CSCI 241

Lecture ~~6~~ 7

~~Quicksort~~

Stability; Non-Comparison Sorts

Radix Sort

Announcements

- Quiz 1 grades and review video out soon

Goals:

- Know what it means for a sorting algorithm to be **stable**
- Understand the distinction between comparison and non-comparison sorts.
- Be prepared to implement radix sort.
- Know the definition of an in-place sorting algorithm.

Stability

Objects can be sorted on **keys** - **different** objects may have the same value.

A **stable** sort maintains the order of distinct elements with the same key.

- Example: sort the following list on the **tens** place only:

Sorted stably:

unstably:

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Examples: all four sorts we've seen so far!

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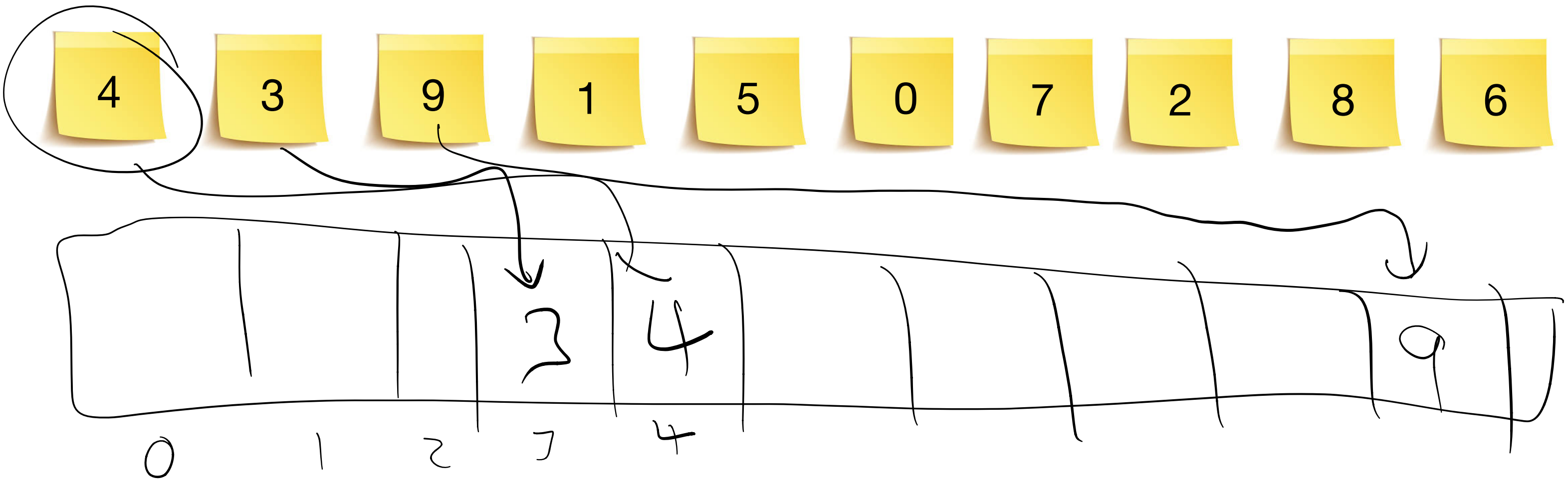
How do you sort without comparing elements?

How do you sort things without comparing them?

Suppose I gave you 10 sticky notes with the digits 0 through 9.

What algorithm would you use to sort them?



How do you sort things without comparing them?

Suppose I gave you 10 sticky notes with the digits 0 through 9.

What algorithm would you use to sort them?

How many times did you need to look at each sticky note?

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What algorithm would you use to sort them?

How many times did you need to look at each sticky note?

What if there are duplicates?

LSD Radix Sort

```
/** least significant digit radix sort A */
LSDRadixSort(A):
max_digits = max # digits in any element of A
for d in 0..max_digits:
   stably sort A on the dth least significant
   digit

// A is now sorted(!)
```

1. ones place, then
2. tens place, then
3. hundreds place,

and so on

Do you believe me?

