



# CSCI 241

Lecture 13b

Balance Factor

Balanced Binary Search Trees

# Goals

- Know why we want our BSTs to be **balanced**.
- Be able to calculate the **balance factor** of a tree.

# Back to BSTs

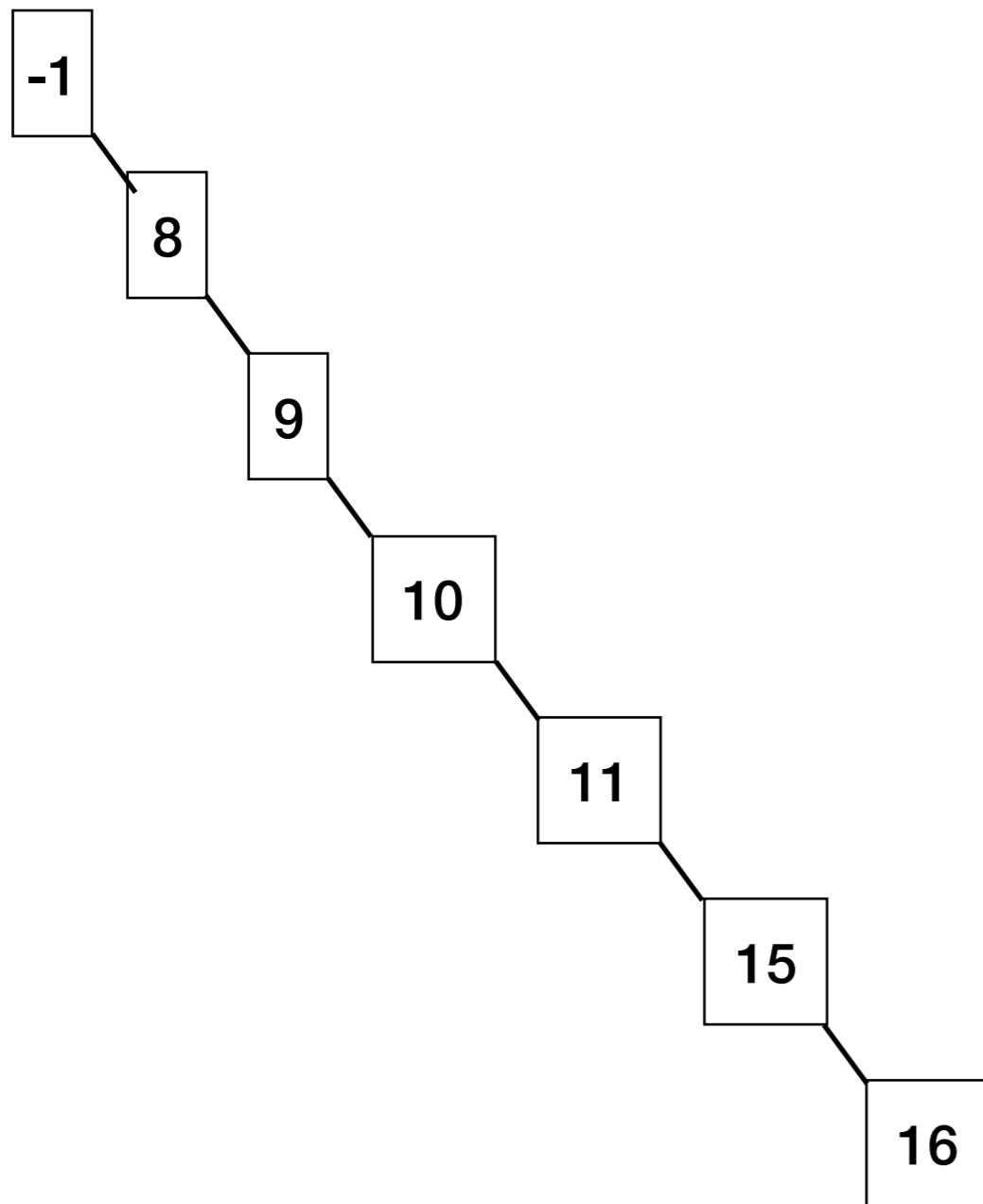
Long ago, we built some trees:

```
t = new BST();  
t.insert(-1)  
t.insert(8)  
t.insert(9)  
t.insert(10)  
t.insert(11)  
t.insert(15)  
t.insert(16)  
t.insert(16)
```

