CSCI 241

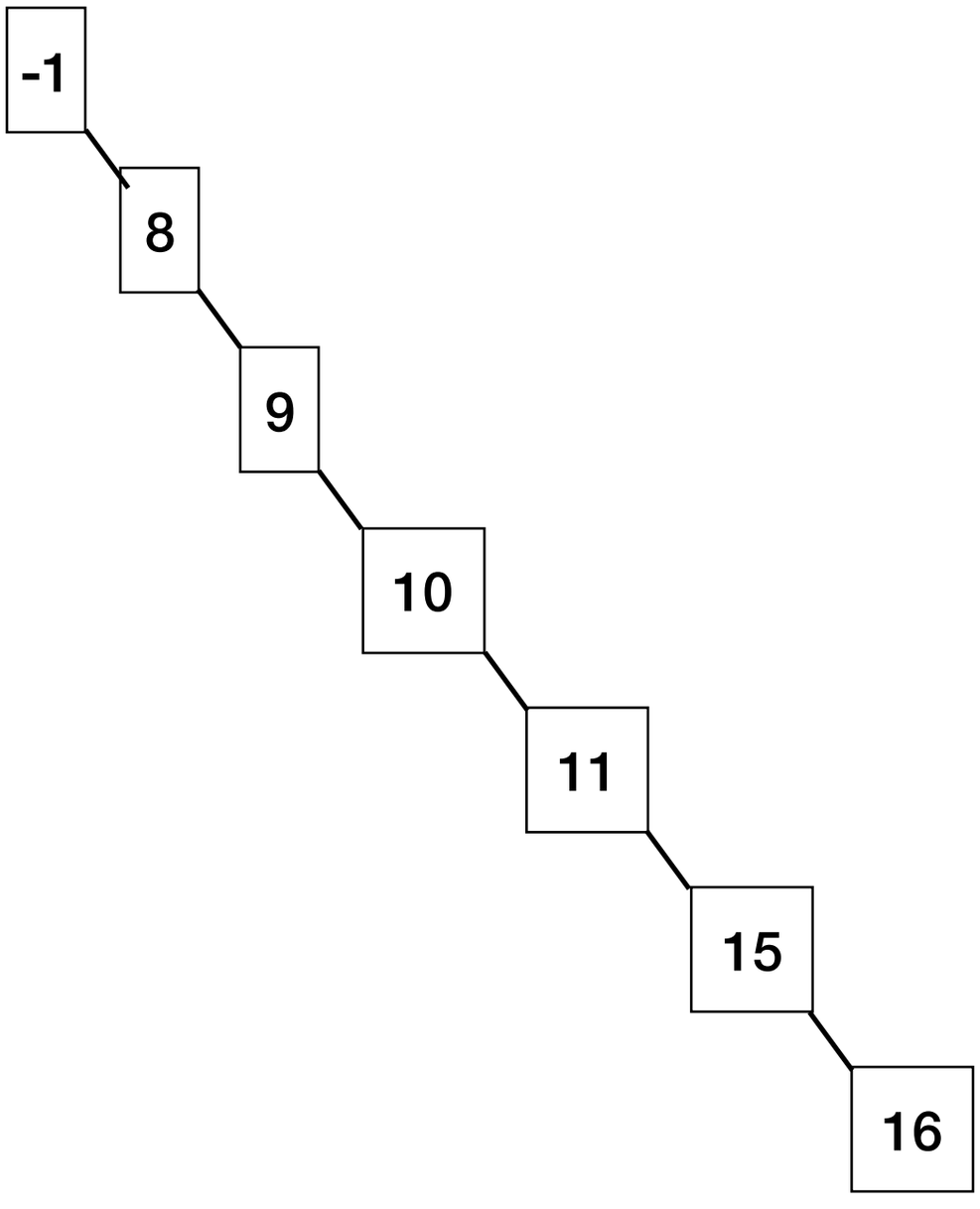
Lecture 13c:
Tree Rotations

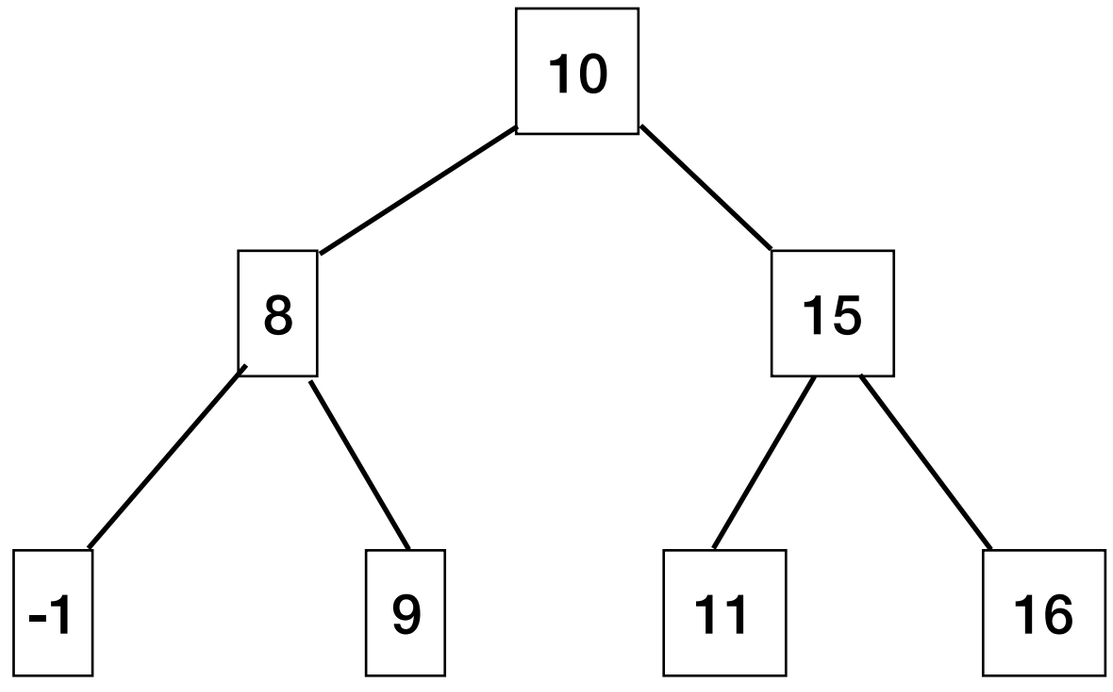
Goals

- Be prepared implement rotations in BSTs

Tree Badness: Balance Factor

Balance(n): height(n.right) - height(n.left)