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[45, 26, 42, 32] 07

42 32 45 26

26 32 42 45

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Still don’t believe me? https://visualgo.net/en/sorting

LSD Radix Sort using queue buckets

Pseudocode from visualgo.net:

```
LSDRadixSort(A):
 create 10 buckets (queues) for each digit (0 to 9)
 for each digit (least- to most-significant):
   for each element in A:
     move element into its bucket based on digit
   for each bucket, starting from smallest digit
     while bucket is non-empty
       restore element to list
```

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LSD Intuition: sort on most-significant digit **last**; if tied, yield to the next most significant digit, and so on.
Only works because **stability** preserves orderings from less significant digits (previously sorted).

Exercise: Radix sort this

[7, 19, 21, 11, 14, 54, 1, 8]

Hint: [07, 19, 21, 11, 14, 54, 01, 08]

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Exercise: Radix sort this

[07, 19, 61, 11, 14, 54, 01, 08]

n^2

$(n \cdot k) d$

Buckets on 1's place:

0	1	2	3	4	5	6	7	8	9
	01 11 61			54 14			07	08	19

Sorted on 1's place: 61 11 01 14 54 07 08 19

Buckets on 10's place:

0	1	2	3	4	5	6	7	8	9
08 07 01	19 14 11				54	61			

Sorted on 10's place: 01 07 08 11 14 19 54 61

LSD Radix Sort
using counting sort

```
/** least significant digit radix sort A */
LSDRadixSort(A):
max_digits = max # digits in any element of A
for d in 0..max_digits:
   counting sort A on the dth least
   significant digit

// A is now sorted(!)
```

Counting Sort

Formalizes what you did with the 0-9 sticky notes:

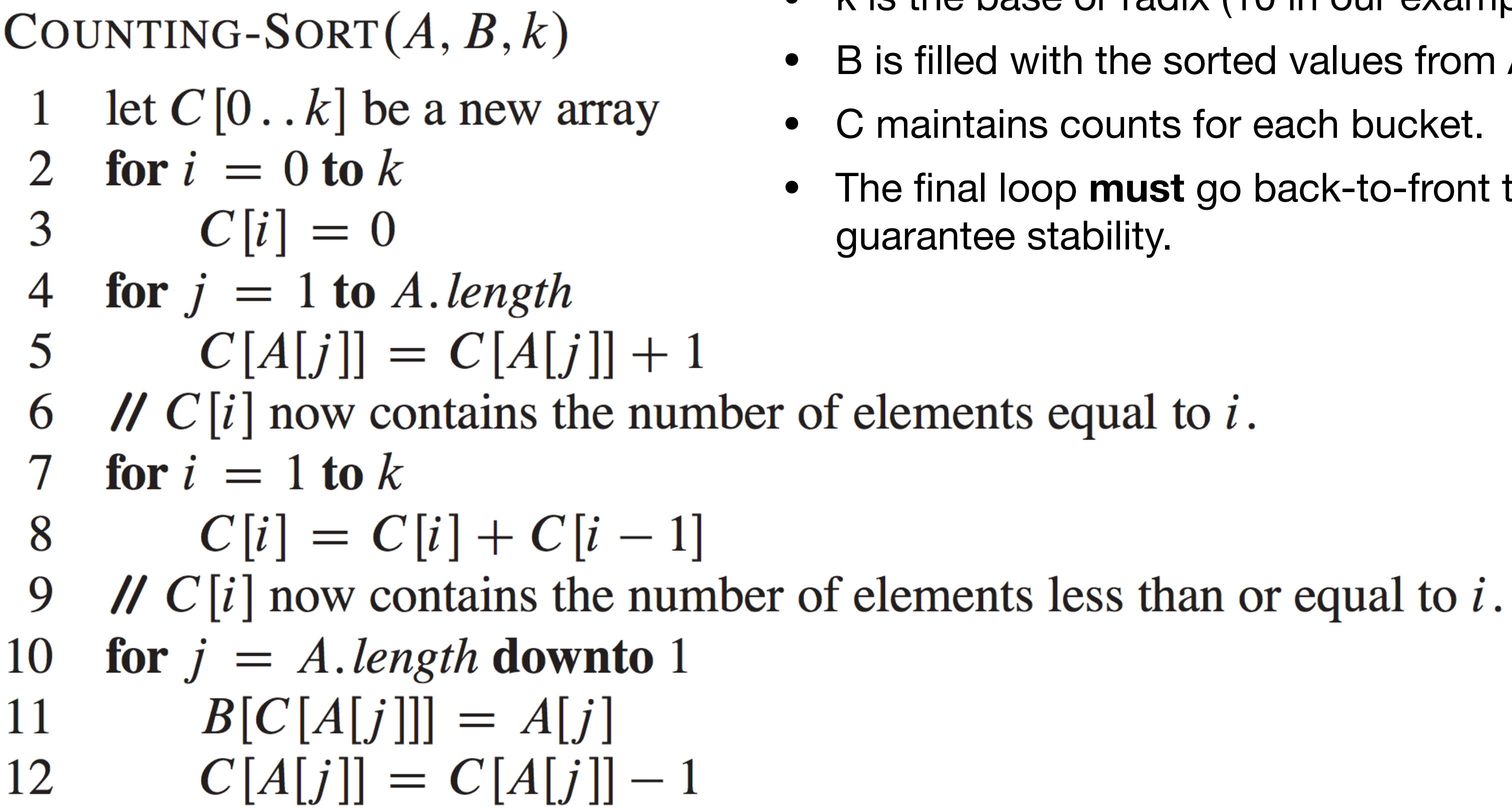
- Handles duplicates
- Stable sort
- Less memory overhead than queue buckets

