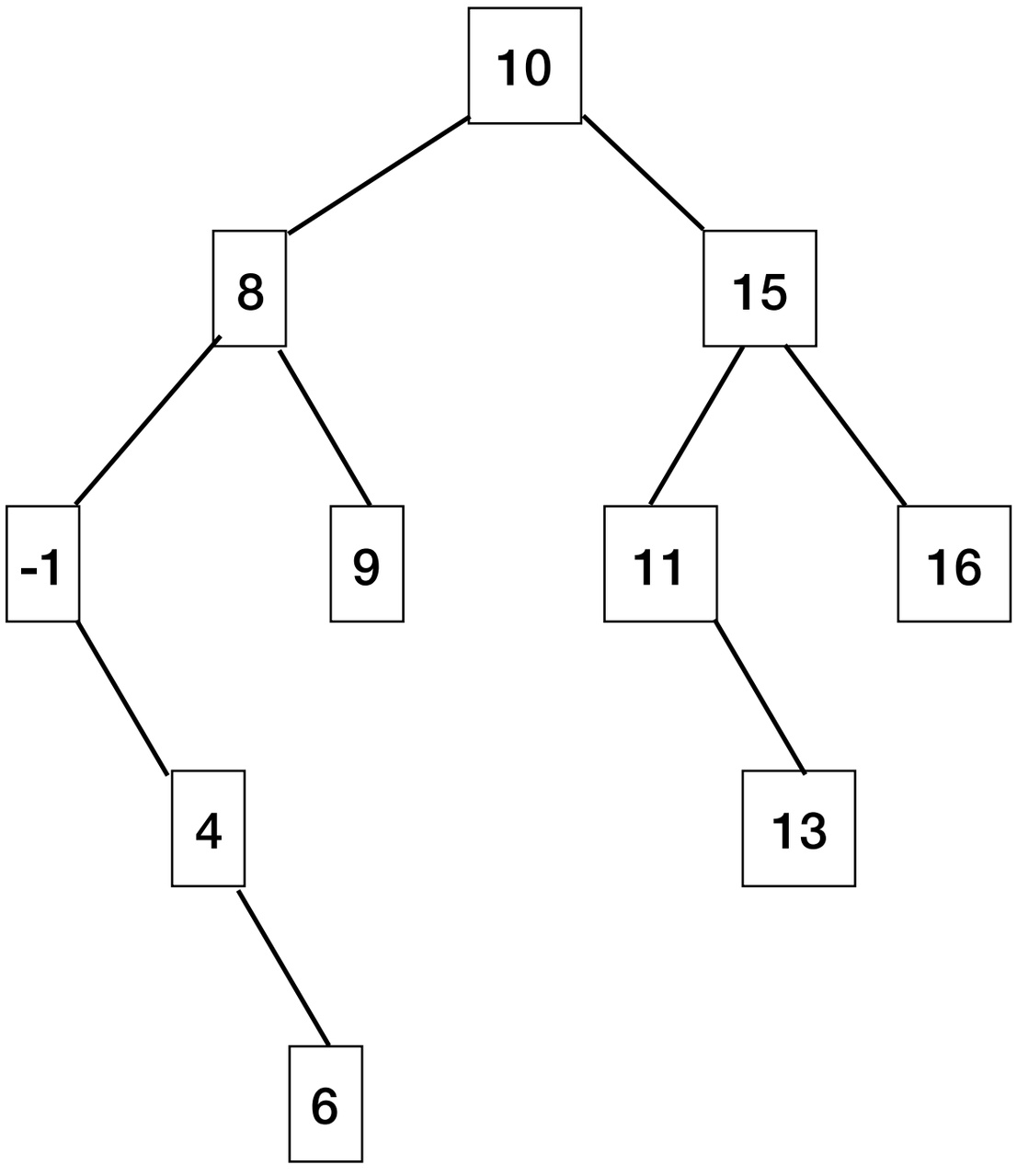
What is the height of

- The root?
- 15's left subtree?
- 15's right subtree?
- 16's left subtree?

What is the balance factor of

- node 16?
- node 15?
- node 10?

