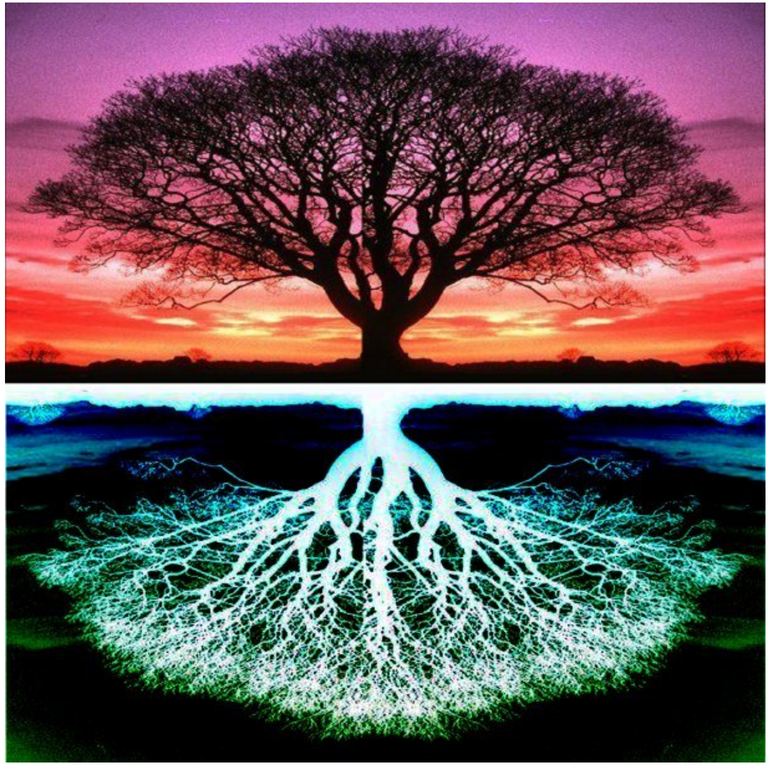


CSCI 241

Lecture ~~8~~: 10:

Abstract Data Types

Introduction to Trees

Announcements

- Submitting late (using slip days or otherwise) requires sending me email **after** you submit.
- Videos of Quicksort and Radix Sort runtime analysis will be posted soon after class.
- Today: onward to trees!
- Survey!
- There is a lab this week

Goals:

- Know the difference between an abstract data type and its implementation.
- Understand the motivation for trees:
  - To model **tree-structured data.**
  - To implement **abstract data types**.
- Understand the definition of a tree.
- Know the basic terminology associated with trees:
  - Root, child, parent, leaf, height, depth, subtree, descendent, ancestor
- Be able to write a tree class and simple recursive methods such as size, height, and traversals (lab 4).