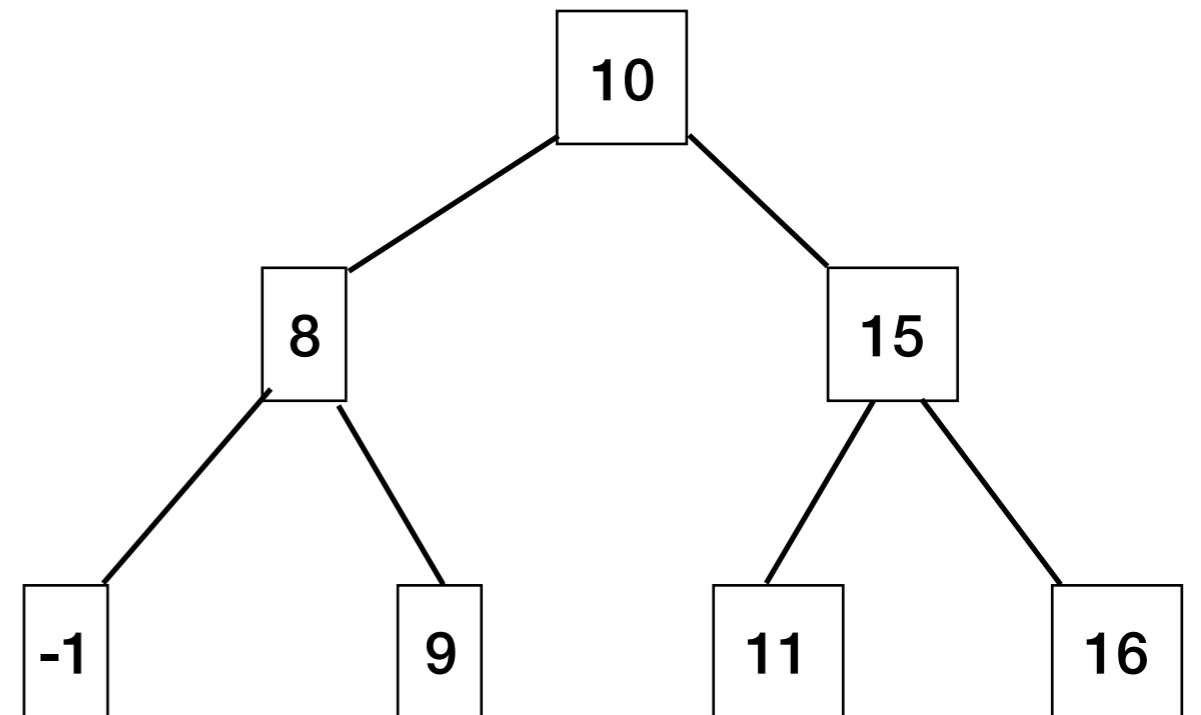
```
t = new BST();  
t.insert(10)  
t.insert(15)  
t.insert(16)  
t.insert(8)  
t.insert(16)  
t.insert(9)  
t.insert(11)  
t.insert(-1)
```

# Same values, different trees

Bad tree =(



Good tree =)