Height, Balance Factor



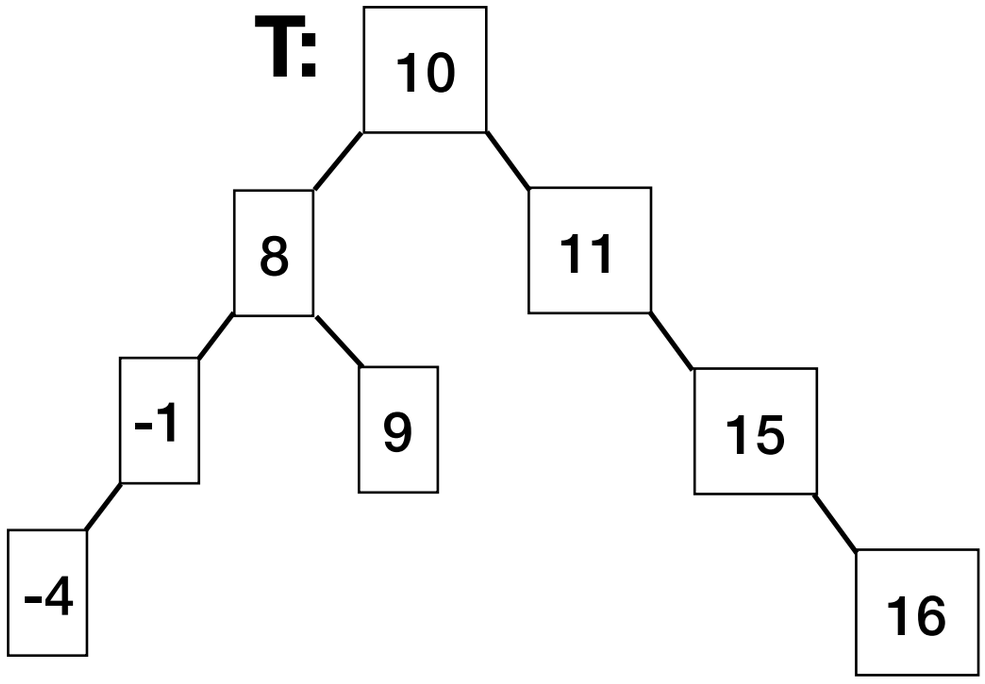
What is the height of

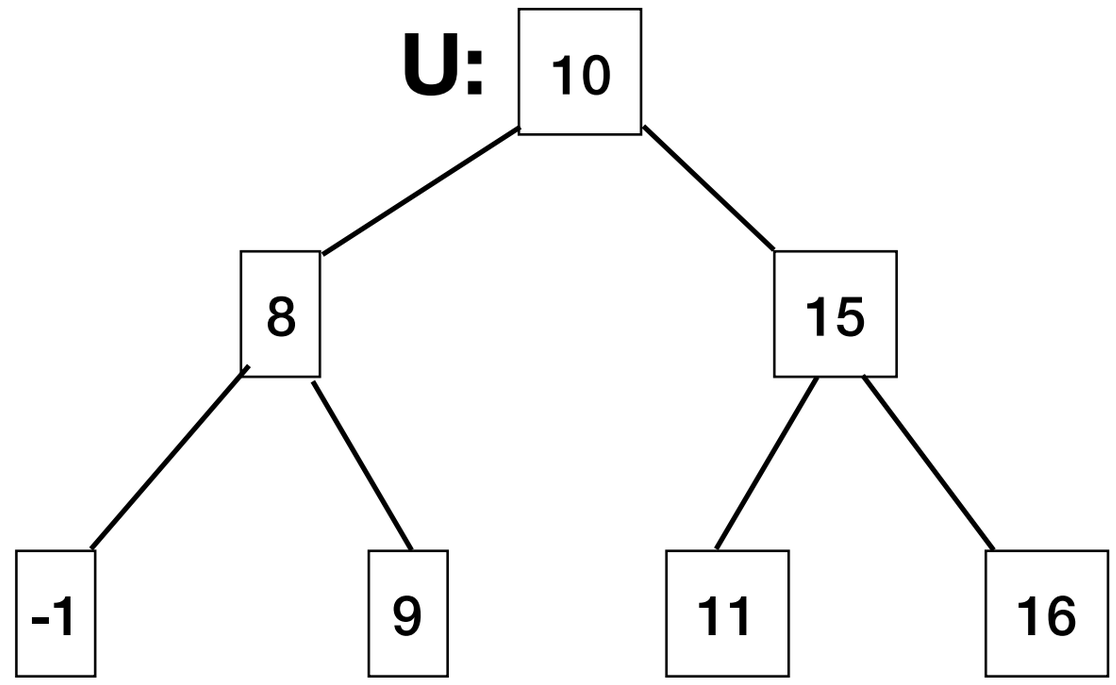
- The root? 4
- 15's left subtree? 1
- 15's right subtree? 1
- 16's left subtree? -1