Last Week:
Big-Deal CS Concept #1: Runtime

Big-Deal CS Concept #2: Interface vs Implementation and Abstract Data Types

An abstract data type specifies only **interface**, not **implementation**

Big-Deal CS Concept #2: Interface vs Implementation and Abstract Data Types

What the operations do

An abstract data type specifies only **interface**, not **implementation**

Big-Deal CS Concept #2: Interface vs Implementation and Abstract Data Types

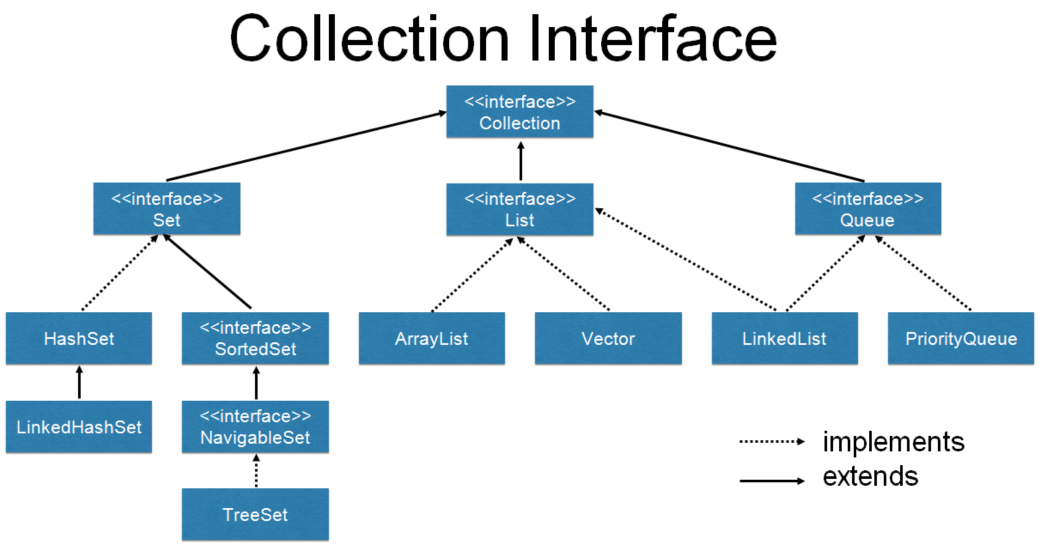
What the operations do

An abstract data type specifies only **interface**, not **implementation**

How they are accomplished

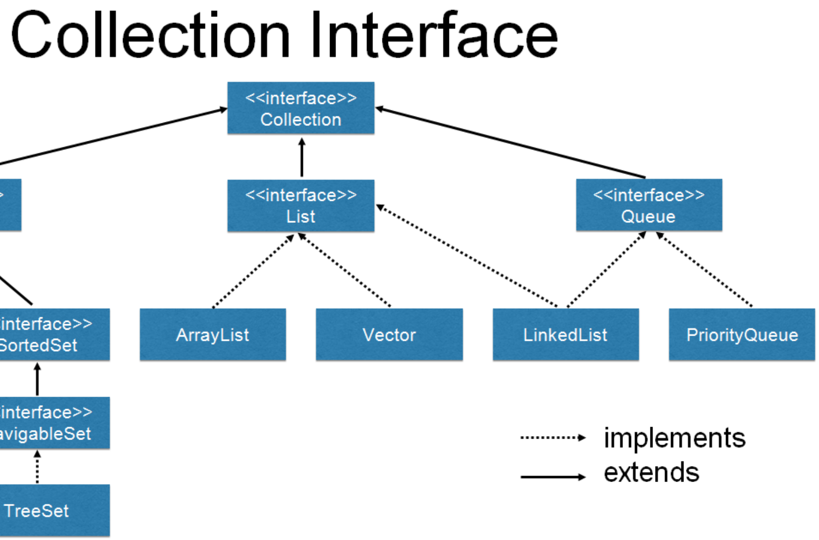
Abstract Data Types: Examples

Abstract Data Types: Examples



Abstract Data Types: Examples

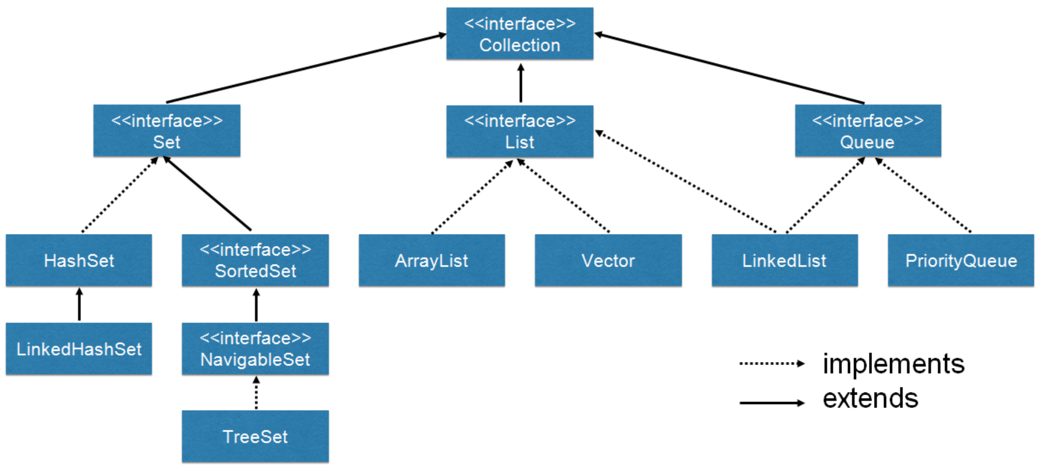
- List, Queue, Stack



Abstract Data Types: Examples

- List, Queue, Stack
- Set

Collection Interface



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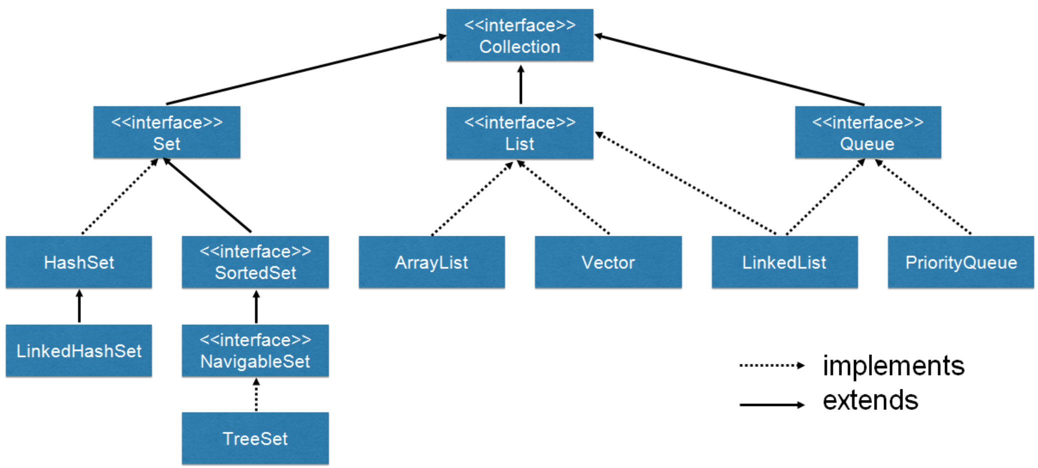
- Set
- Tree

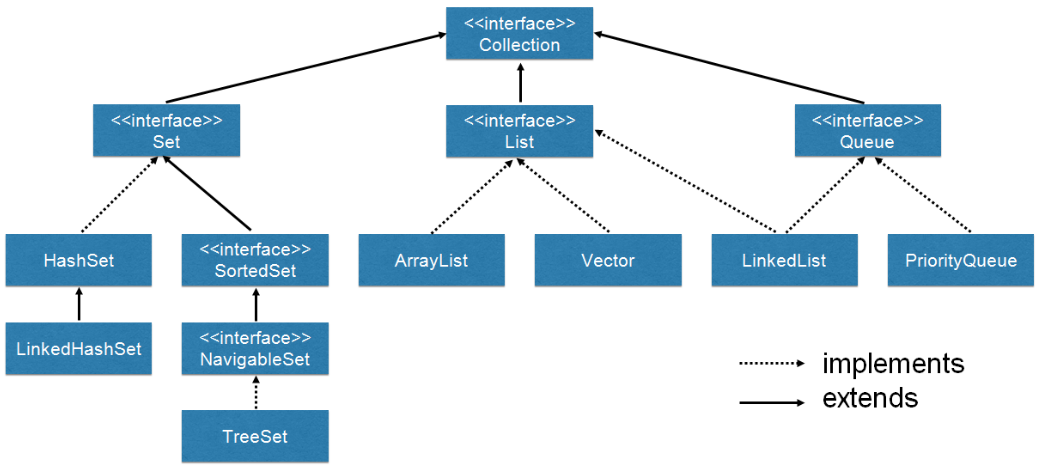
Collection Interface



Abstract Data Types: Examples

- List, Queue, Stack
- Set
- Tree
- Priority Queue

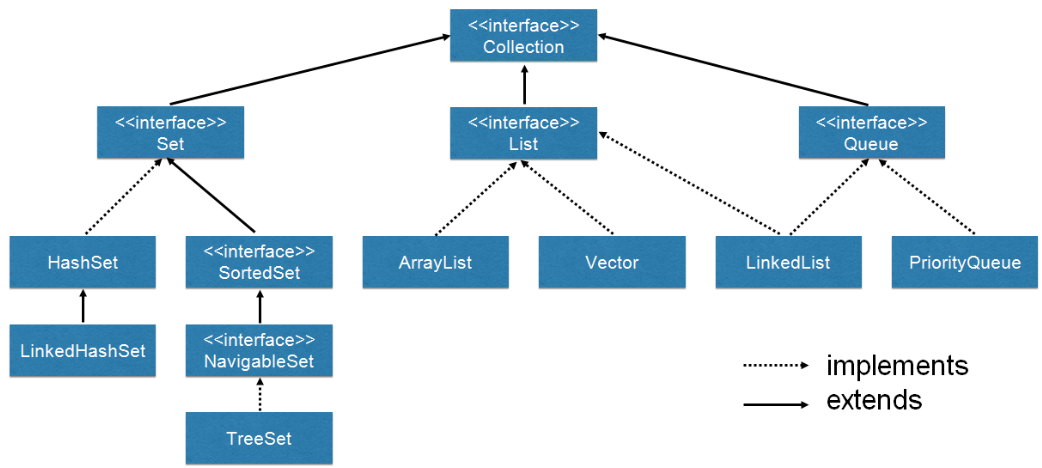
Collection Interface



Abstract Data Types: Examples

- List, Queue, Stack
- Set
- Tree
- Priority Queue
- Map

Collection Interface



Abstract Data Types: Examples

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- Set
- Tree
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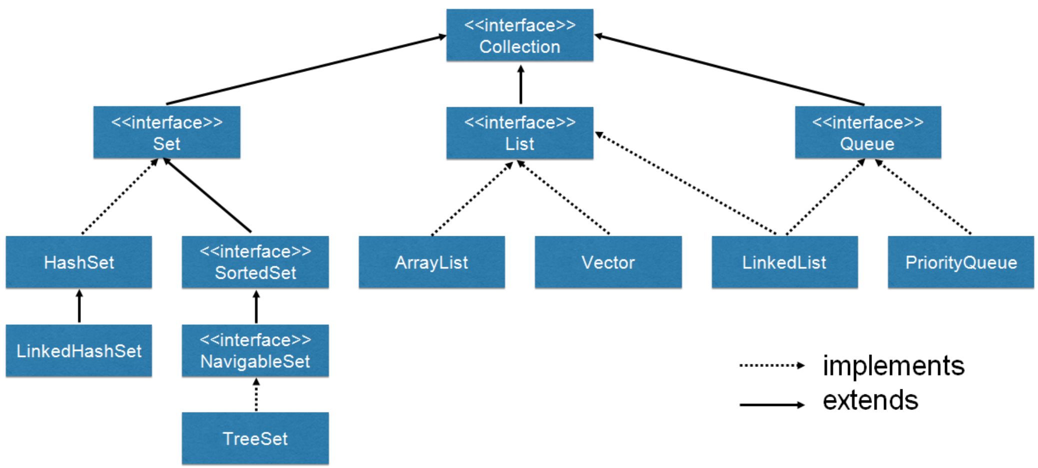
- Map
- Graph