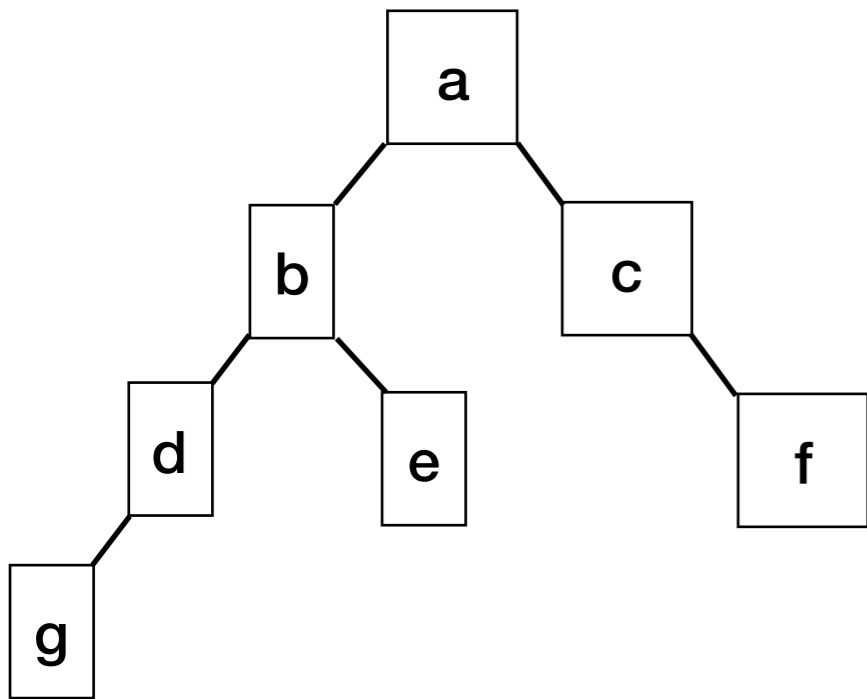


# Quantifying Badness: Balance Factor

**Height**(t): path length from t's deepest descendant (leaf) to t's root.

**Height**(n): height of the subtree rooted at n

**Balance**(n):  $\text{height}(\text{n.right}) - \text{height}(\text{n.left})$



$\text{height}(\text{null}) = -1$

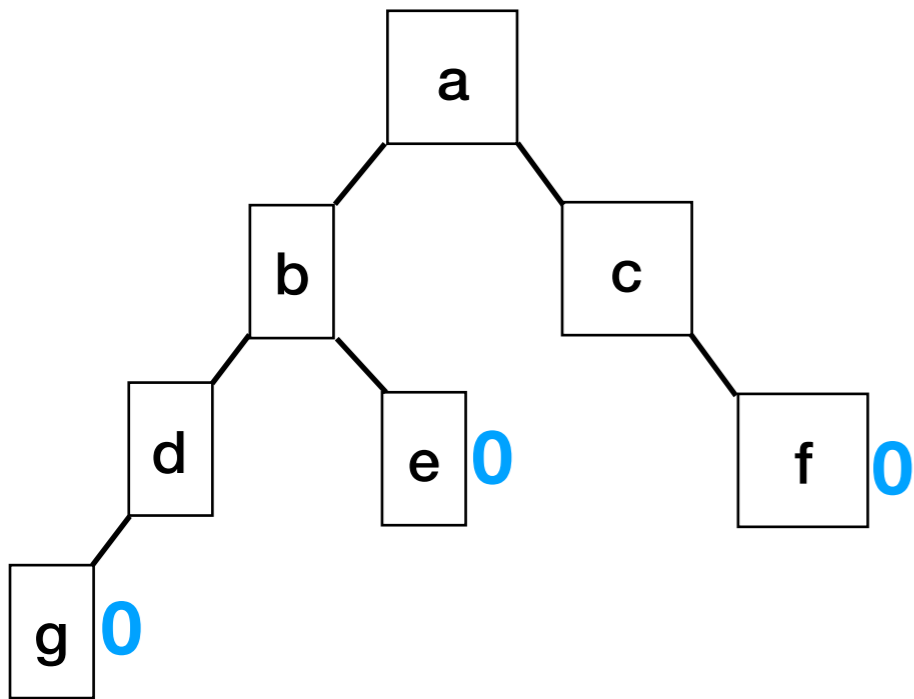
$\text{height}(n) = 1 + \max(\text{height}(n.\text{left}), \text{height}(n.\text{right}))$

# Quantifying Badness: Balance Factor

**Height**(t): path length from t's deepest descendant (leaf) to t's root.

**Height**(n): height of the subtree rooted at n

**Balance**(n):  $\text{height}(\text{n.right}) - \text{height}(\text{n.left})$



$\text{height}(\text{null}) = -1$

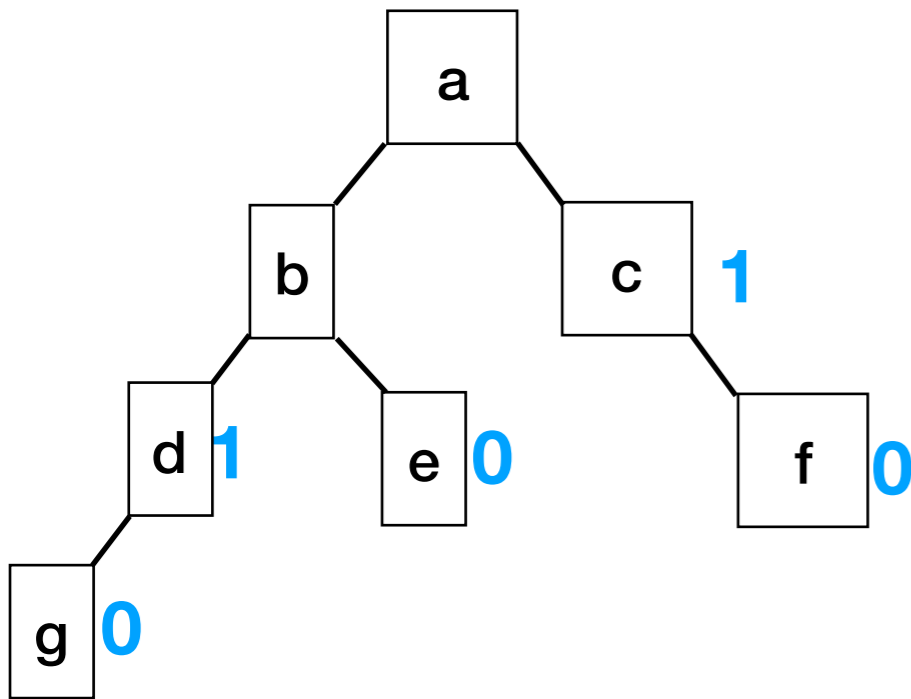
$\text{height}(n) = 1 + \max(\text{height}(n.\text{left}), \text{height}(n.\text{right}))$