What is the balance factor of

- node 16? 0
- node 15? -1
- node 10? -1

Balance Factor

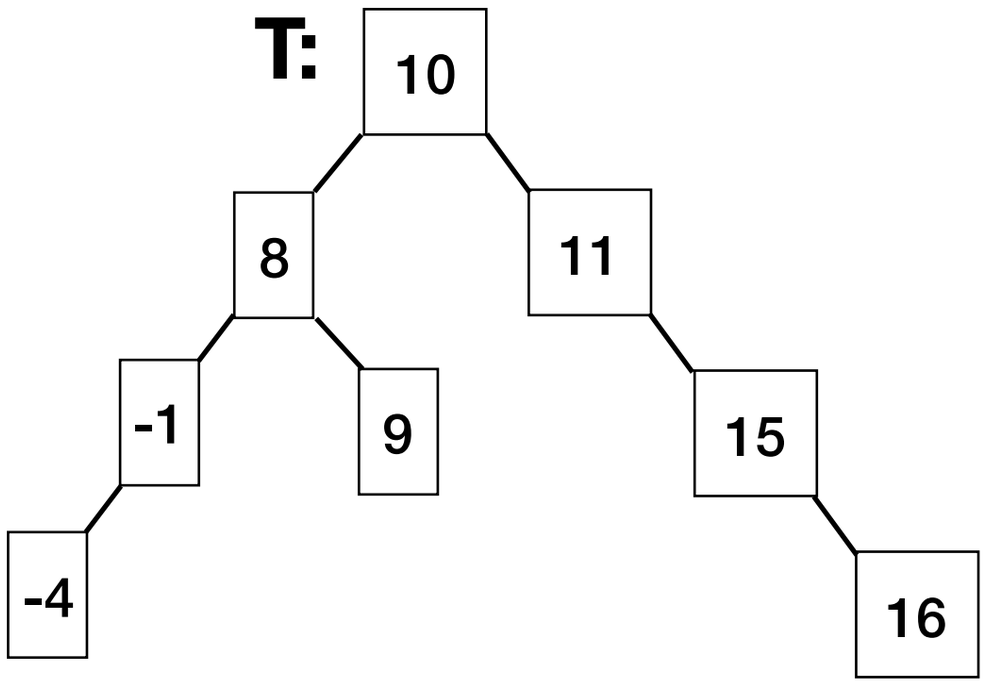


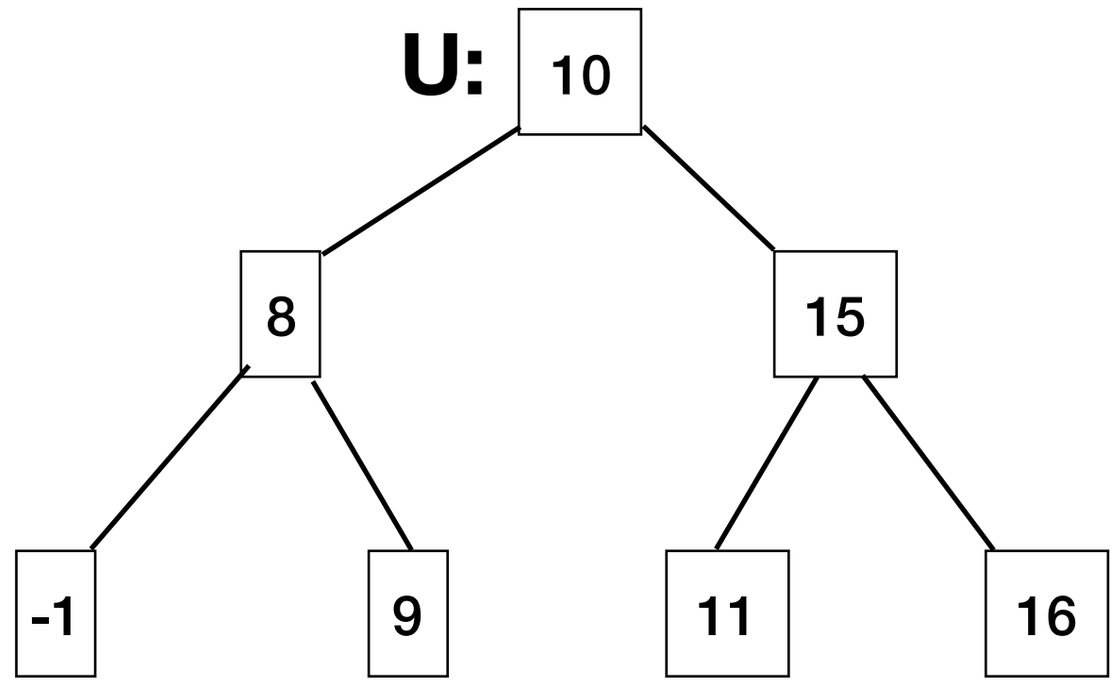


What's the largest *absolute* balance factor of any node in each tree?