Collection Interface



Abstract Data Types: Examples

(145)

(Weeks 4,5,7)

(Weeks 4-6; A2)

(Week 6; A3)

(Week 7; A3)

(Weeks 8-9; A4)

Abstract Data Types: Examples

- List, Queue, Stack (145)

(Weeks 4,5,7)

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Abstract Data Types: Examples

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Abstract Data Types: Examples

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Abstract Data Types: Examples

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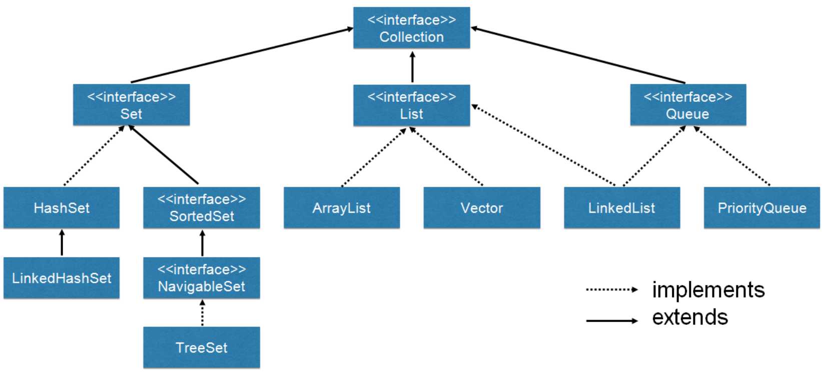
Abstract Data Types: Examples

Collection Interface



- List, Queue, Stack (145)
- Set (Weeks 4,5,7)
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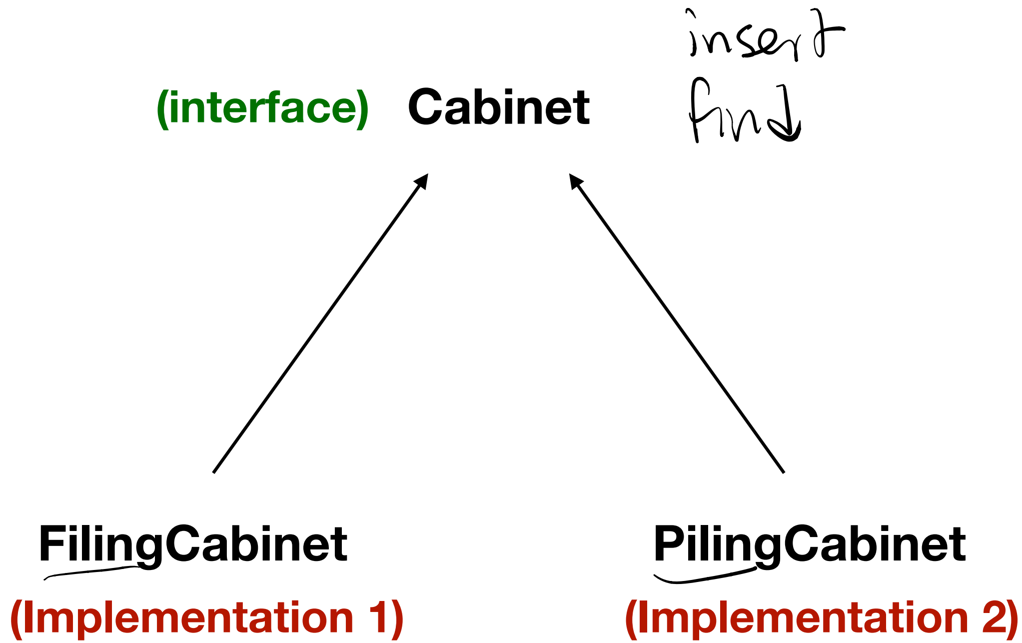
Interface vs Implementation: Example



Interface vs Implementation: Example

Interface

Cabinet:

(short for "if and only if")

- Contains(item) - returns true iff item is in the cabinet
- Add(item) - adds item to the cabinet
- Remove(item) - removes item from the cabinet if it exists

Implementation

FilingCabinet implements Cabinet:

Contains(item):

```
look up drawer by first letter range
find folder by first letter
search folder for item
return true if item is found, false otherwise
```

Comparing Implementations

class FilingCabinet:

- Contains(item):

```
    look up drawer by first letter range
    find folder by first letter
    search folder for item
    return true if item is found, false otherwise
```

class PilingCabinet:

- Contains(item):

```
  for each drawer:
    exhaustively search drawer
    if found, return true
  return false
```

Comparing Implementations

class FilingCabinet:

- Add(item):

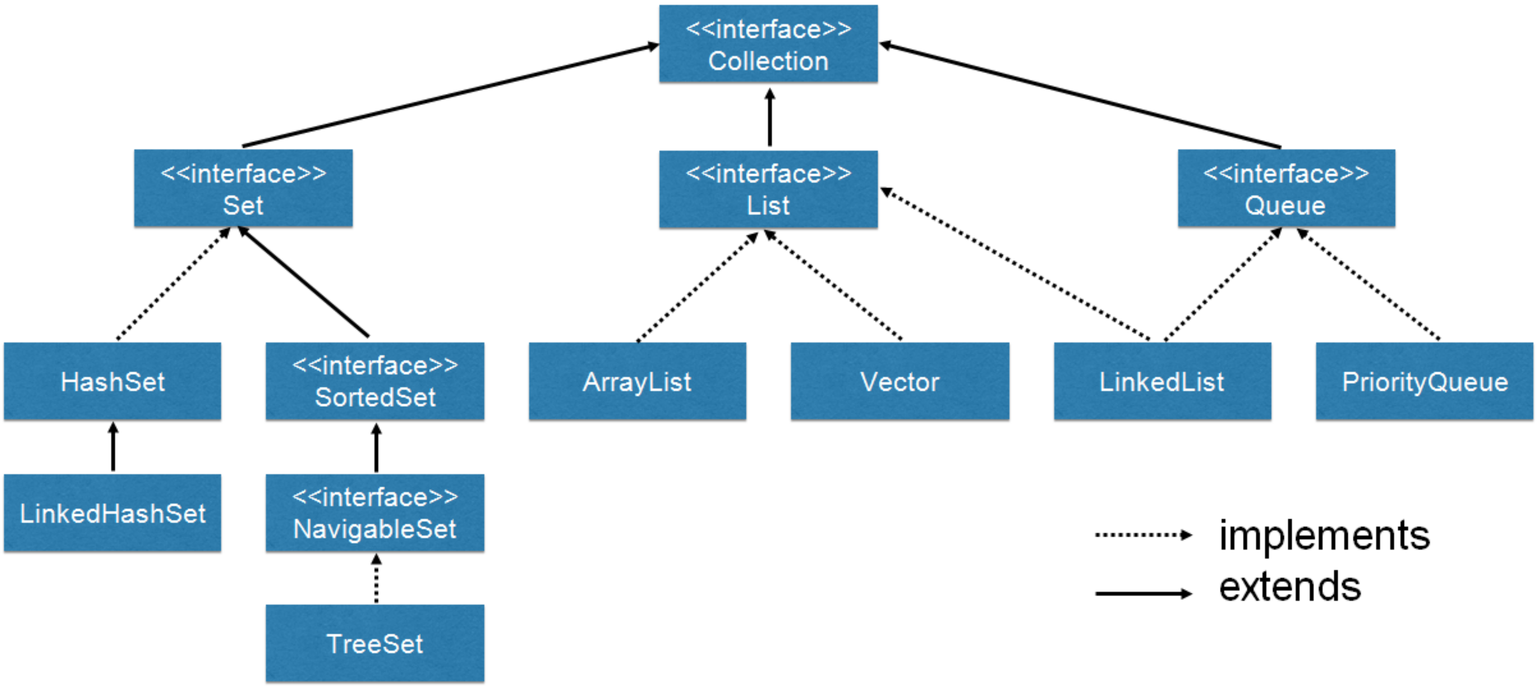
```
look up drawer by first letter range
find folder by first letter
insert item into folder
```