Intuition:

http://www.cs.miami.edu/home/burt/learning/Csc517.091/workbook/countingsort.html

Pseudocode in CLRS (reproduced on the next slide).

Counting Sort - from CLRS



```
COUNTING-SORT$(A, B, k)$
let $C[0..k]$ be a new array
for $i = 0$ to $k$
    $C[i] = 0$
for $j = 1$ to $A.length$
    $C[A[j]] = C[A[j]] + 1$
// $C[i]$ now contains the number of elements equal to $i$.
for $i = 1$ to $k$
    $C[i] = C[i] + C[i-1]$
// $C[i]$ now contains the number of elements less than or equal to $i$.
for $j = A.length$ downto 1
    $B[C[A[j]]] = A[j]$
    $C[A[j]] = C[A[j]] - 1$
```

Notes:

- k is the base or radix (10 in our examples)
- B is filled with the sorted values from A.
- C maintains counts for each bucket.
- The final loop **must** go back-to-front to guarantee stability.

in-place sorts

One more property of sorting algorithms aside from runtime.

A sorting algorithm is considered in-place if:

the extra storage used doesn't depend on the size of the input.

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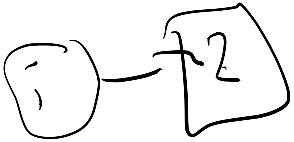
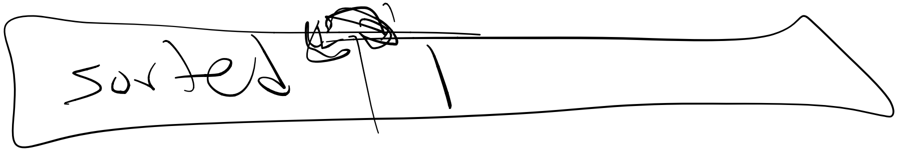
↑ i.e., not part of the input array

↑ i.e., the array being sorted





	in-place?
insertion	Y
selection	Y
merge	?
quick	?
radix	?