	T	U
A	0	0
B	2	1
C	2	0
D	1	1

Balance Factor





What's the largest *absolute* balance factor of any node in each tree?

	T	U
A	0	0
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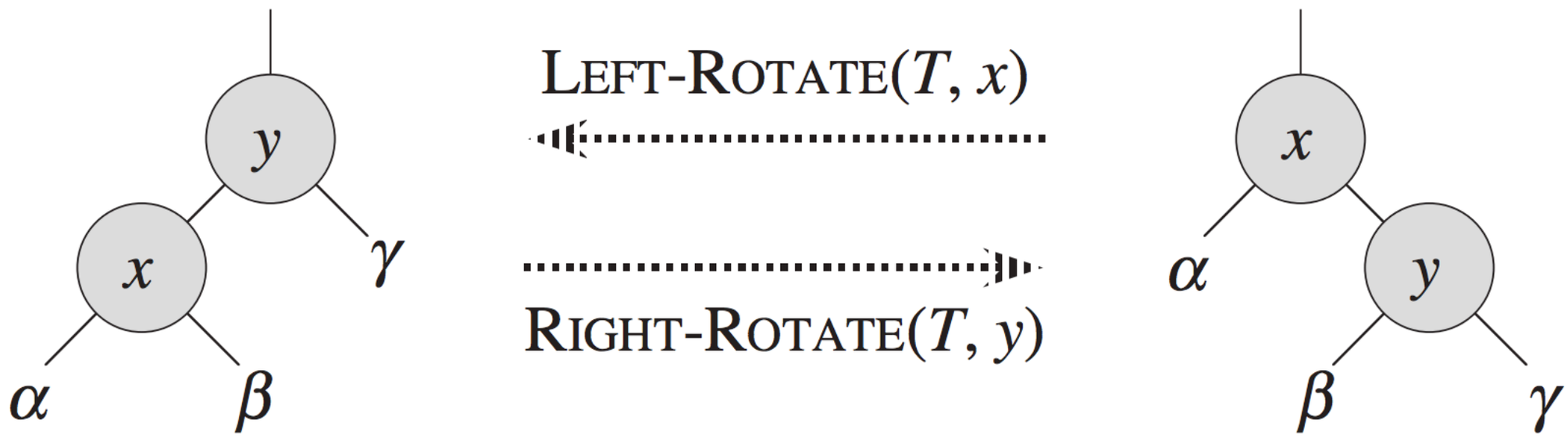
Tree Rotations

modify the structure without violating the BST property.

Steps in left rotation (move y up to its parent's position):