class PilingCabinet:

- Add(item):

```
open random drawer
insert item into drawer
```

Collection Interface



Is an array an $\underbrace{\text{ADT}}_{\text{abstract data type}}$?

ADTs and Runtime: Why we care

Runtime comparison of **List** implementations:

Class:	ArrayList	LinkedList
Backing storage:	array	chained nodes
addAt(i, val)	O(n)	O(n)
addFirst(val)	O(n)	O(1)
addLast(val)	O(1)	O(1)
get(i)	O(1)	O(n)
getFirst()	O(1)	O(1)
getLast()	O(1)	O(1)

Assume: i = arbitrary index   n = array's length

Linked List

```
public class ListNode {
   int value;
   ListNode next;
}
```

Linked List

```
public class List {
  int value;
  List next;
}
```

Linked List

```
public class List {
  int value;
  List next;
}
```

The node ***is*** *the list*.
Next points to the **tail** of the list (also a list!)

Binary Tree

```
public class Tree {
  int value;
  Tree left;
  Tree right;
}
```

Binary Tree

```
public class Tree {
  int value;
  Tree left;
  Tree right;
}
```

The node ***is*** *the tree*.

`left` points to the **left child** of the tree (also a tree!)

`right` points to the **right child** of the tree (also a tree!)

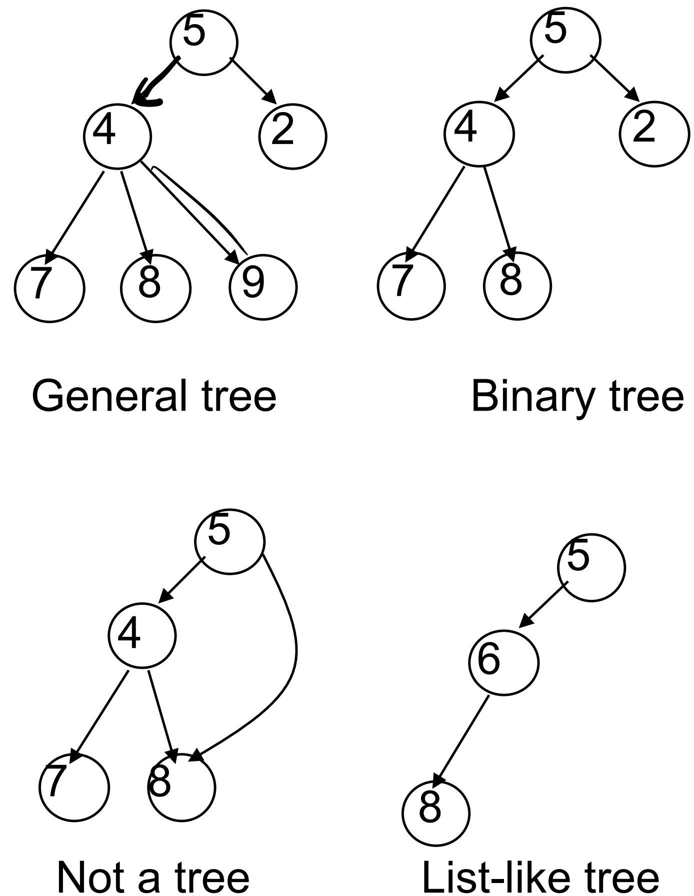
Tree - Definition

Tree: like a linked list, but:

- Each node may have zero or more *successors* (**children**)
- Each node has exactly one *predecessor* (**parent**) except the **root**, which has none
- All nodes are reachable from *root*

Binary tree: A tree, but:

- Each node can have at most **two** children (**left child**, **right child**)