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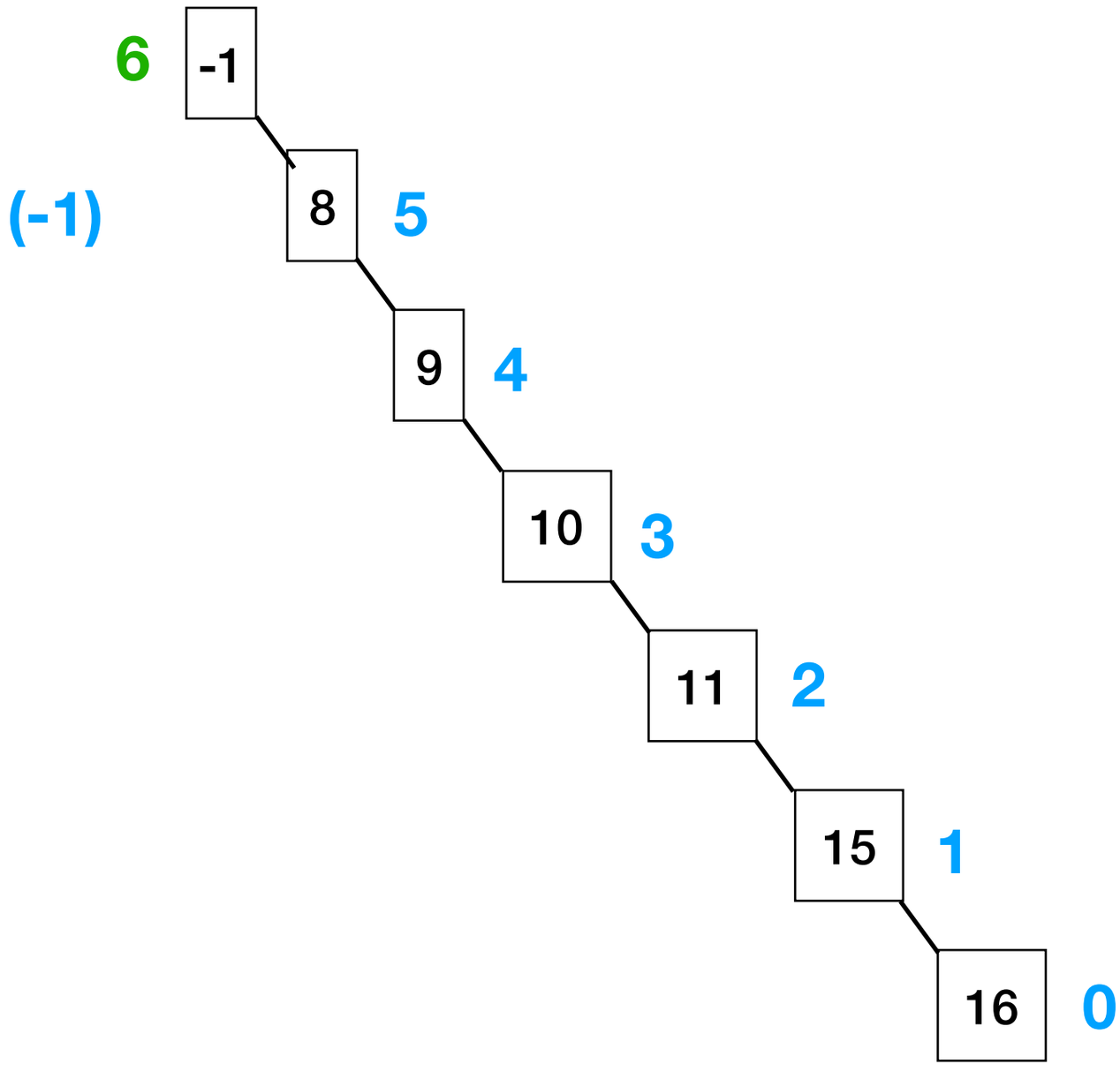
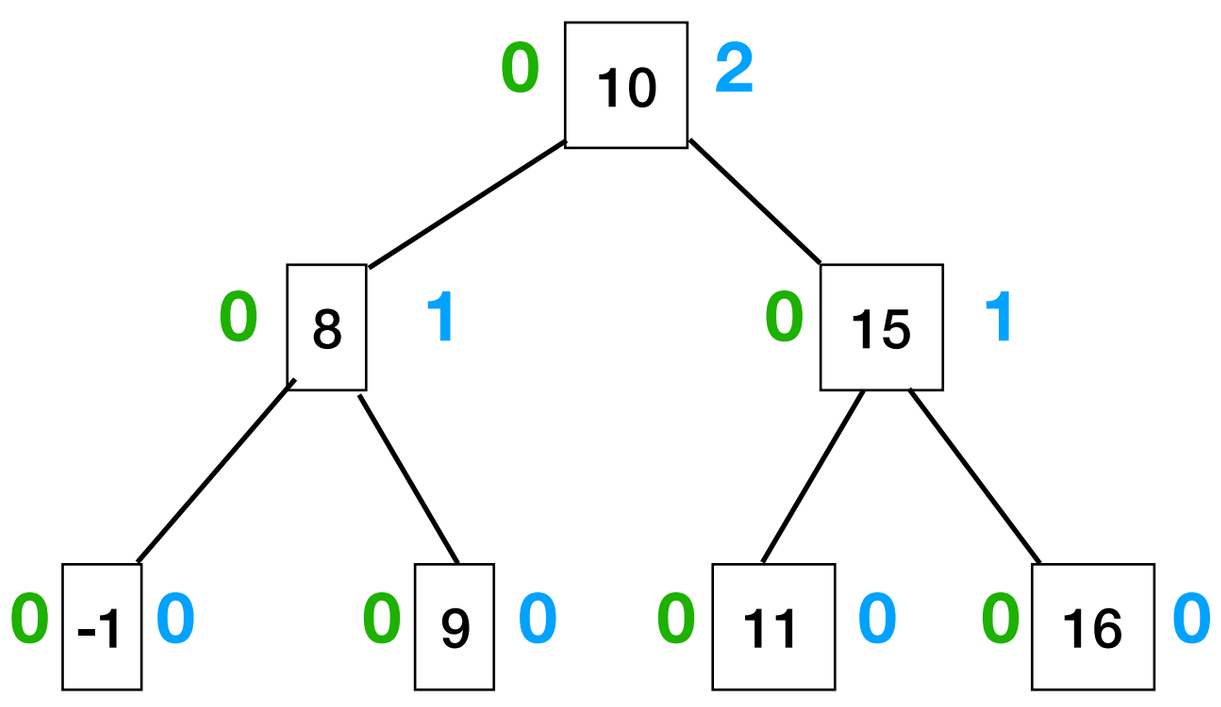
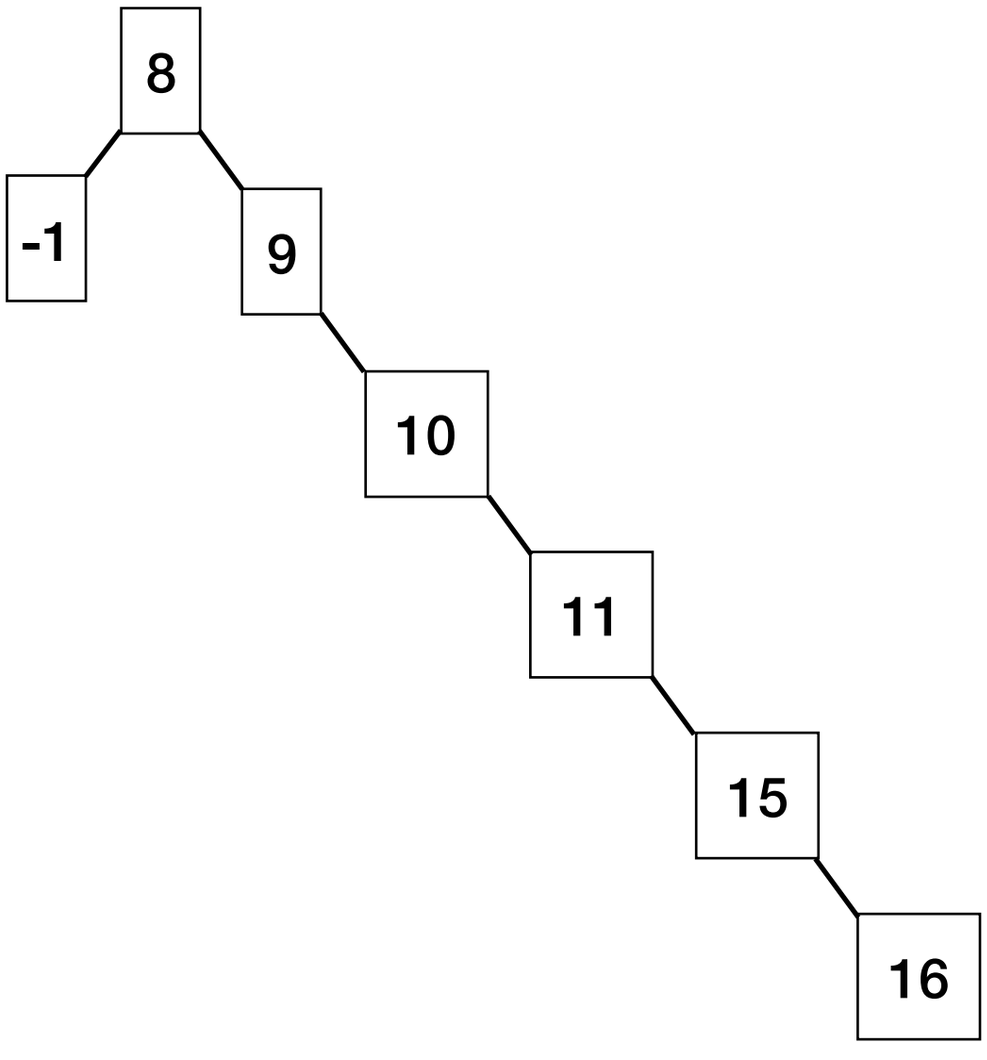
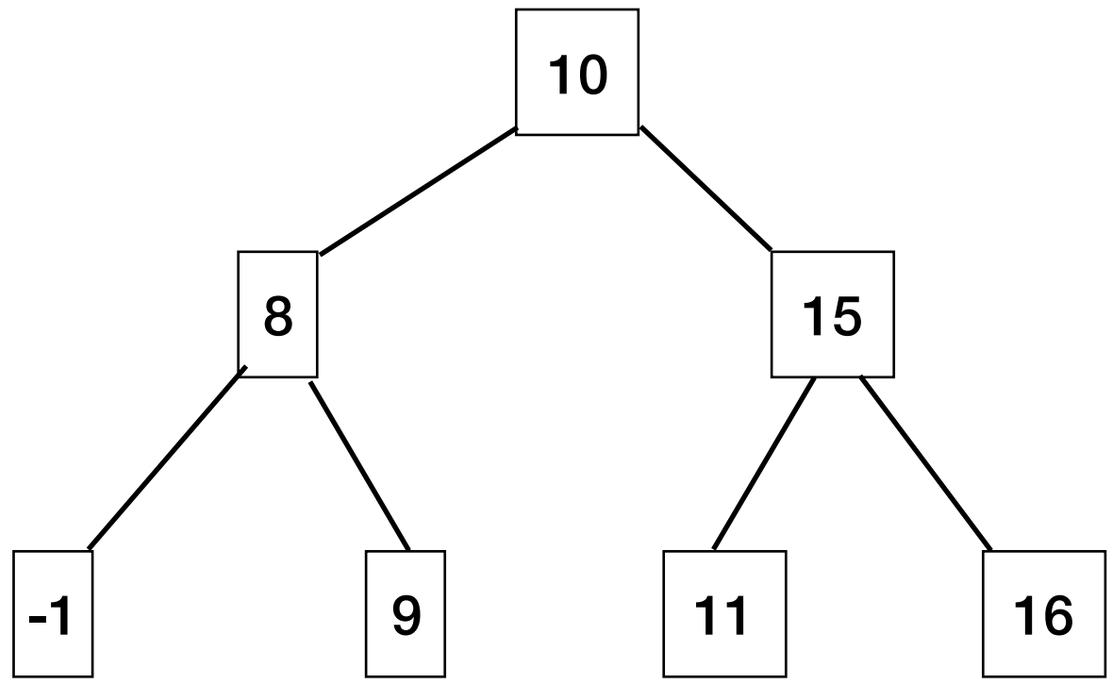




Can we make a tree less bad?





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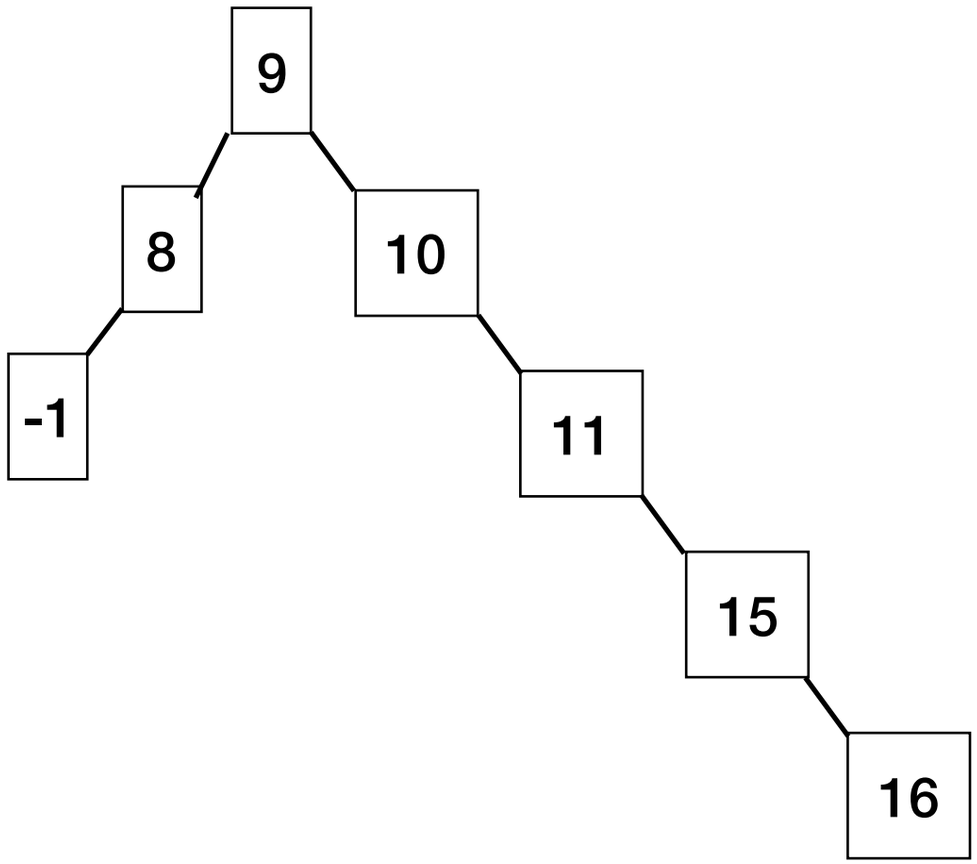