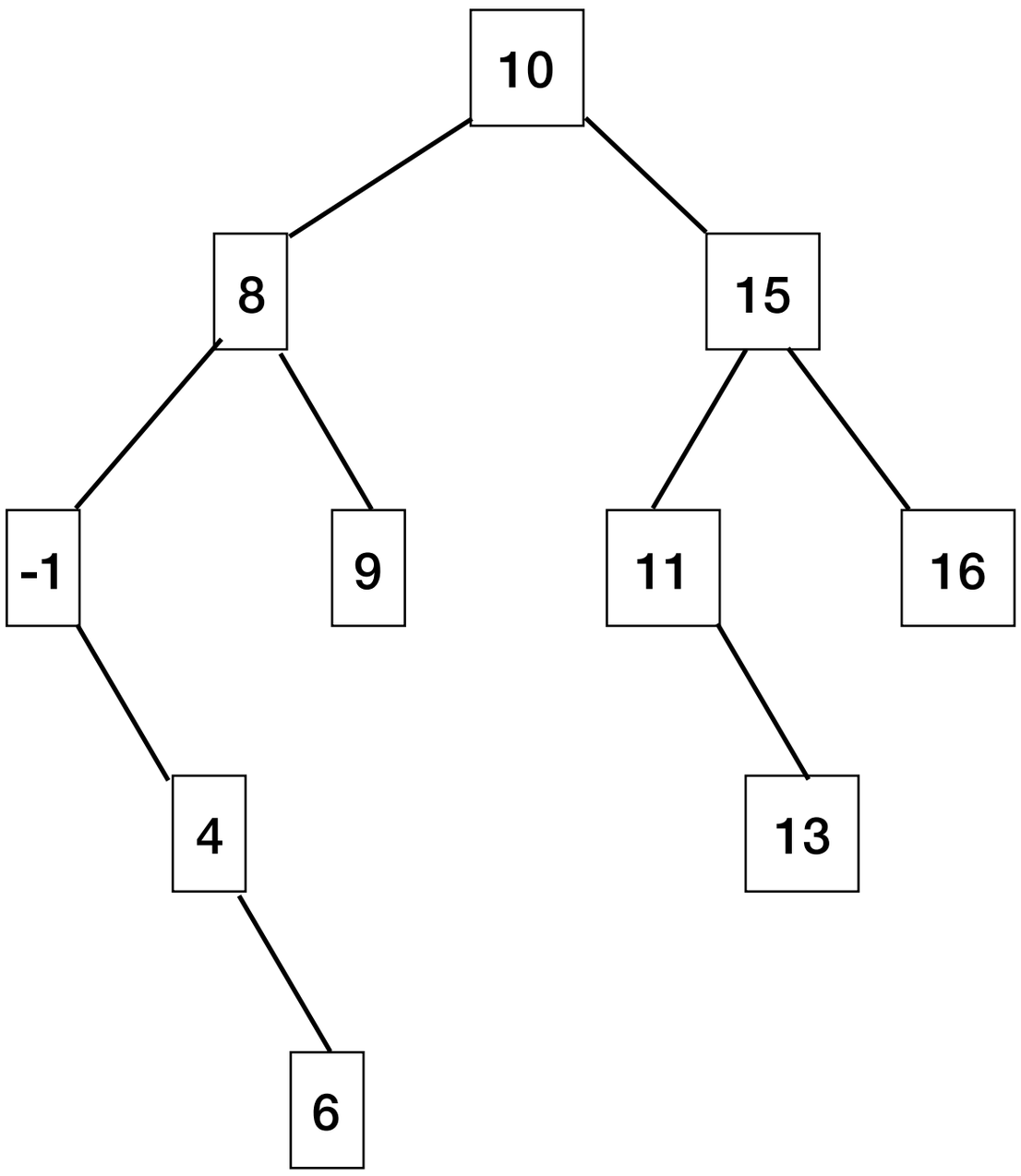
1. Transfer β: x's right subtree becomes y's old left subtree (β)
2. Transfer the parent: y's parent becomes x's old parent
3. Transfer x itself: x becomes y's left subtree