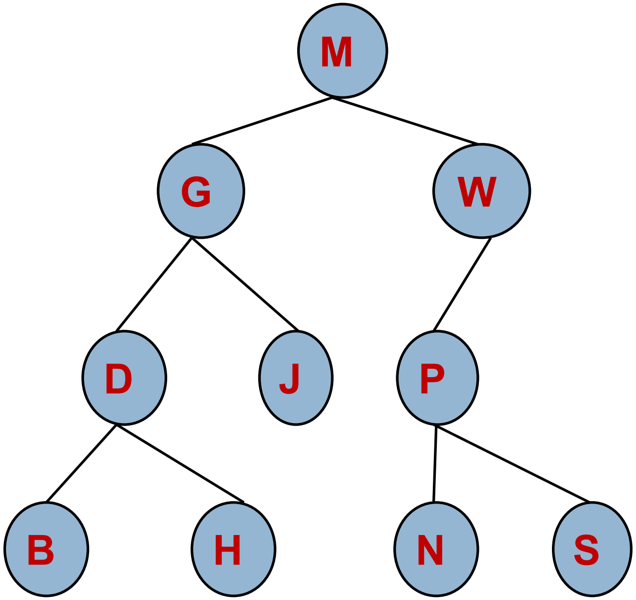
Tree Terminology

M is the **root** of this tree

N is the **left child** of P

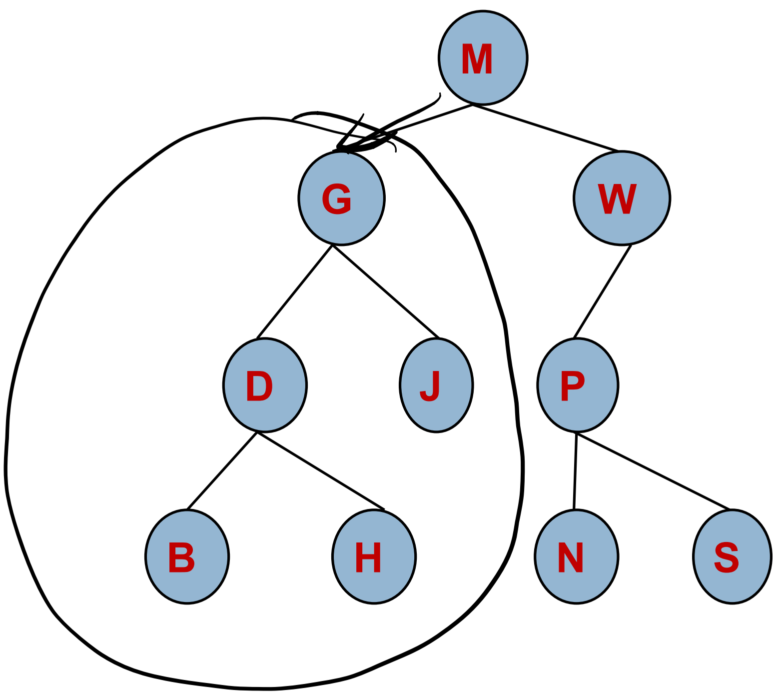
S is the **right child** of P

P is the **parent** of N



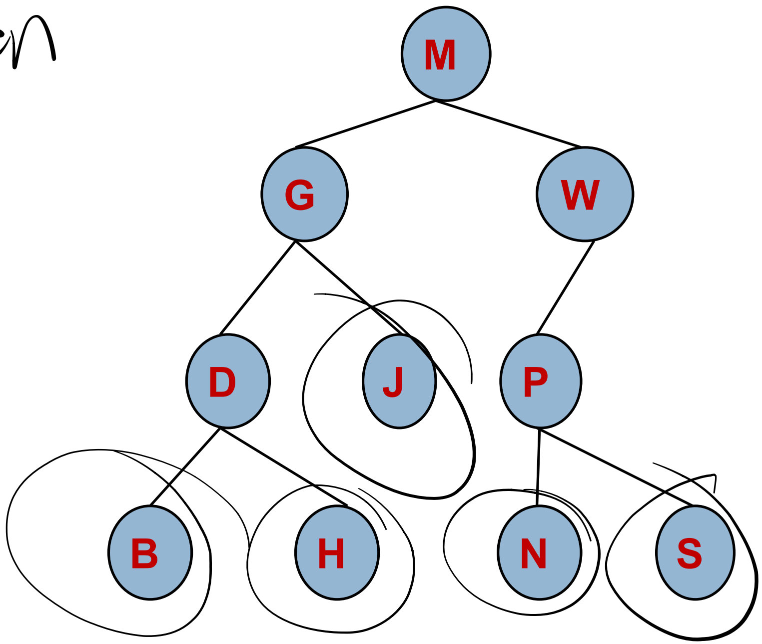
Subtree

G is the root of
M's left subtree



Leaf

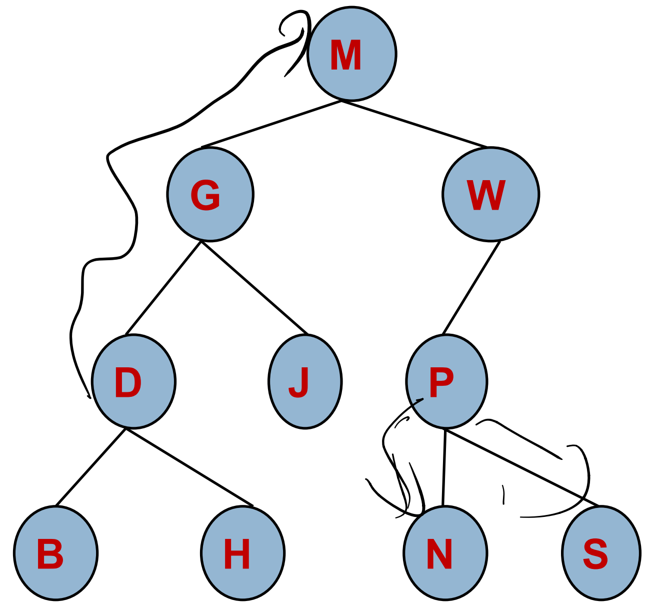
A leaf has no children



Ancestor, Descendent

B's ancestors are
D, G, M

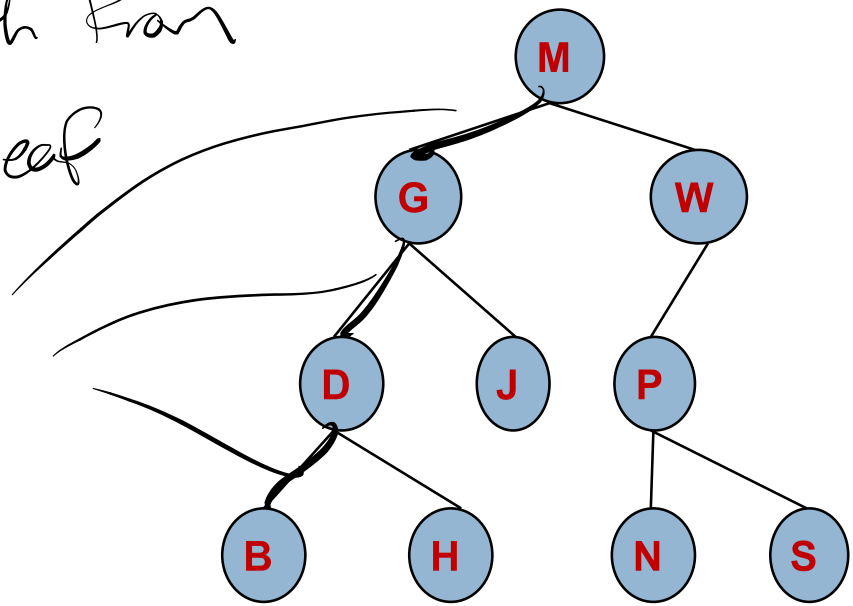
P's descendants are
N, S



Height of a tree

length of the path from root to deepest leaf

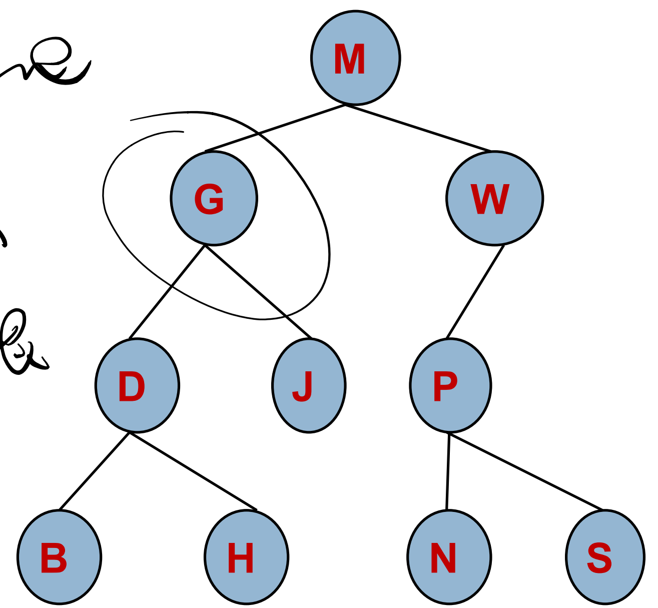
Height 3



Depth of a node

Depth of a node is the length of the path from the root to the node

G has depth 1



Tree Terminology

M is the **root** of this tree

N is the **left child** of P

S is the **right child** of P

P is the **parent** of N

G is the **root** of the **left subtree** of M

B, H, J, N, S are **leaves**