# Quantifying Badness: Balance Factor

**Height**(t): path length from t's deepest descendant (leaf) to t's root.

**Height**(n): height of the subtree rooted at n

**Balance**(n):  $\text{height}(n.\text{right}) - \text{height}(n.\text{left})$



$\text{height}(\text{null}) = -1$

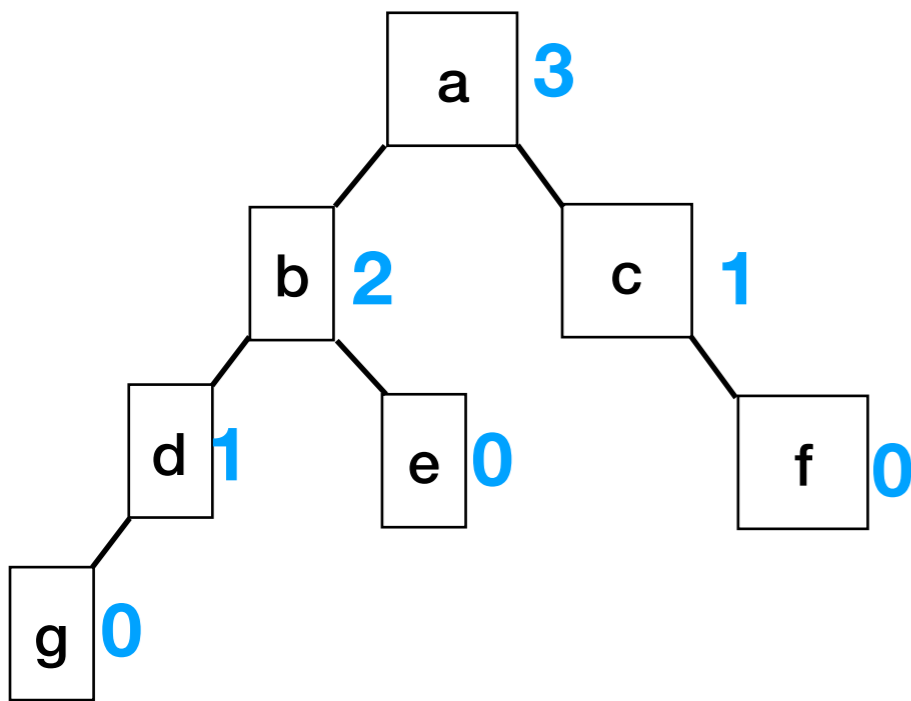
$\text{height}(n) = 1 + \max(\text{height}(n.\text{left}), \text{height}(n.\text{right}))$

# Quantifying Badness: Balance Factor

**Height**(t): path length from t's deepest descendant (leaf) to t's root.

**Height**(n): height of the subtree rooted at n

**Balance**(n):  $\text{height}(\text{n.right}) - \text{height}(\text{n.left})$



$\text{height}(\text{null}) = -1$

$\text{height}(n) = 1 + \max(\text{height}(n.\text{left}), \text{height}(n.\text{right}))$