CLRS Fig 13.2, pg 313

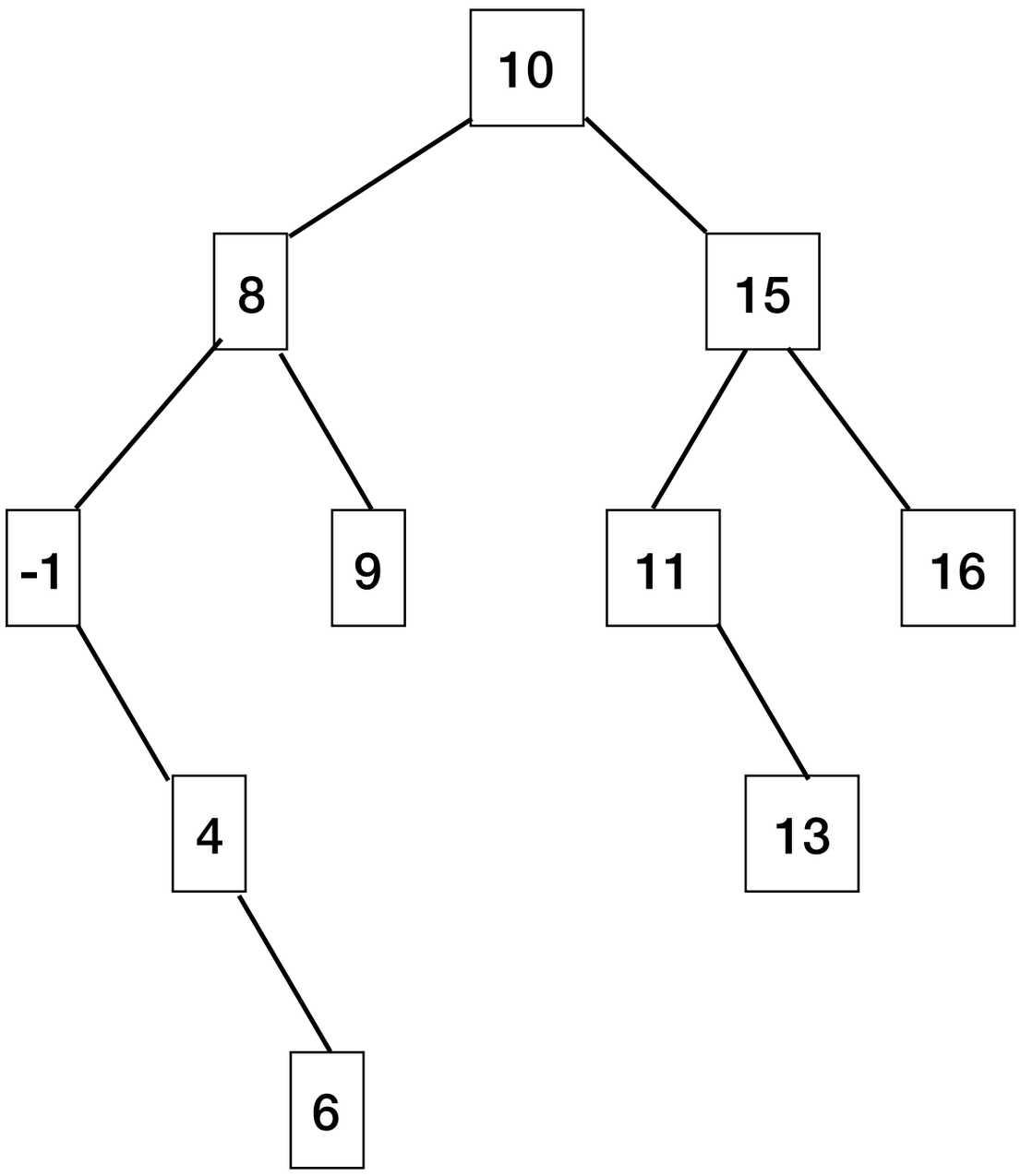
Rotations

Write the tree after a left rotation on the node with value 11.