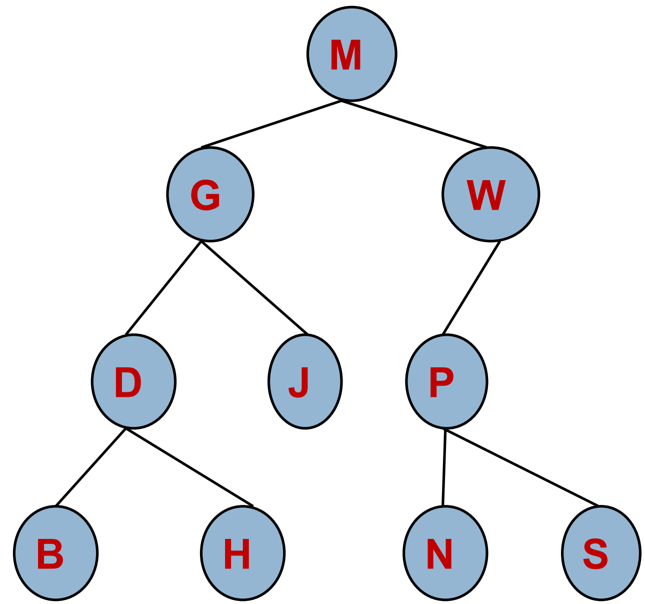
M and G are **ancestors** of D

P, N, S are **descendants** of W

J is at **depth** 2

The subtree rooted at W has **height** 2

A collection of several trees is called a forest?



```
public class BinaryTreeNode {
  private int value;
  private BinaryTreeNode parent;  (null if no left child)
  private BinaryTreeNode left;   // left subtree
  private BinaryTreeNode right;  // right subtree
                                 (null if no right child)
}
```

```
public class GeneralTreeNode {
  private int value;
  private GeneralTreeNode parent;
  private List<GeneralTreeNode> children;
}
```

Why do we need these?

Why do we need these?

to represent **hierarchical structure.**

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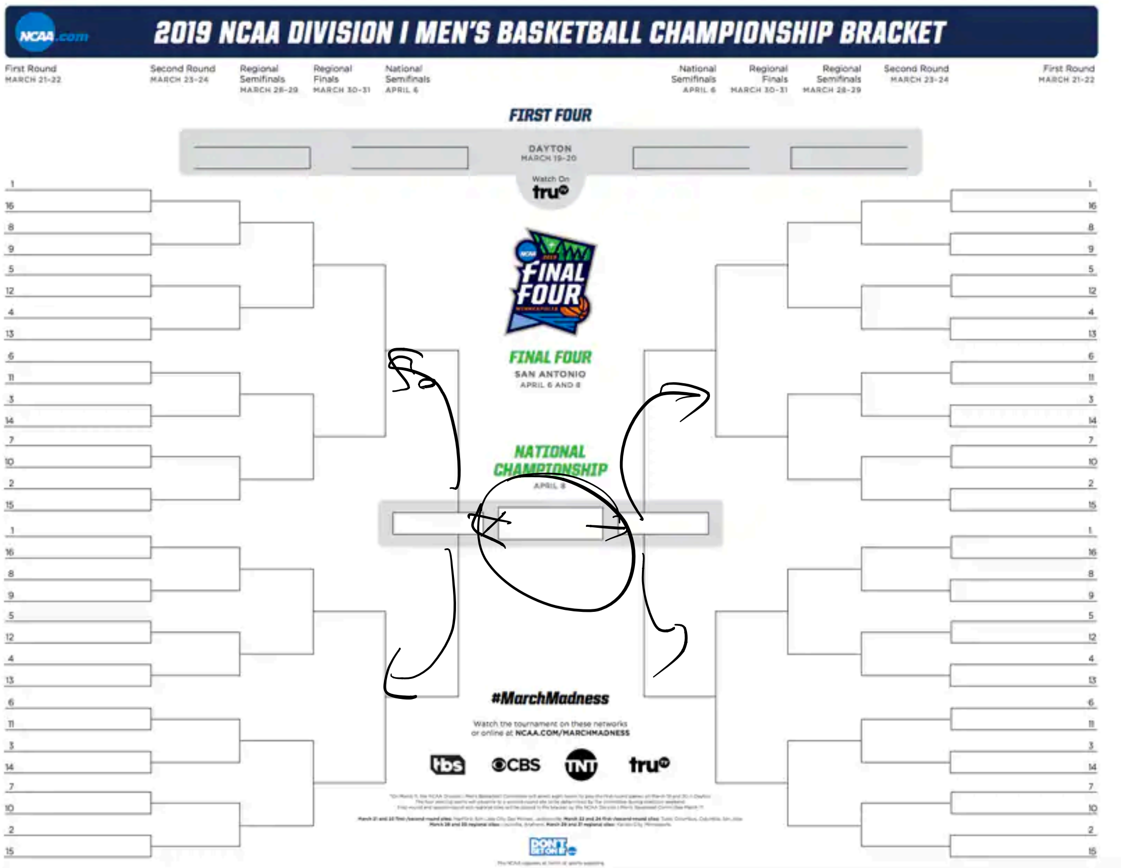

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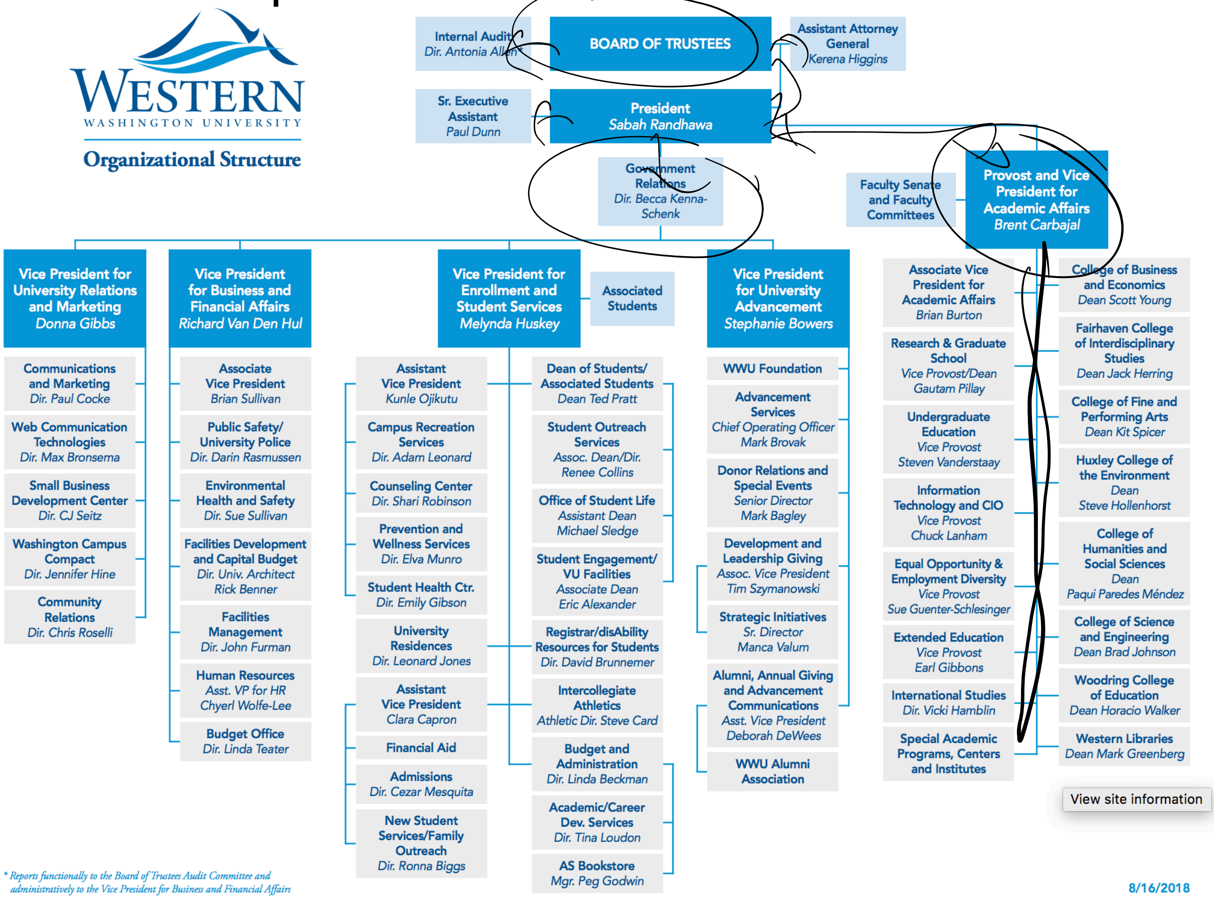