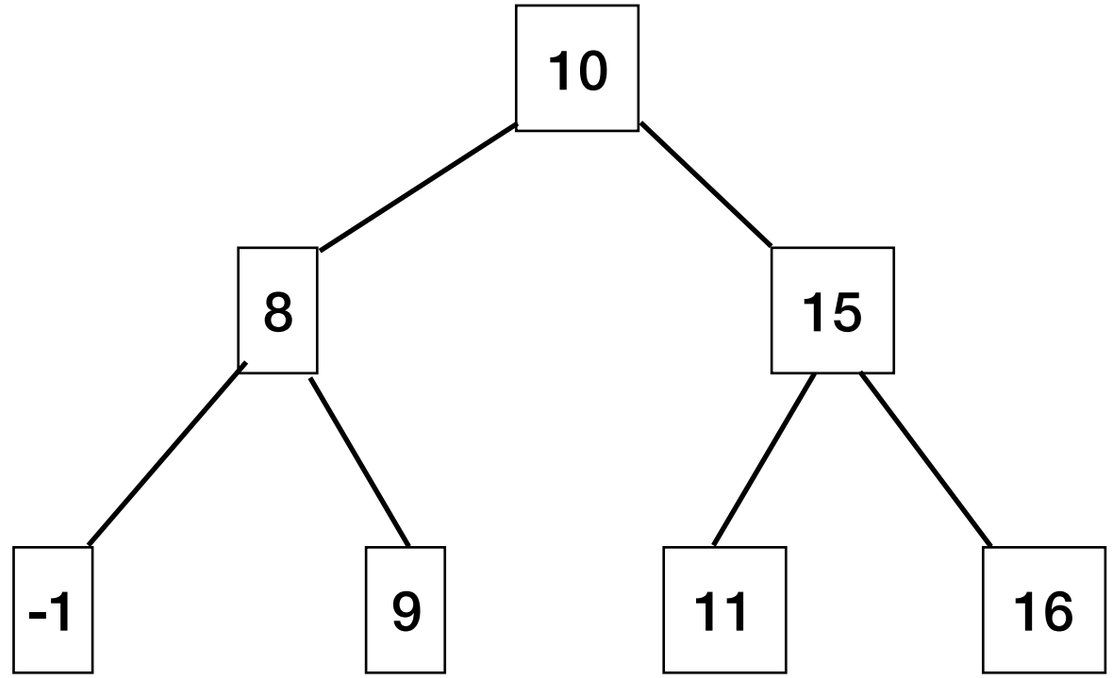

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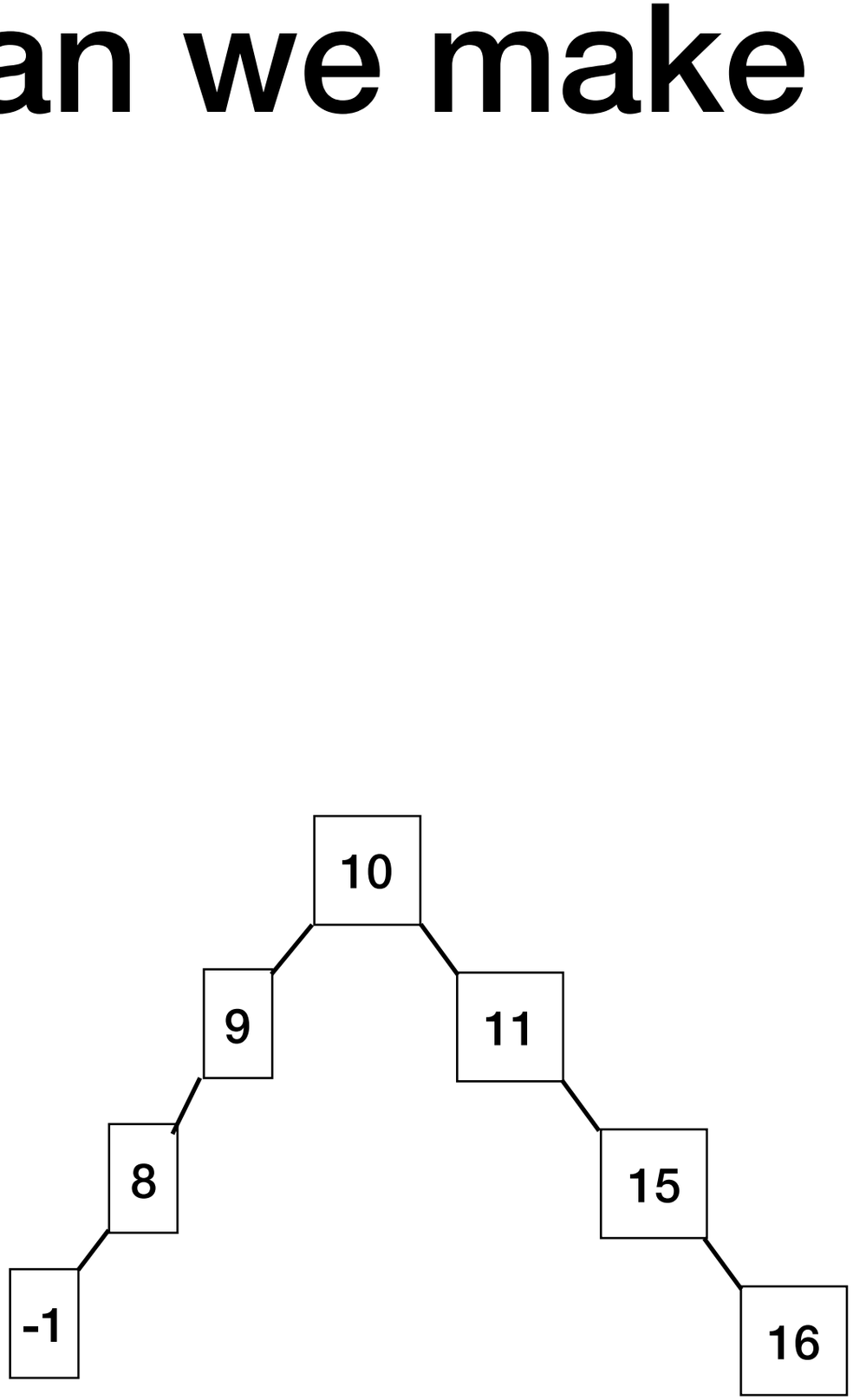

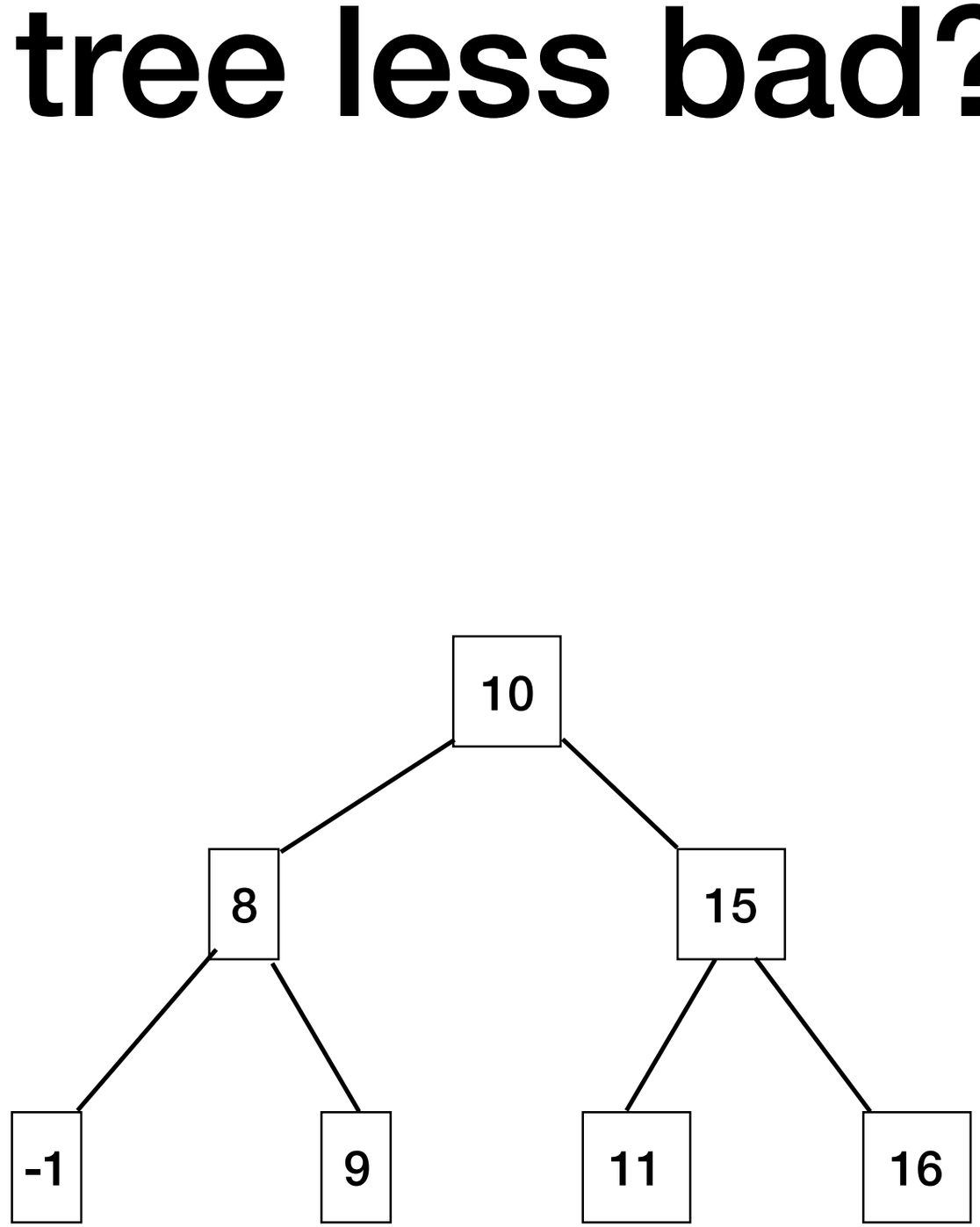

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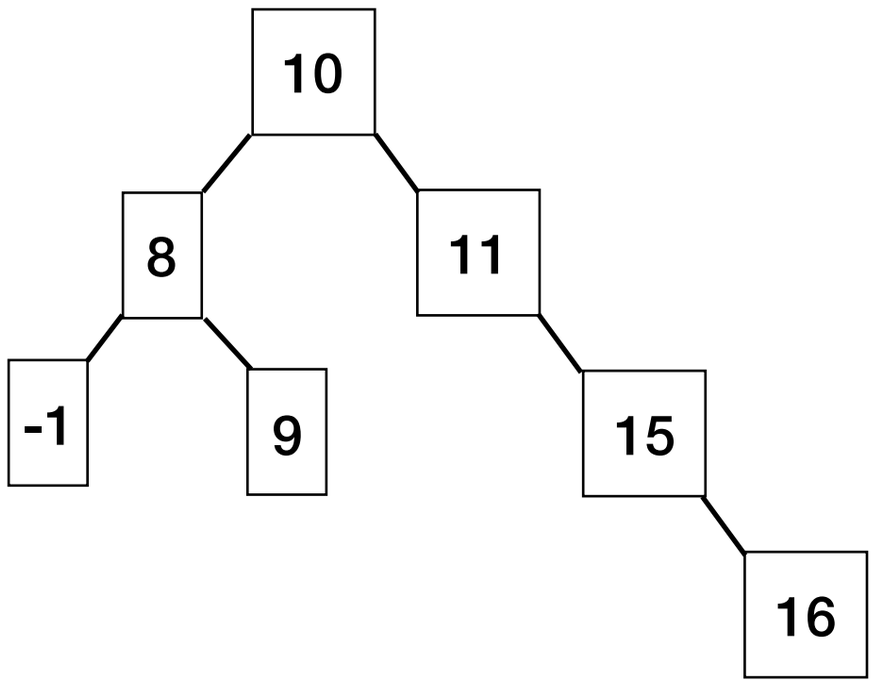