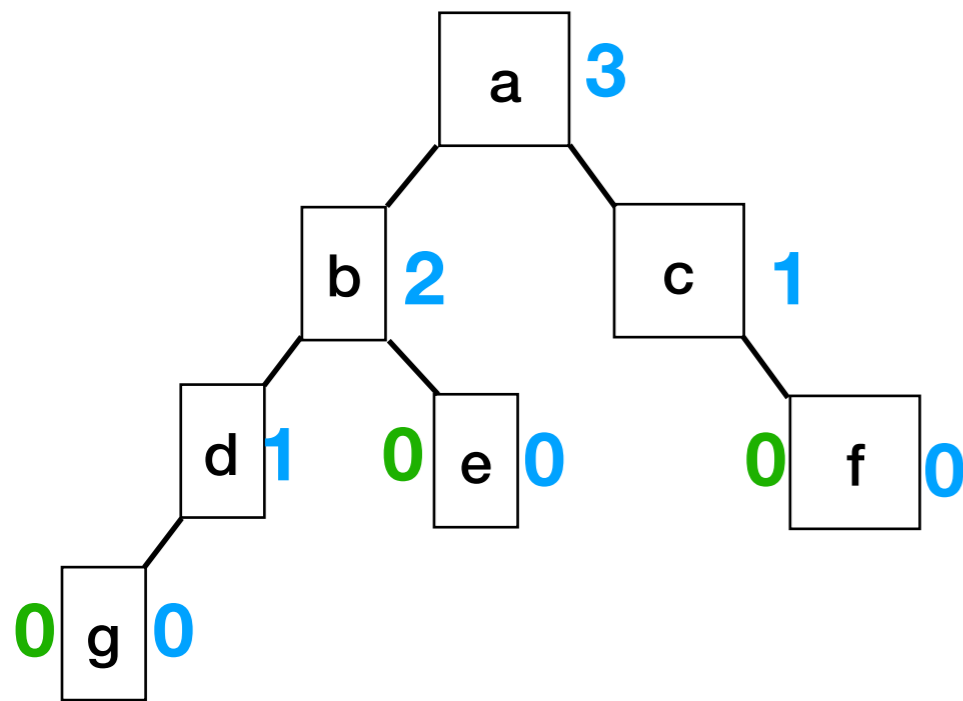


# Quantifying Badness: Balance Factor

**Height**(t): path length from t's deepest descendant (leaf) to t's root.

**Height**(n): height of the subtree rooted at n

**Balance**(n):  $\text{height}(\text{n.right}) - \text{height}(\text{n.left})$



$\text{height}(\text{null}) = -1$

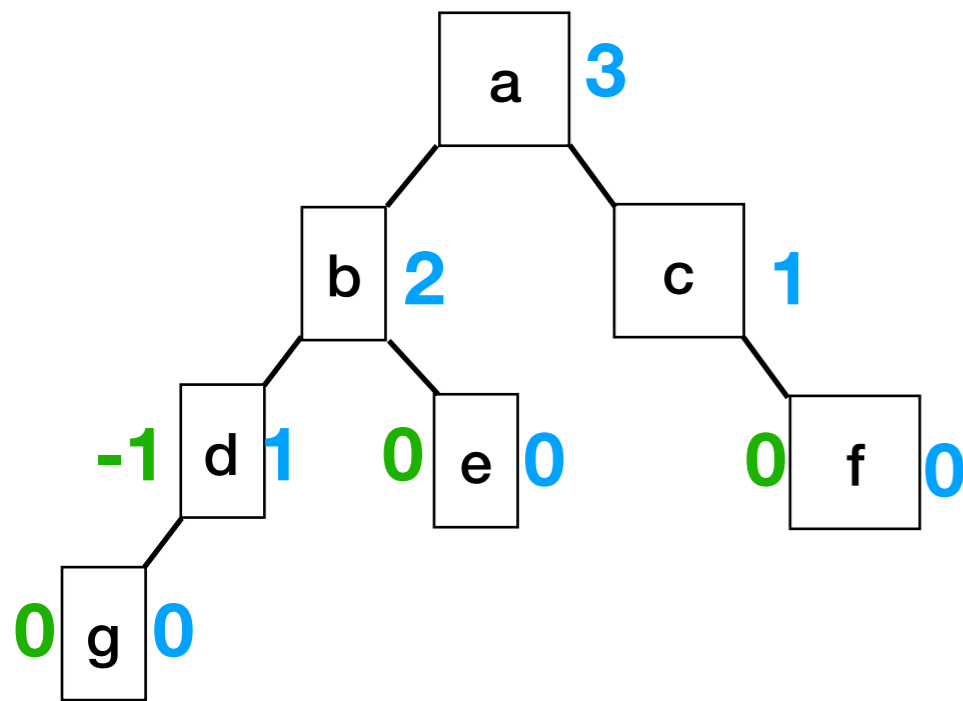
$\text{height}(n) = 1 + \max(\text{height}(n.\text{left}), \text{height}(n.\text{right}))$