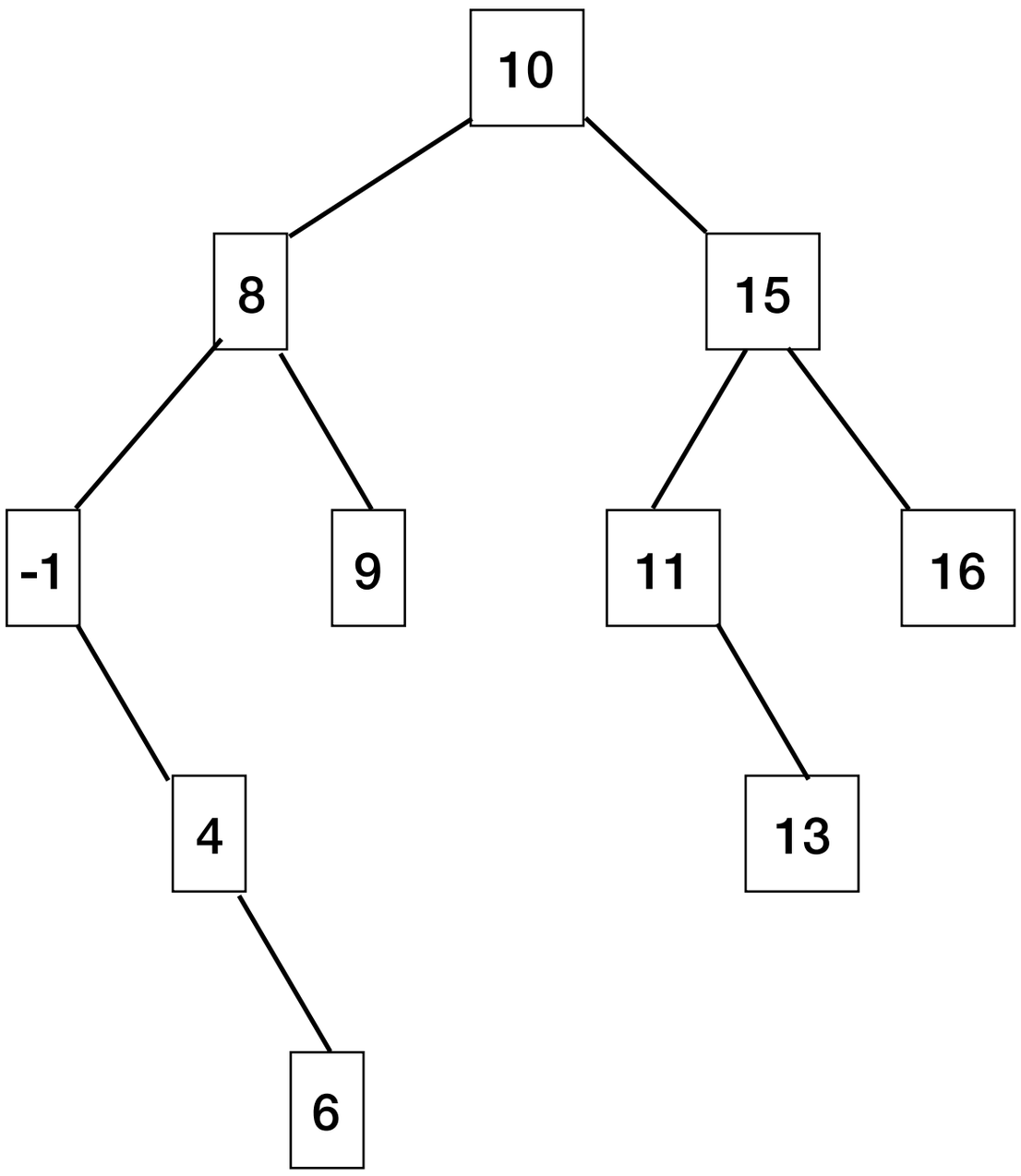
Rotations

Write the tree after a left rotation on the node with value 15.

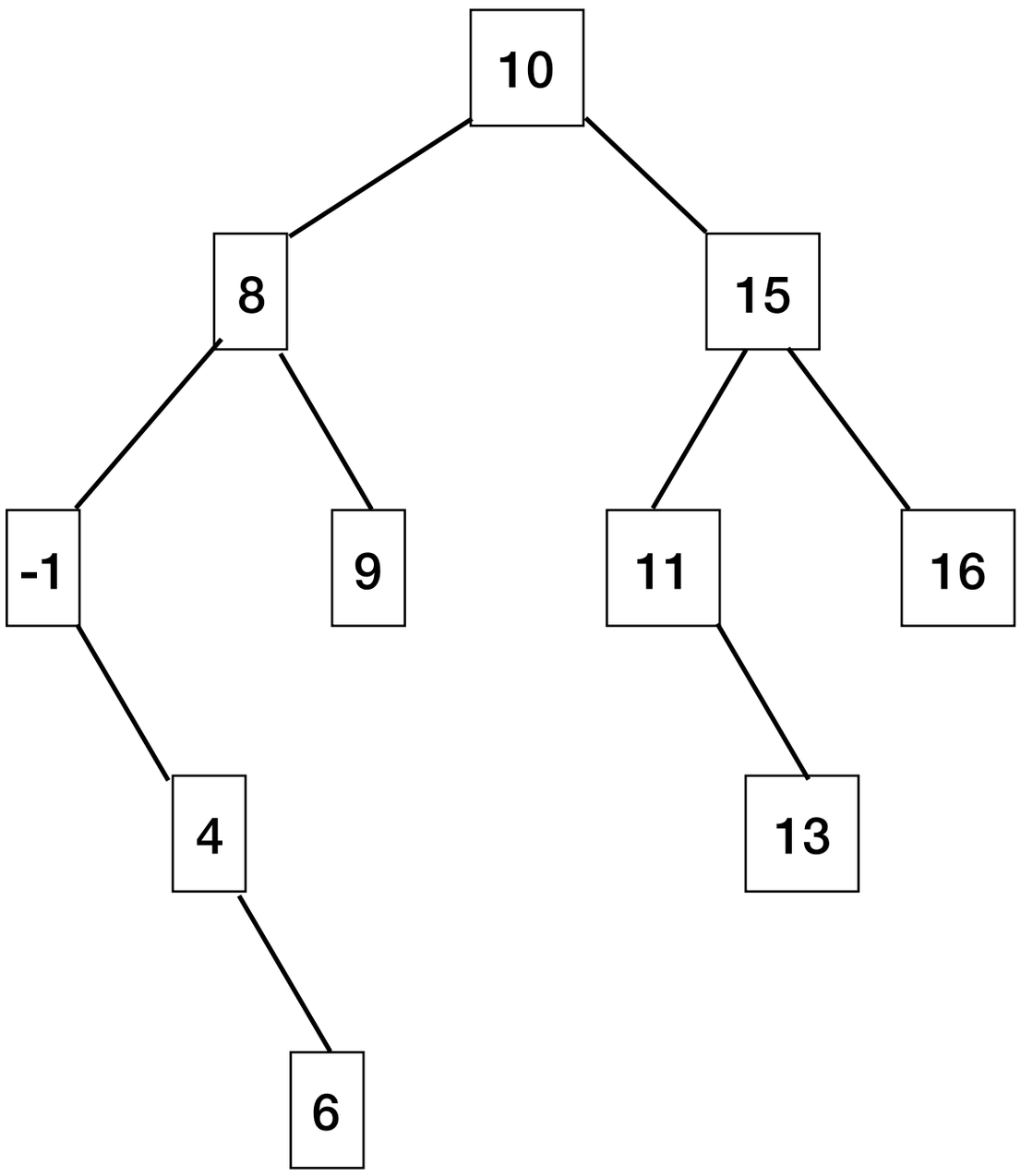


Rotations

Write the tree after a right rotation on the node with value 15.