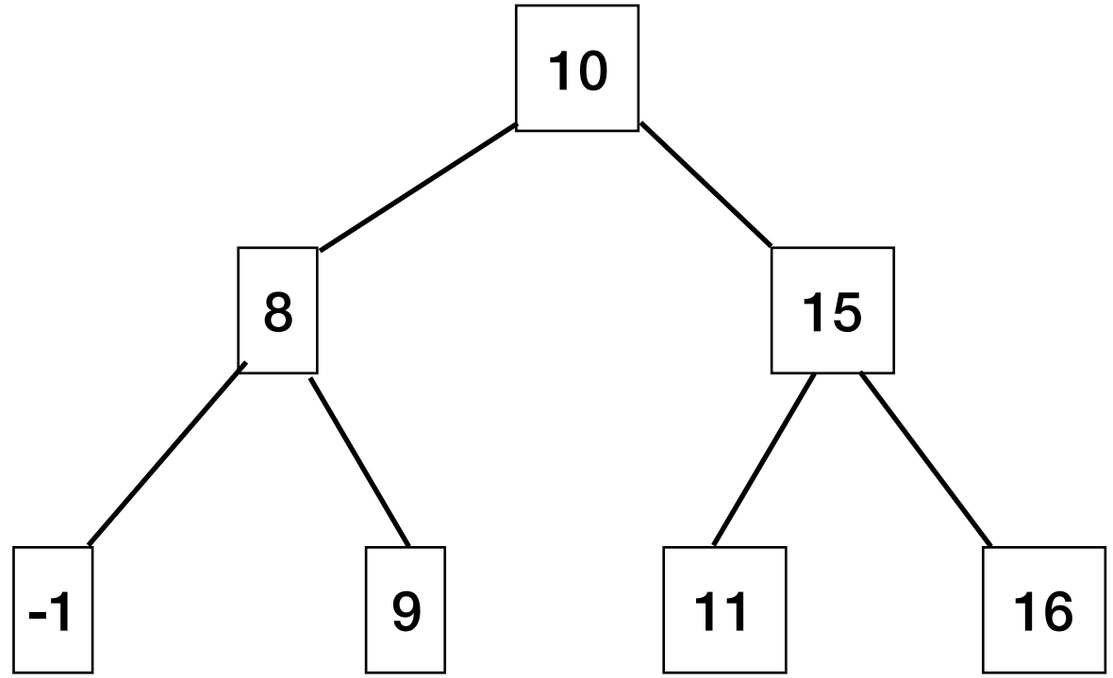

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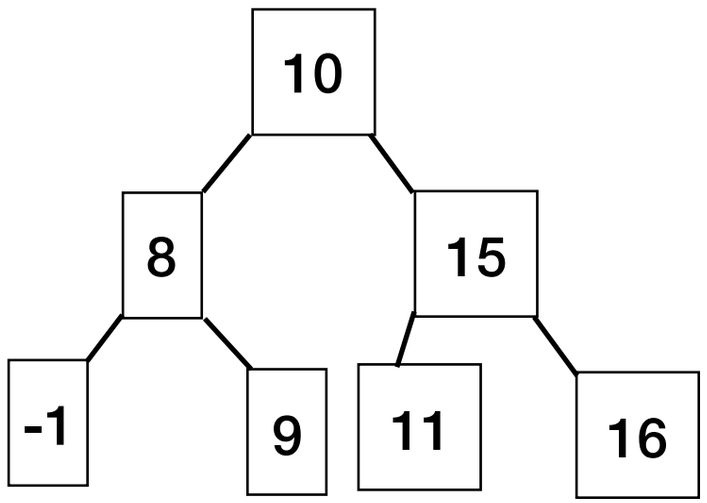

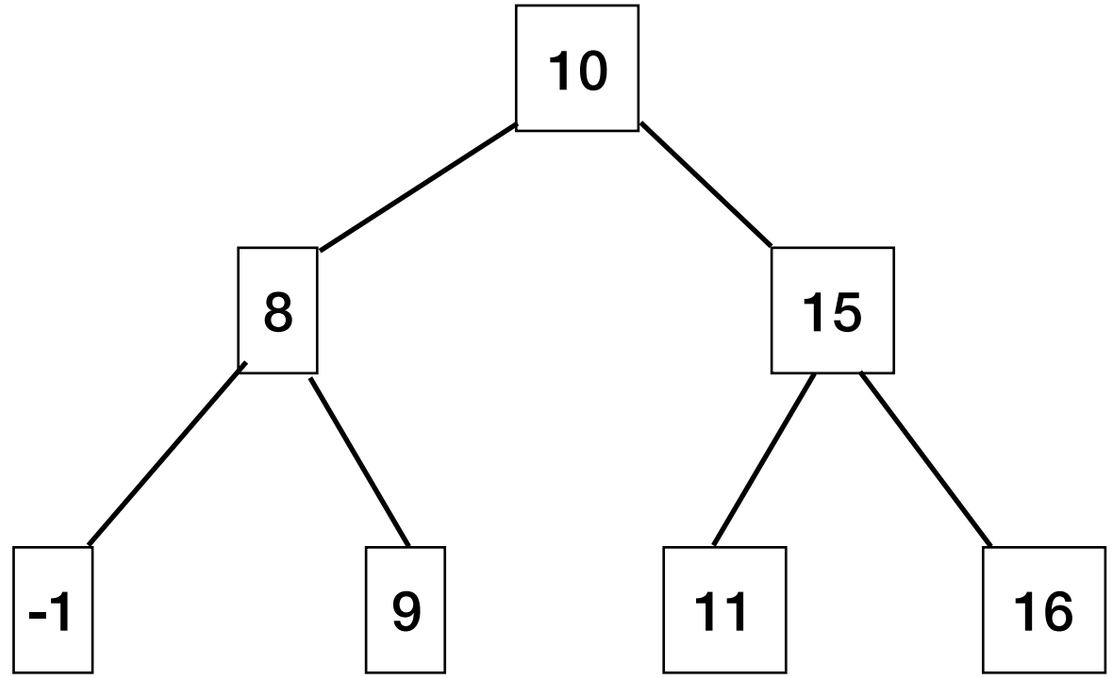

Tree Rotations

modify the structure without violating the BST property.

Steps in left rotation (move y up to its parent's position):

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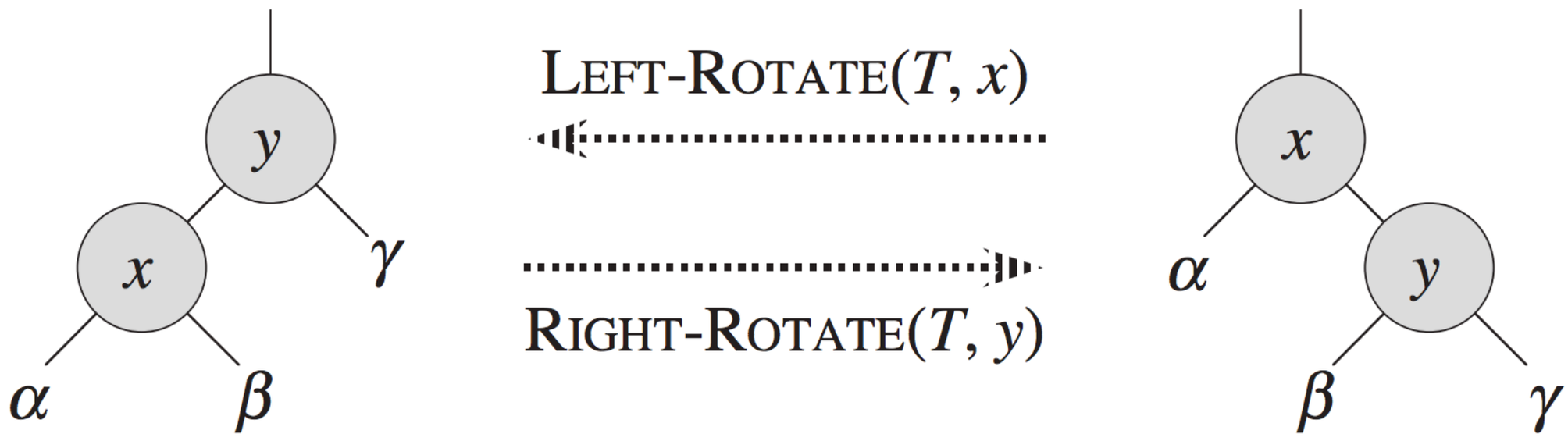

CLRS Fig 13.2, pg 313

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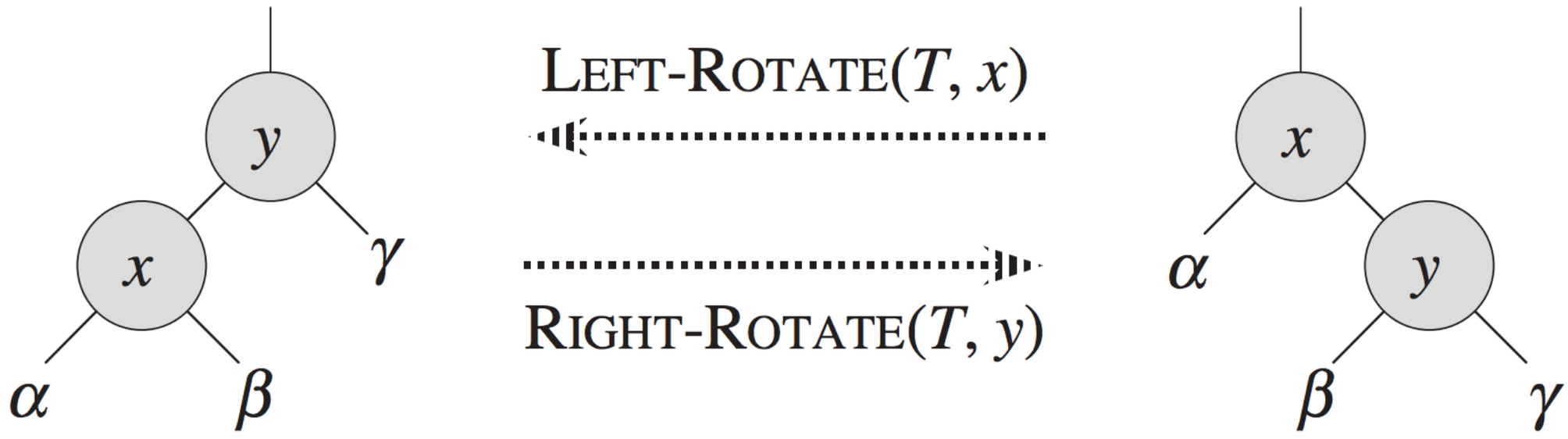
Details: need to update child, parent, and (possibly) root pointers.