# Quantifying Badness: Balance Factor

**Height**(t): path length from t's deepest descendant (leaf) to t's root.

**Height**(n): height of the subtree rooted at n

**Balance**(n):  $\text{height}(\text{n.right}) - \text{height}(\text{n.left})$



$\text{height}(\text{null}) = -1$

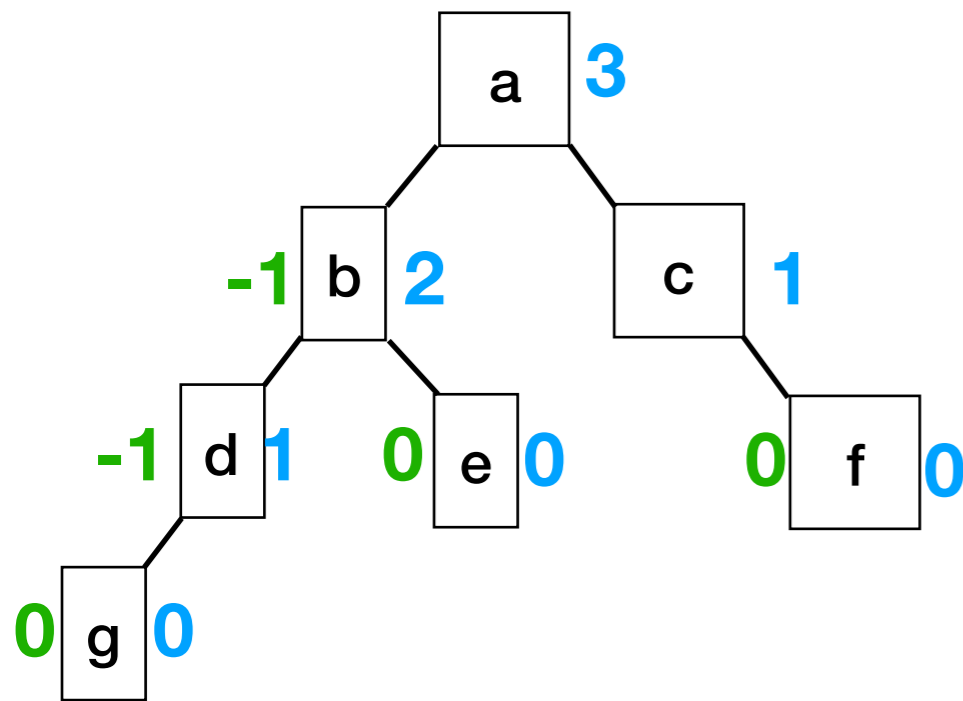
$\text{height}(n) = 1 + \max(\text{height}(n.\text{left}), \text{height}(n.\text{right}))$

# Quantifying Badness: Balance Factor

**Height**(t): path length from t's deepest descendant (leaf) to t's root.

**Height**(n): height of the subtree rooted at n

**Balance**(n):  $\text{height}(\text{n.right}) - \text{height}(\text{n.left})$



$\text{height}(\text{null}) = -1$

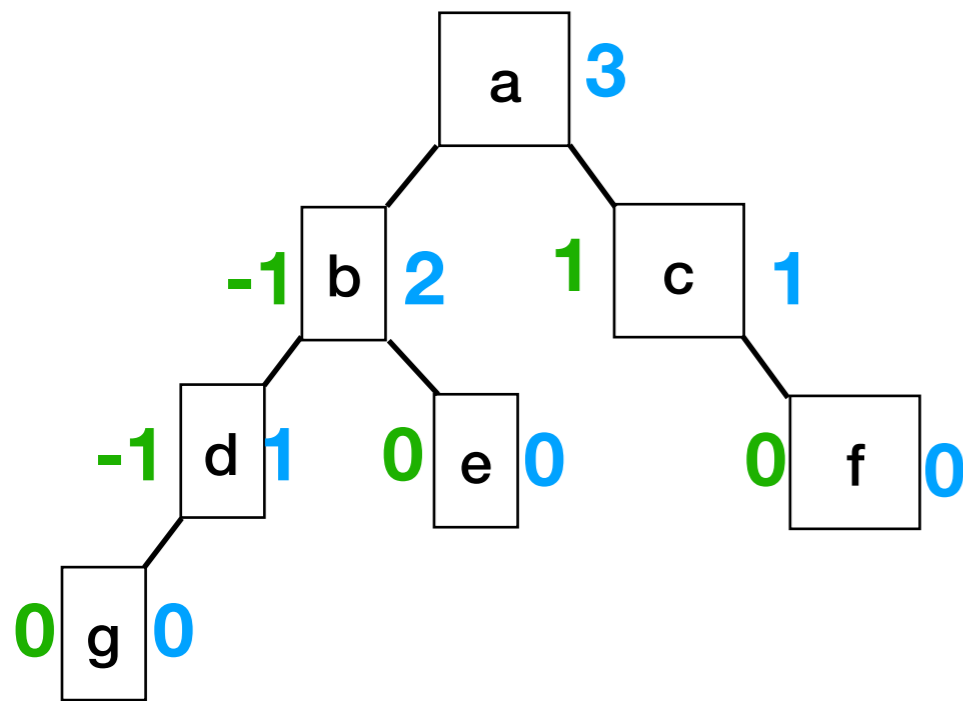
$\text{height}(n) = 1 + \max(\text{height}(n.\text{left}), \text{height}(n.\text{right}))$