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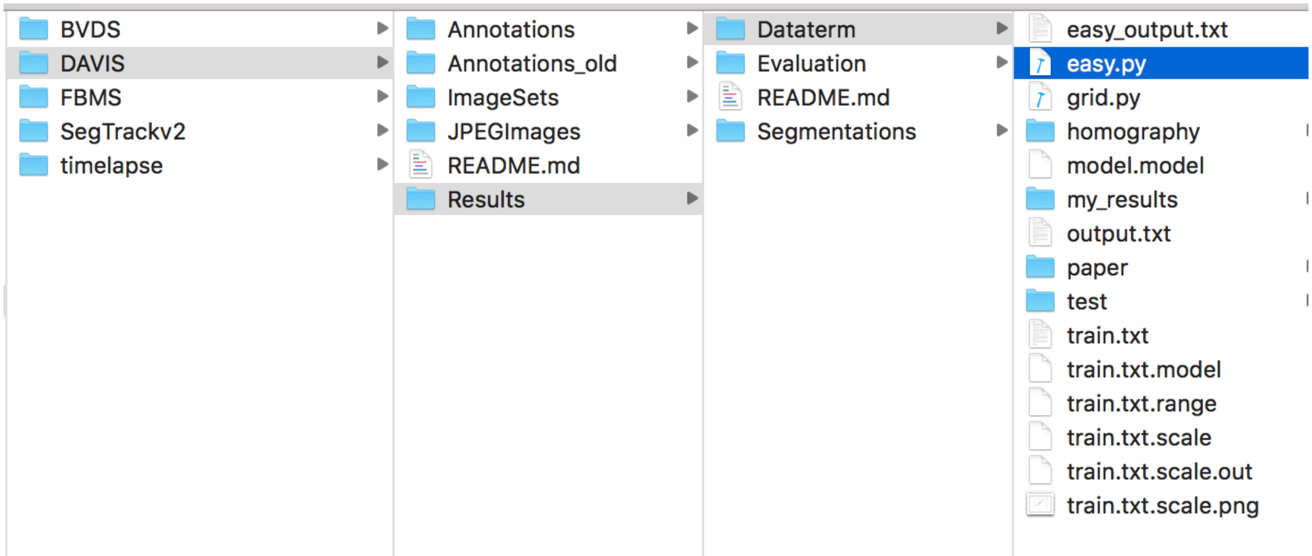


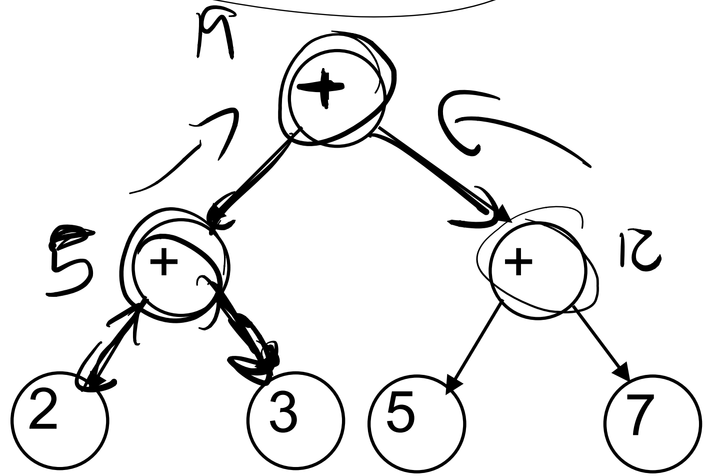
Why do we need these?

to represent **hierarchical structure**.

Syntax Trees:

((2+3) + (5+7))

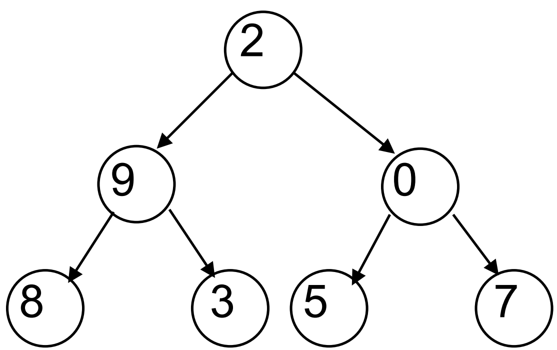
- In textual representation, **parentheses** show hierarchical structure
- In tree representation, hierarchy is explicit in the tree's **structure**



Also used for **natural languages** and **programming languages**!

Why do we need these?

to implement various ADTs **efficiently.**



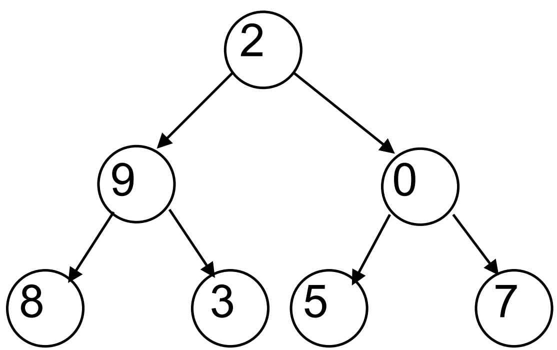
Why do we need these?

to implement various ADTs **efficiently.**

Tree**Set**, Tree**Map**

unordered collection of unique items

unordered collections of key-value pairs



Why do we need these?