CLRS Fig 13.2, pg 313

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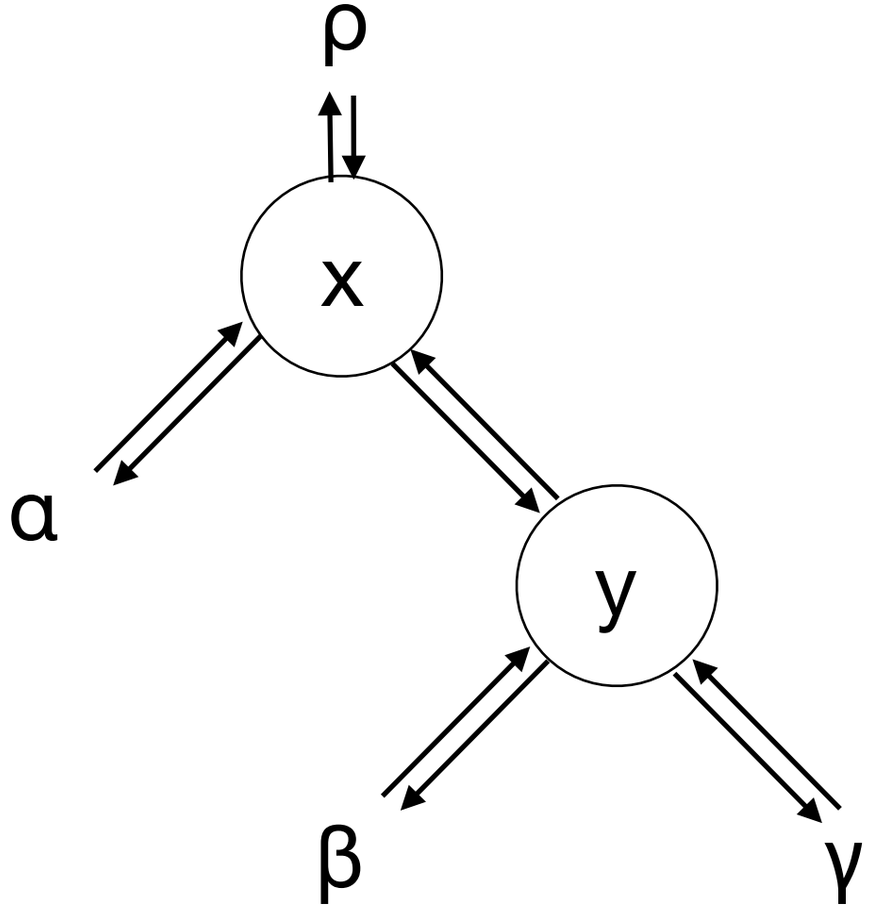

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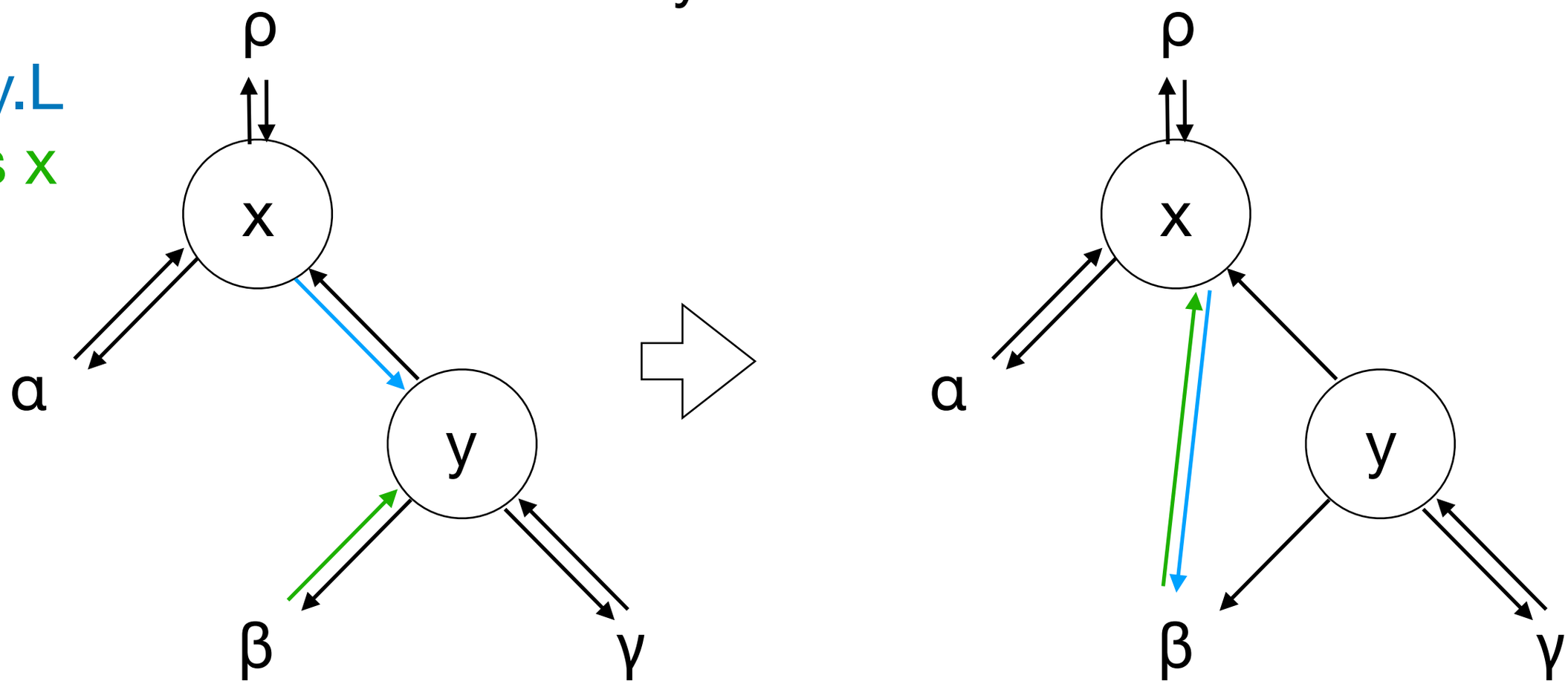

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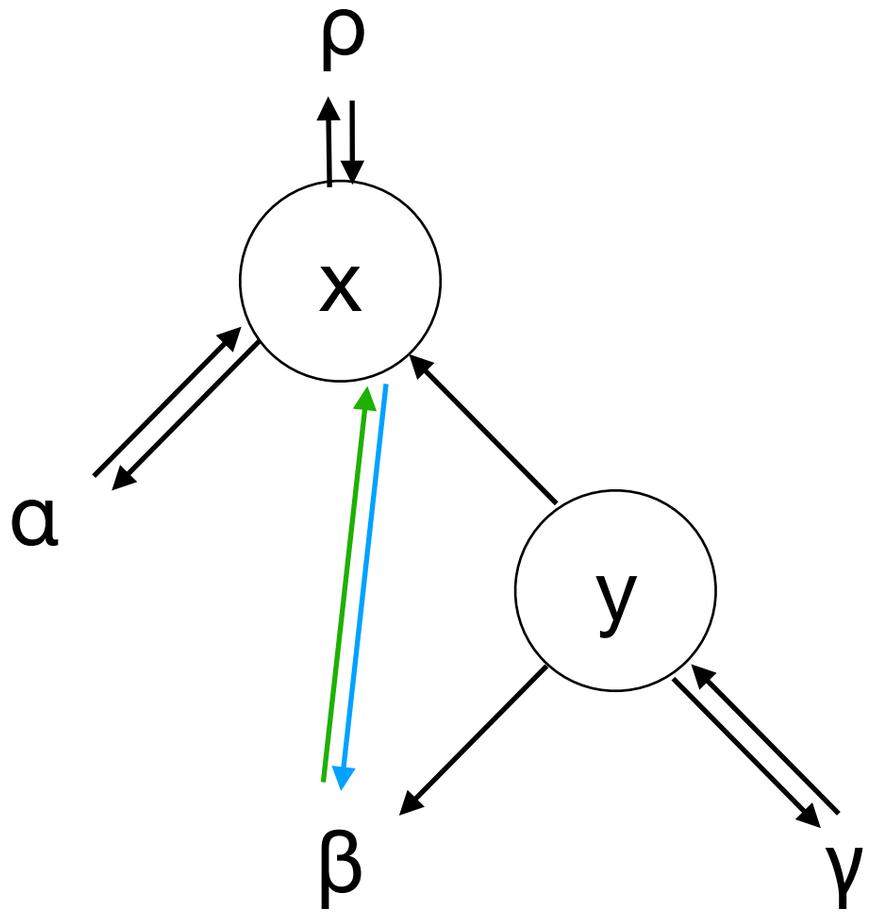