# Quantifying Badness: Balance Factor

**Height**(t): path length from t's deepest descendant (leaf) to t's root.

**Height**(n): height of the subtree rooted at n

**Balance**(n):  $\text{height}(n.\text{right}) - \text{height}(n.\text{left})$



$\text{height}(\text{null}) = -1$

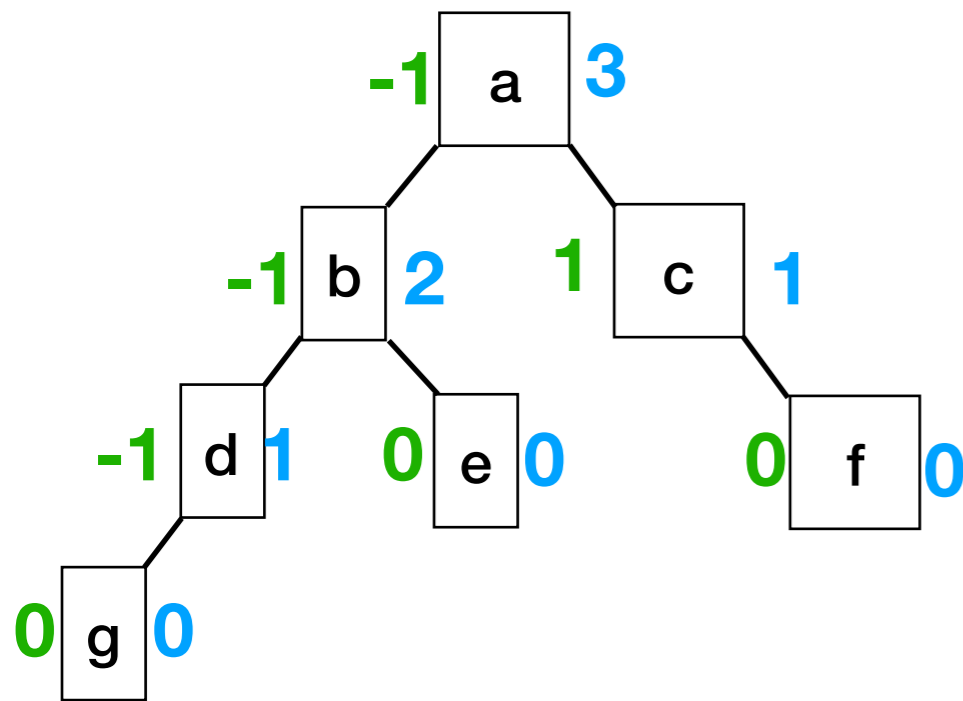
$\text{height}(n) = 1 + \max(\text{height}(n.\text{left}), \text{height}(n.\text{right}))$

# Quantifying Badness: Balance Factor

**Height**(t): path length from t's deepest descendant (leaf) to t's root.

**Height**(n): height of the subtree rooted at n

**Balance**(n):  $\text{height}(n.\text{right}) - \text{height}(n.\text{left})$



$\text{height}(\text{null}) = -1$

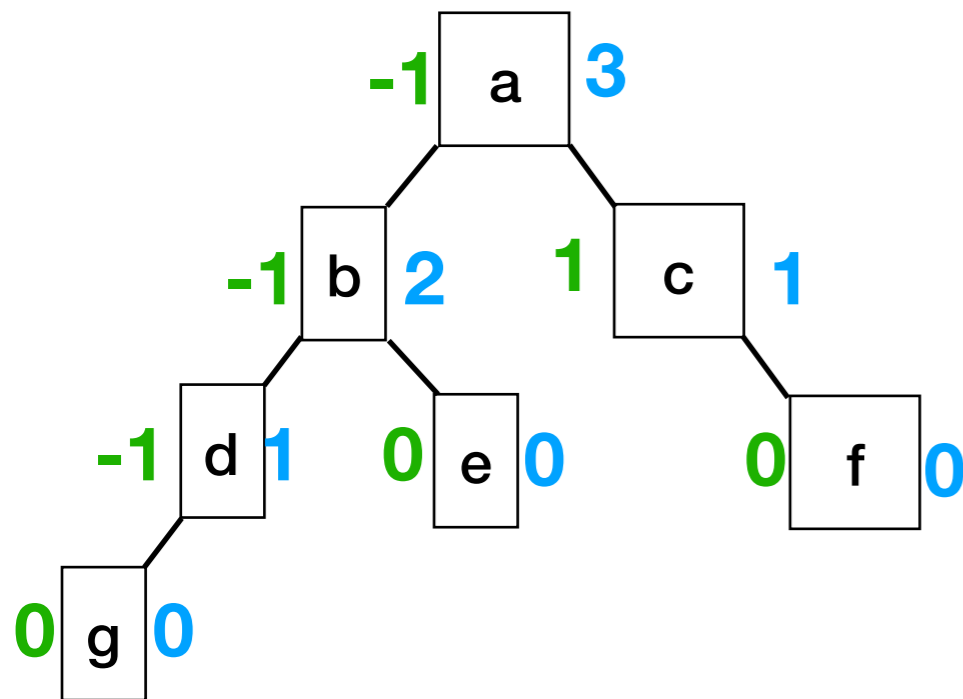
$\text{height}(n) = 1 + \max(\text{height}(n.\text{left}), \text{height}(n.\text{right}))$