Rotations

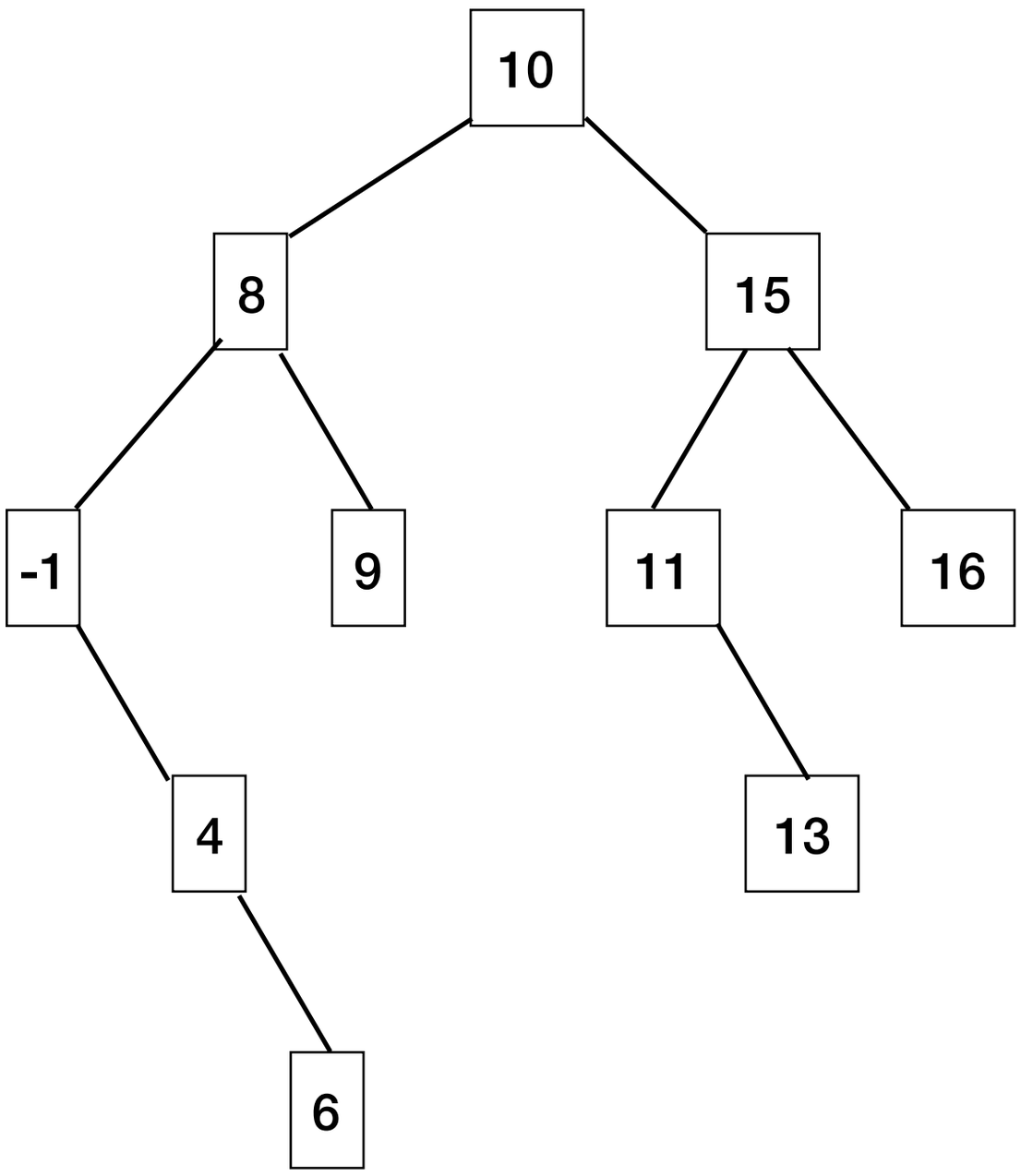


What is the **precondition** for performing a left rotation on a node?

What is the **precondition** for performing a right rotation on a node?

Hint: try performing a right rotation on node 11.

Rotations



What is the **precondition** for performing a left rotation on a node?

n.right != null

What is the **precondition** for performing a right rotation on a node?

n.left != null

Hint: try performing a right rotation on node 11.