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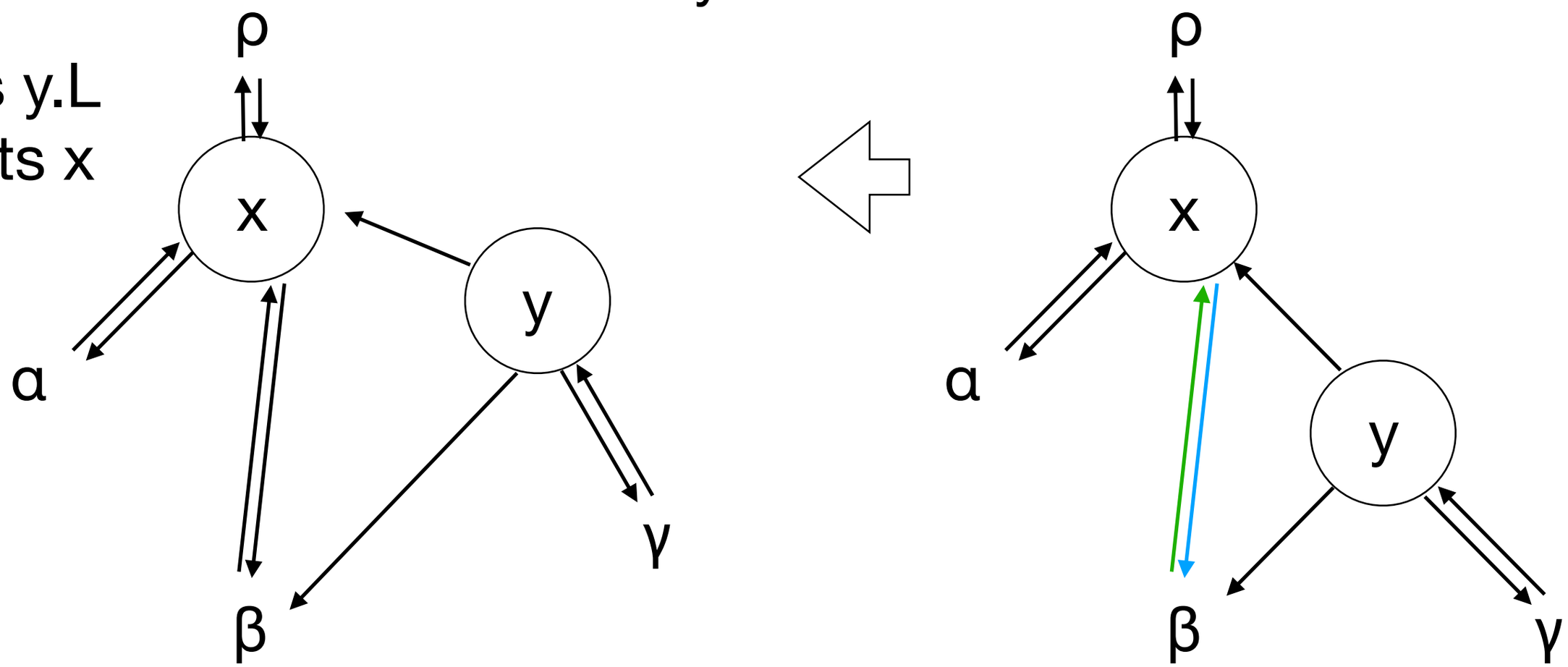

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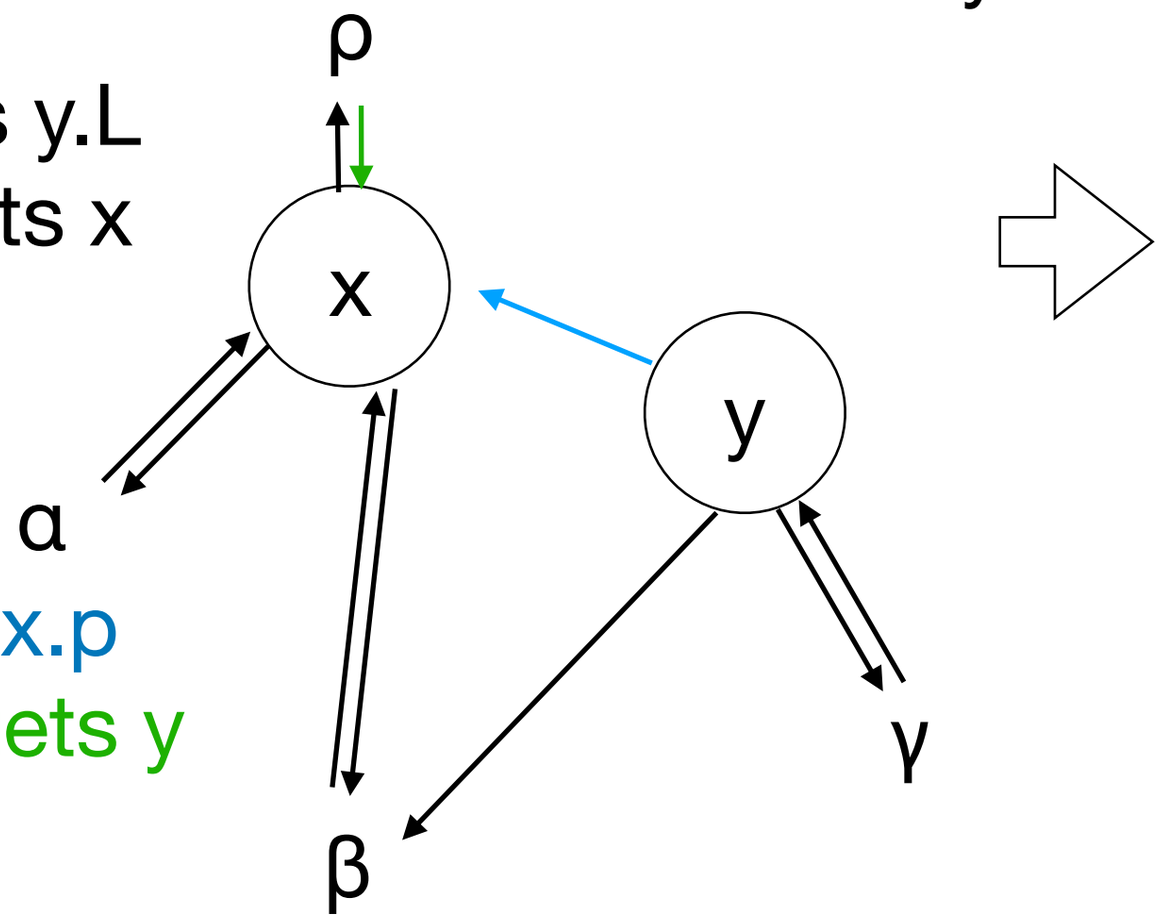

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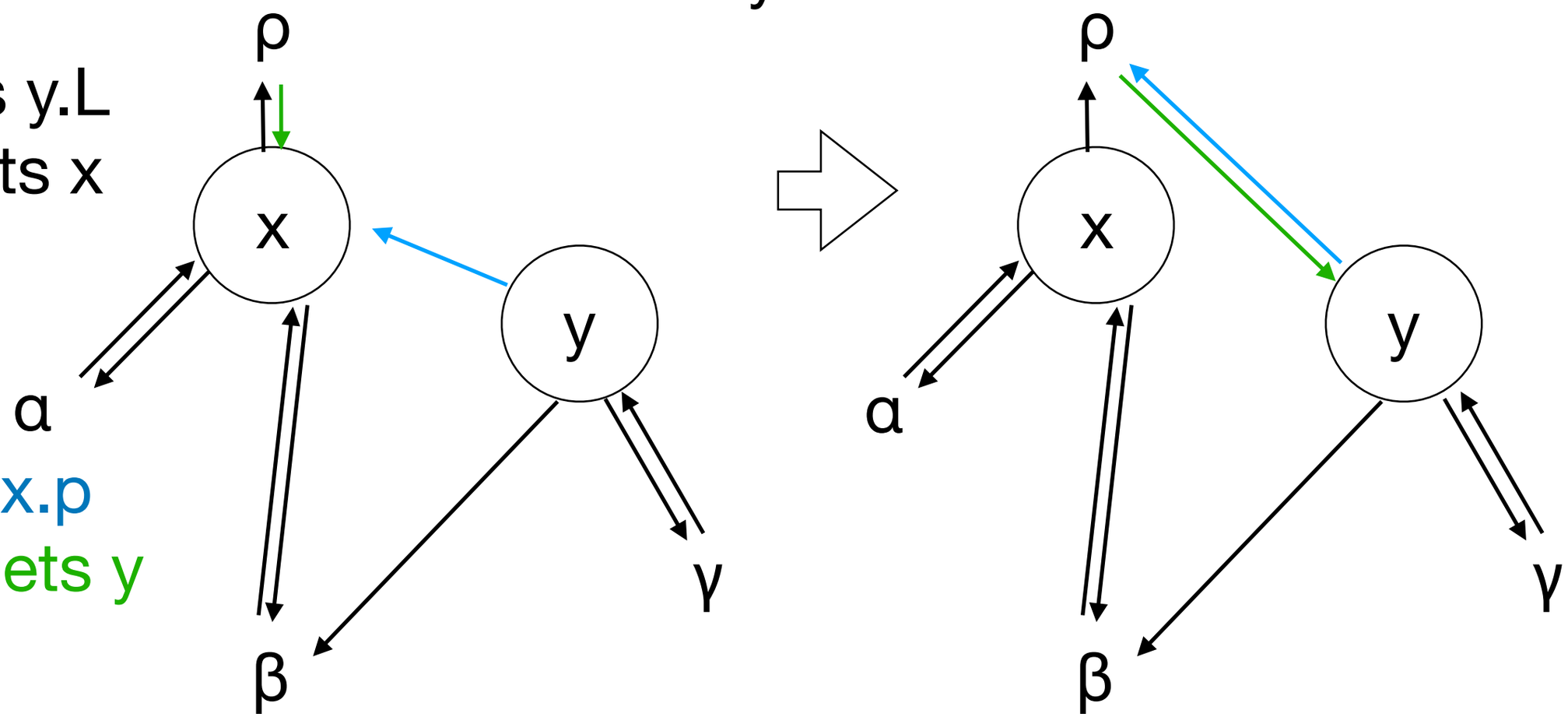

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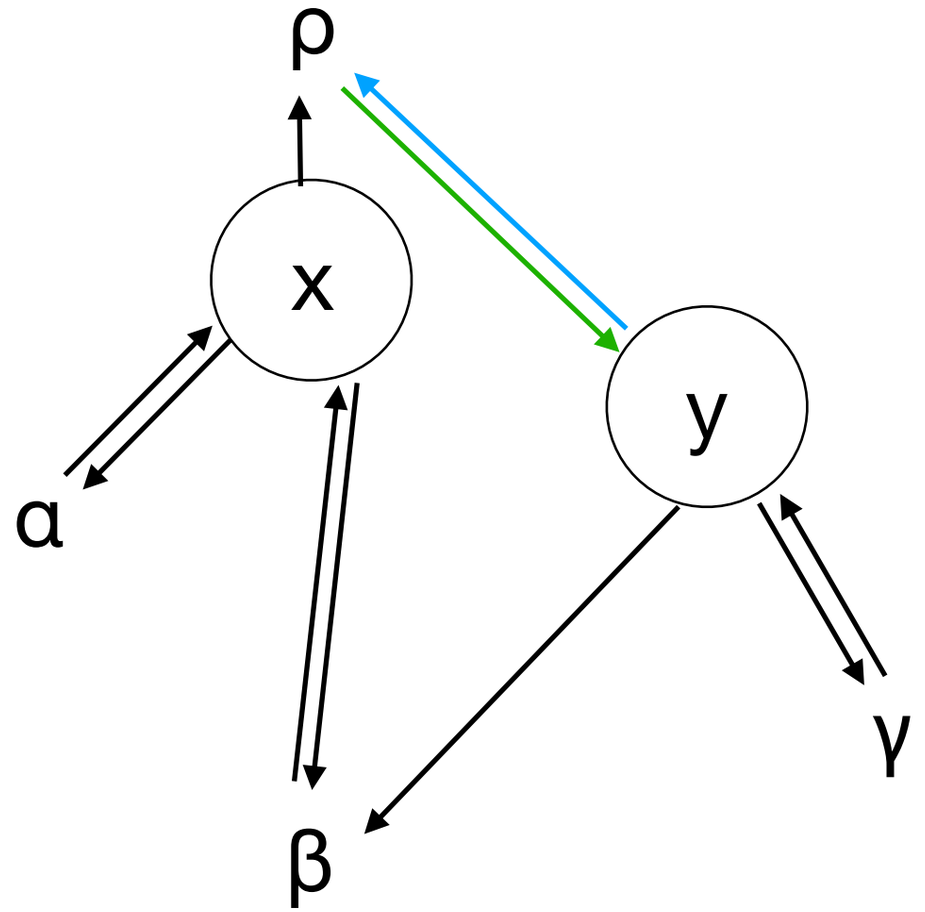