# Quantifying Badness: Balance Factor

**Height**(t): path length from t's deepest descendant (leaf) to t's root.

**Height**(n): height of the subtree rooted at n

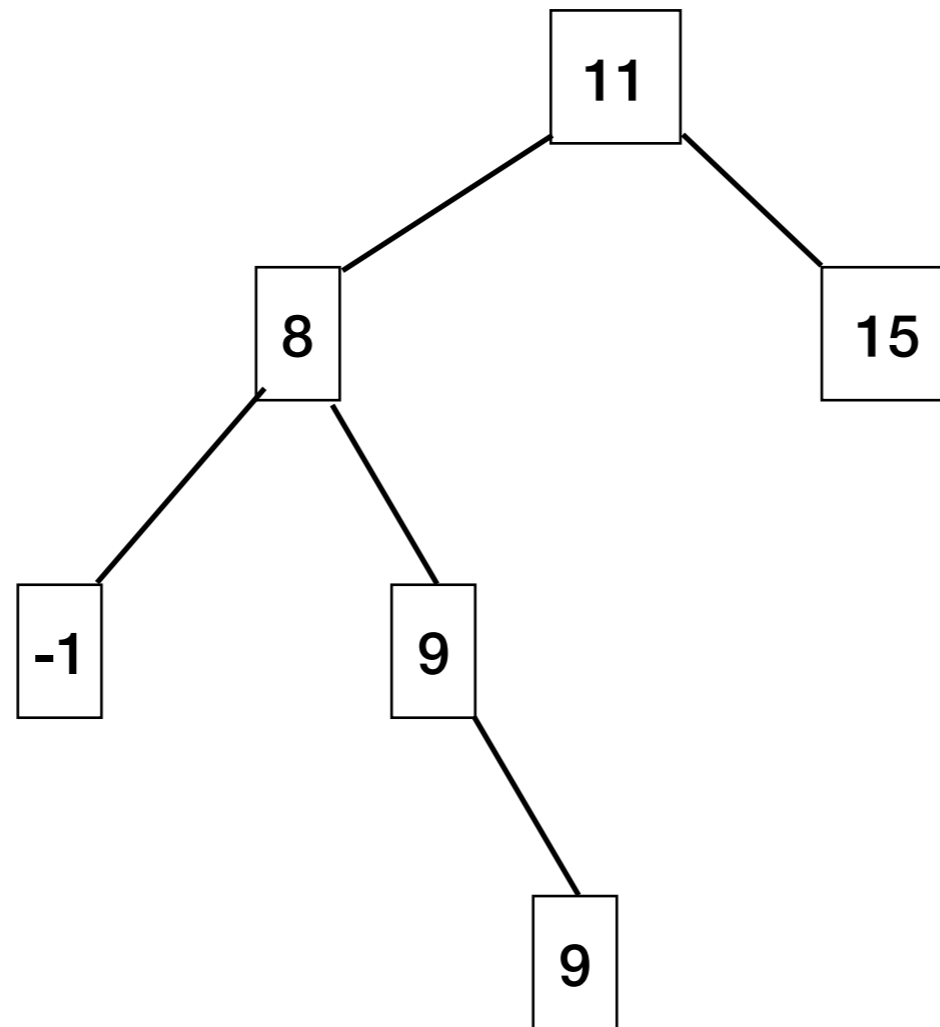
**Balance**(n):  $\text{height}(n.\text{right}) - \text{height}(n.\text{left})$



$\text{height}(\text{null}) = -1$

$\text{height}(n) = 1 + \max(\text{height}(n.\text{left}), \text{height}(n.\text{right}))$

# Balance Factor: Another Example



# Balance Factor: Another Example