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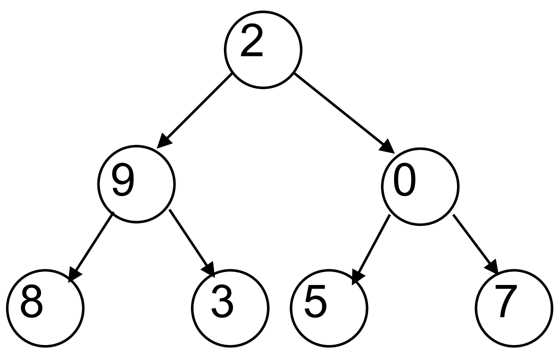
Tree**Set**, Tree**Map**

Height of a balanced binary tree is O(log n)

Consequence: Many operations (find, insert, …) can be done in **O(log n)** in carefully-designed trees.



Thinking about trees recursively

- A **binary tree** is

```
public class BinaryTreeNode {
  private int value;
  private BinaryTreeNode parent;
  private BinaryTreeNode left;
  private BinaryTreeNode right;

}
```

  - Empty, or
  - Three things:
    - value
    - a left **binary tree**
    - a right **binary tree**

Thinking about trees recursively

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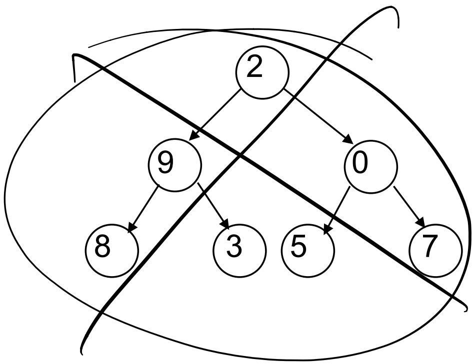

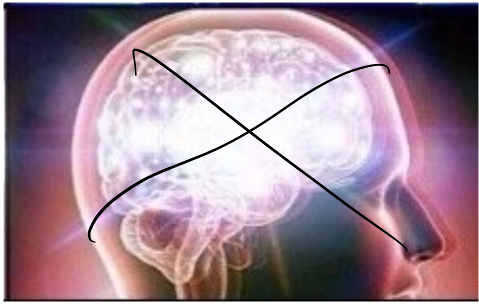

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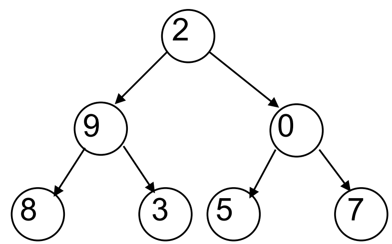

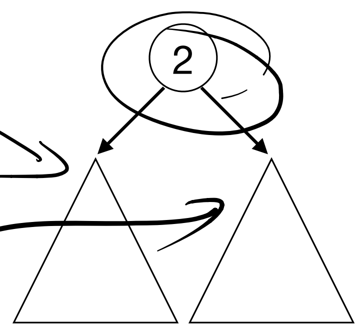

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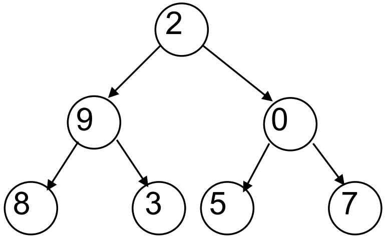

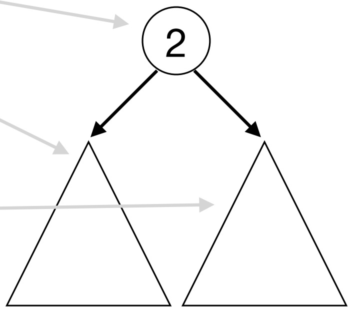

Operations on trees

often follow naturally from the definition of a tree:

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Find v in a binary tree:

Operations on trees

often follow naturally from the definition of a tree:

- A **binary tree** is

Find v in a binary tree:

  - Empty, or

(base case - not found!)

  - Three things:
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(recursive call - is v in left?)

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Find v in a binary tree:

```
boolean findVal(Tree t, int v):
```

(base case - not found!)

(base case - is this v?)

(recursive call - is v in left?)

(recursive call - is v in right?)

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Find v in a binary tree:

```
boolean findVal(Tree t, int v):
```

(base case - not found!)

```
if t == null:
   return false
```

(base case - is this v?)

(recursive call - is v in left?)

(recursive call - is v in right?)

Operations on trees

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    if t.value == v: return true
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(recursive call - is v in left?)

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boolean findVal(Tree t, int v):
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(base case - not found!)

```
if t == null:
   return false
```

(base case - is this v?)

```
if t.value == v: return true
```

(recursive call - is v in left?)

```
return findVal(t.left)
    || findVal(t.right)
```

(recursive call - is v in right?)

Tree Traversals

Print (or otherwise process) every node in a tree:

- A **binary tree** is
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Tree Traversals

Print (or otherwise process) every node in a tree:

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Print all nodes in a binary tree:

```
boolean printTree(Tree t):
```

Tree Traversals

Print (or otherwise process) every node in a tree:

- A **binary tree** is
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    - a left **binary tree**
    - a right **binary tree**

Print all nodes in a binary tree:

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boolean printTree(Tree t):
```

(base case - nothing to print)

```
if t == null:
    return
```

Tree Traversals

Print (or otherwise process) every node in a tree:

- A **binary tree** is
  - Empty, or
  - Three things:
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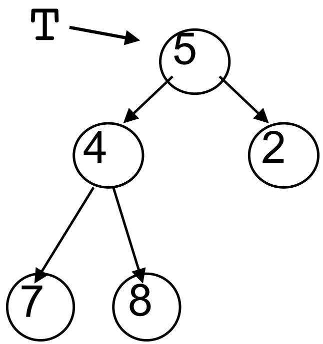

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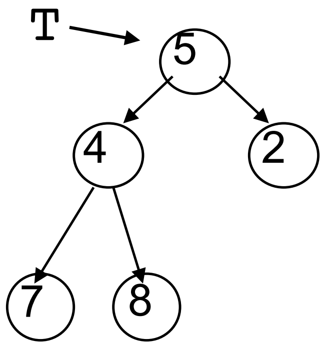

ABCD: `T` is a reference to the node with value 5. What is printed by the call `printTree(T)`?

A. 5 4 2 7 8
B. 7 4 8 5 2
C. 7 8 4 2 5
D. 5 4 7 8 2

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```

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"Walking" over the whole tree is called a tree traversal
This is done often enough that there are standard names.
Previous example was a **pre-order traversal**:

1. Process root
2. Process left subtree
3. Process right subtree

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Other common traversals:

in-order traversal:

1. Process left subtree
2. Process root
3. Process right subtree

post-order traversal:

1. Process left subtree
2. Process right subtree
3. Process root

Why do we need these?

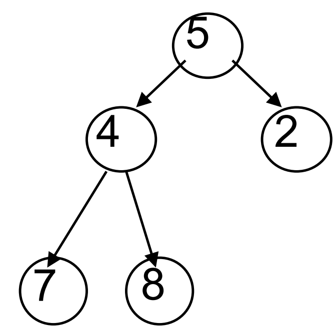
to represent **hierarchical structure.**

Quadtrees in graphics and simulation:
https://www.youtube.com/watch?v=fuexOsLOfl0

Practice Exercise

- Write the values printed by a:
  - pre-order
  - in-order
  - post-order

  traversal of this tree.



Terminology - Self-Quiz

root

subtree

leaf

child

parent

ancestor

descendant

depth

height