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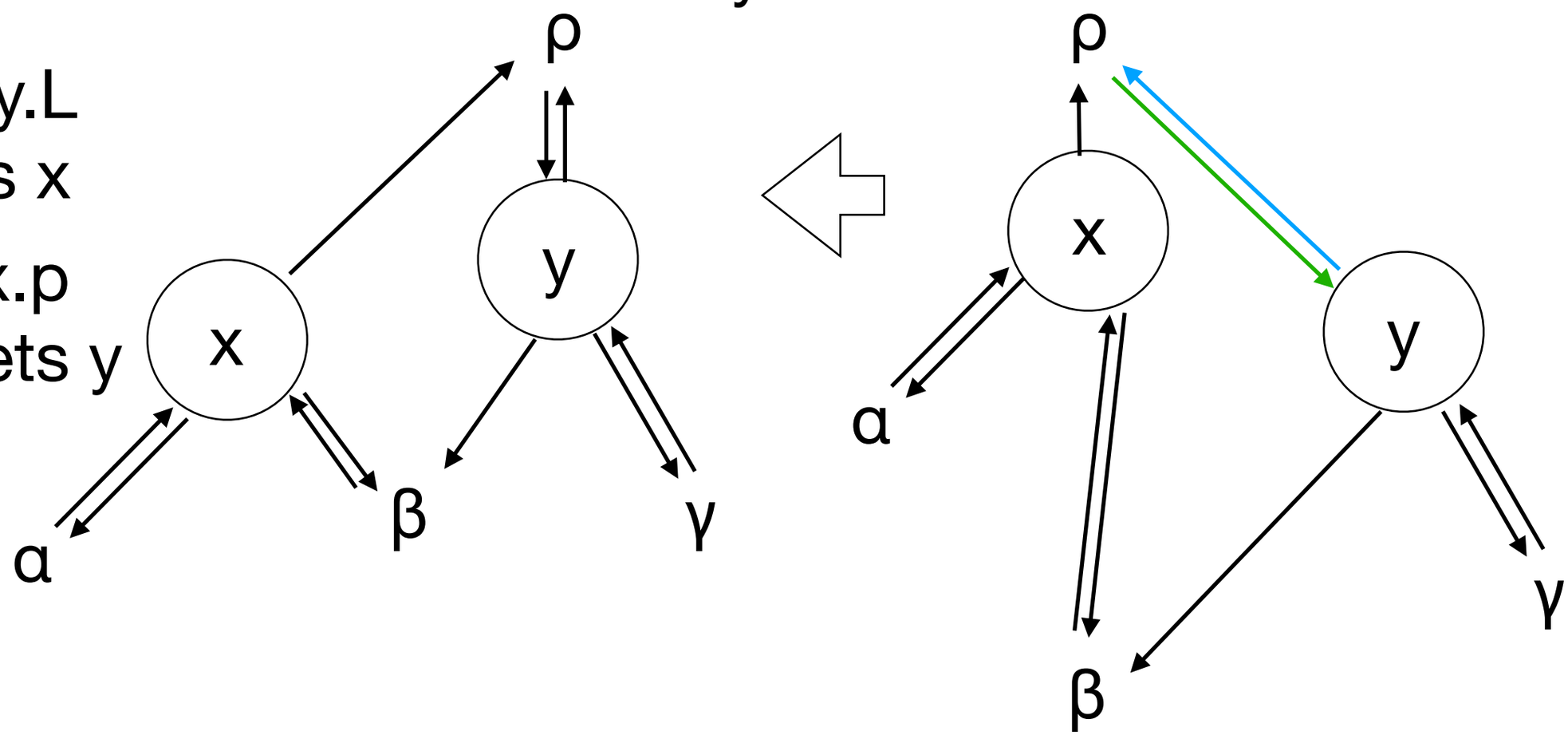

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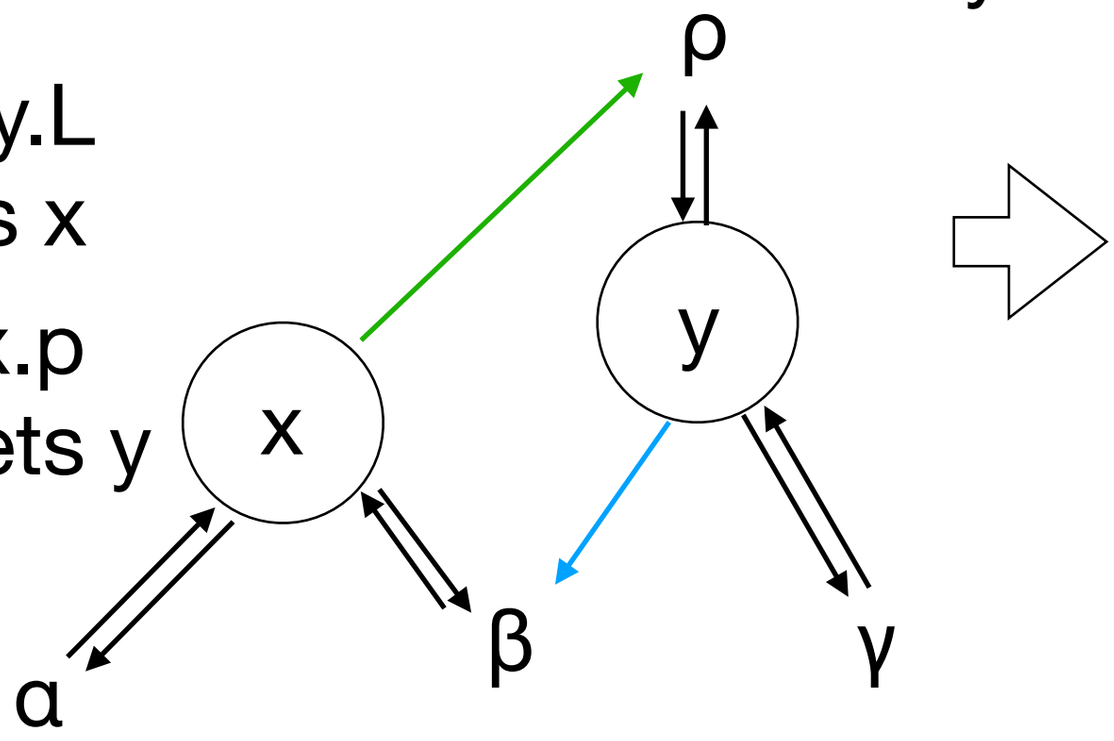

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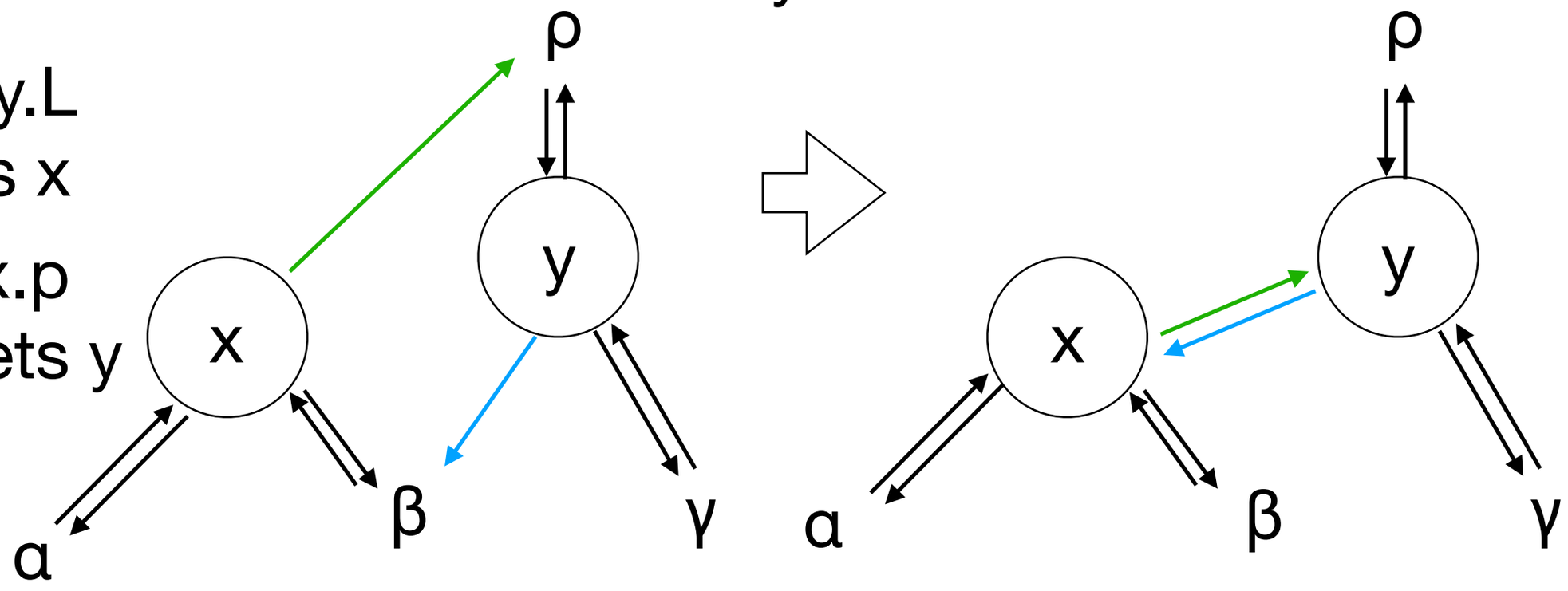

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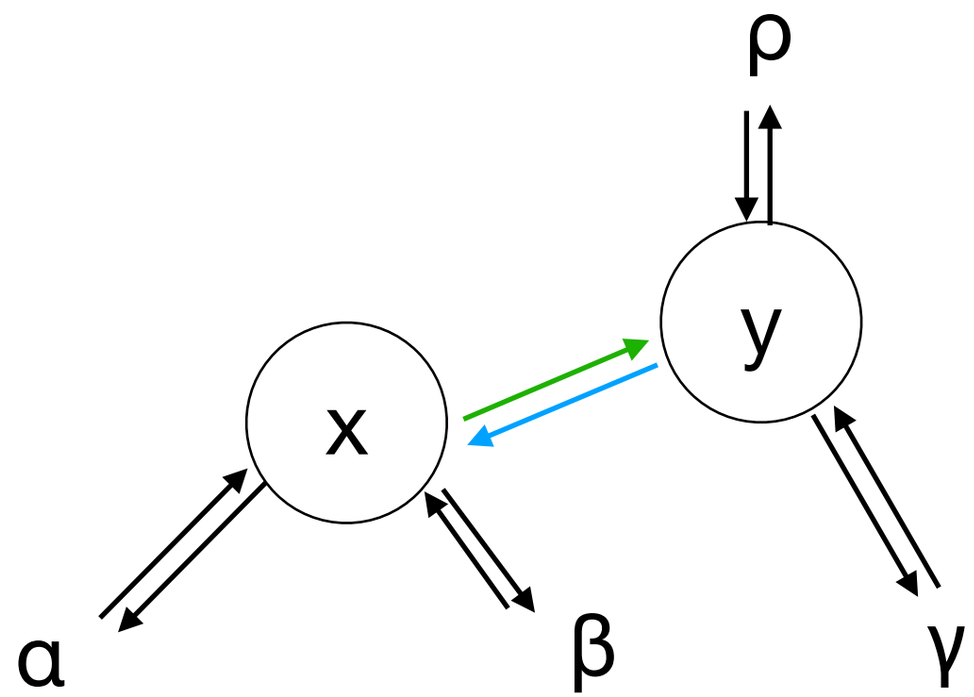

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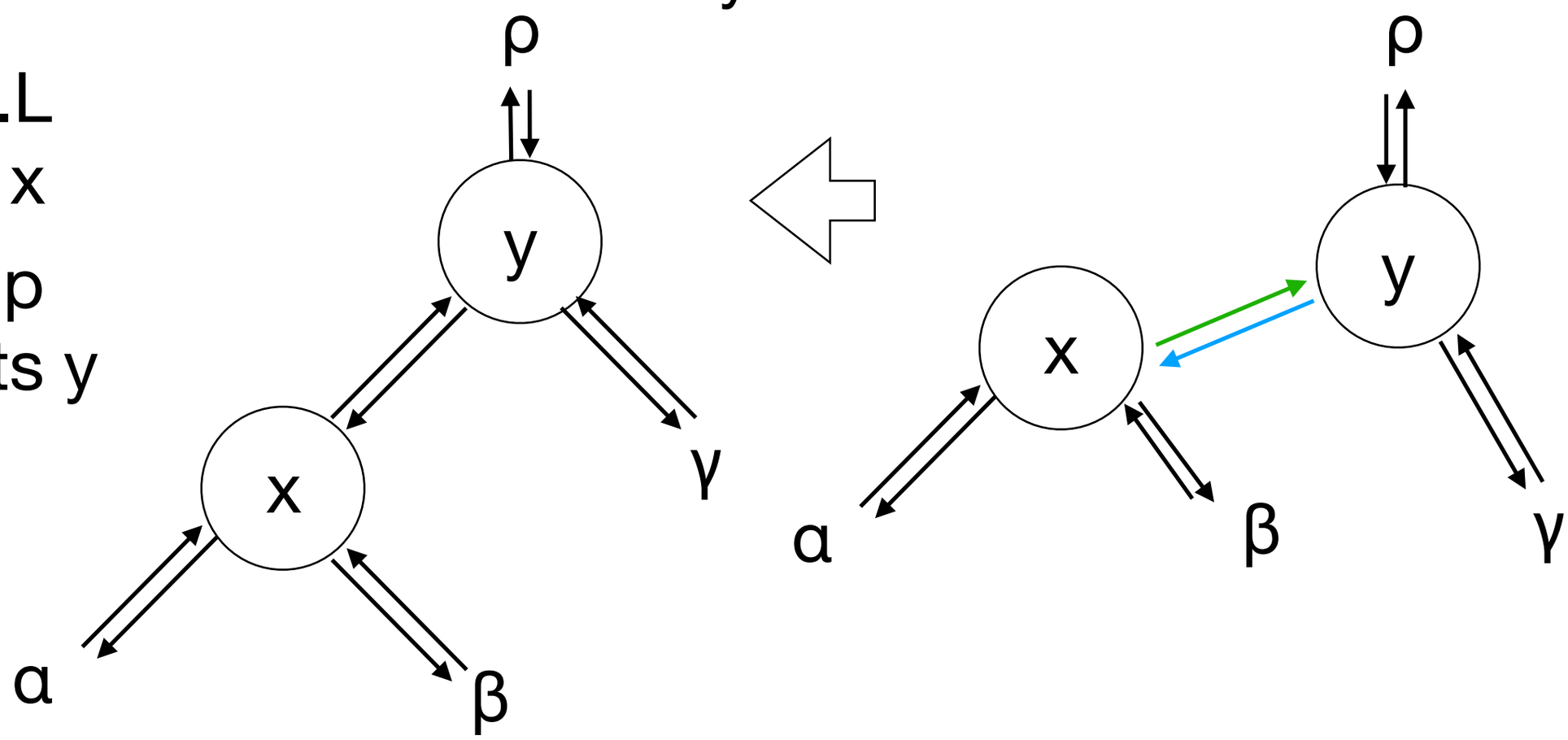

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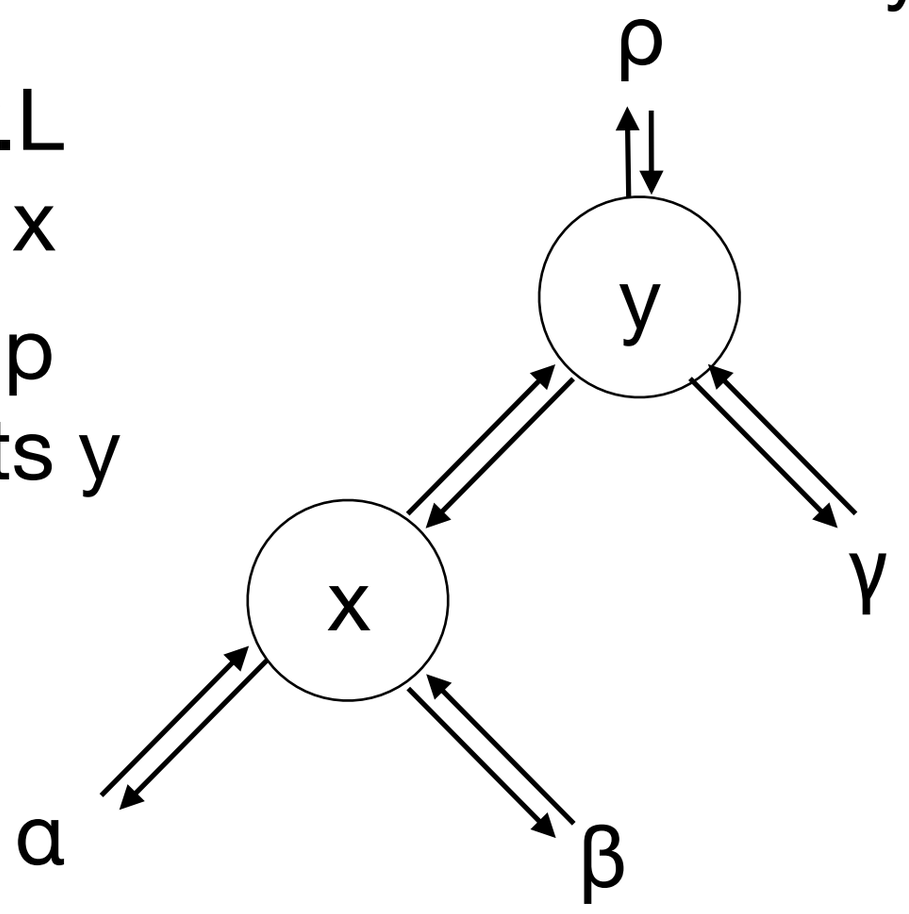

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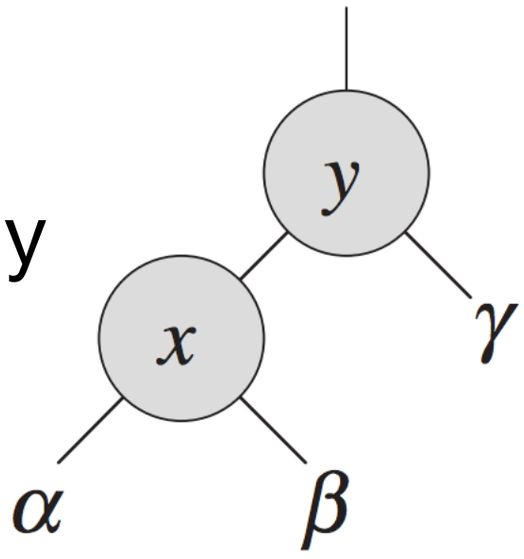
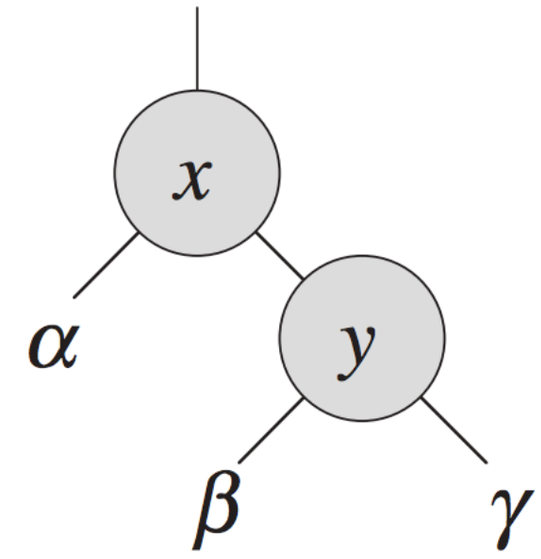


LEFT-ROTATE(T, x)

RIGHT-ROTATE(T, y)



Overall Transformation

Pseudocode from CLRS

```
LEFT-ROTATE(T, x)
1   y = x.right                    // set y
2   x.right = y.left               // turn y's left subtree into x's right subtree
3   if y.left ≠ T.nil
4       y.left.p = x
5   y.p = x.p                      // link x's parent to y
6   if x.p == T.nil
7       T.root = y
8   elseif x == x.p.left
9       x.p.left = y
10  else x.p.right = y
11  y.left = x                     // put x on y's left
12  x.p = y
```

1. xfer β (lines 1–4)

2. xfer parent (lines 6–10)

3. xfer x (lines 11–12)

Notational quirk: assume T.nil means “null”

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