CSCI 241

Lecture 3:
Insertion and Selection Sort
Intro to Runtime Analysis
Recursion

Announcements

- First programming assignment out Sunday.
  - We'll cover all the sorting algorithms you need by next Wednesday.
- Lab 2 also out Sunday
  - Done in the same repository as A1 - writing test code
- Norms

Quiz 0

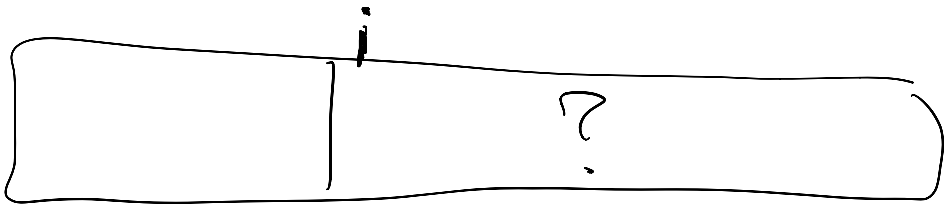
- Quiz 0 is today. Covers only review material.
    - Will be scored but grading is based only on completion.
    - Taken on gradescope.com between 10am to 10pm today.
    - 15 minute time limit
    - This is your trial run: make sure you can login and take the quiz, etc. Later quizzes will count towards your grade.

Goals

- Be able to execute insertion sort and selection sort on paper.
- Be able to implement insertion sort and selection sort.
- Know how to count primitive operations to determine the runtime of an algorithm.
- Understand how recursive methods are **executed**.

Insertion Sort

Insert A[i] into the sorted sublist A[0..i-1].



Selection Sort

Find the smallest element in A[i..n] and place it at A[i].

https://visualgo.net/bn/sorting

Insertion Sort

Insert A[i] into the sorted sublist A[0..i-1].

		i
Invariant: `A`	`sorted`	?

Selection Sort

Find the smallest element in A[i..n] and place it at A[i].

		i
Invariant: `A`	`sorted, <= A[i..n]`	?

https://visualgo.net/bn/sorting

```
insertionSort(A):
  i = 0;
  while i < A.length:
    // push A[i] to its sorted position by repeatedly
    //   swapping with the element to its left
    // increment i
```

		i
Invariant: A	sorted	?

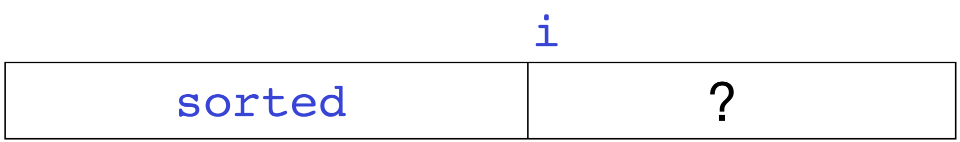
```
selectionSort(A):
  i = 0;
  while i < A.length:
    // find min of A[i..A.length]
    // swap it with A[i]
    // increment i
```

		i
Invariant: A	sorted, <= A[i..n]	?

Insertion sort: Pseudocode

```
// Sorts A using insertion sort
insertionSort(A):
  i = 0;
  while i < A.length:
    j = i;
    while j > 0 and A[j] > A[j-1]:
      swap(A[j], A[j-1])
      j--
    i++
```

Invariant: A



Insertion Sort: Exercise

```
// Sorts A using insertion sort
insertionSort(A):
  i = 0;
  while i < A.length:
    j = i;
    while j > 0 and A[j] < A[j-1]:
      swap(A[j], A[j-1])
      j--
    i++
```

Sort the following array using **insertion** sort:

[1 4 8 2 6]

How many times did you swap two elements?

A. 3
B. 4
C. 6
D. 8

i

Invariant: A	sorted	?

```
// Sorts A using insertion sort
insertionSort(A):
  i = [redacted];
  while i < A.length:
    j = i;
    while j > 0 and A[j] < A[j-1]:
      swap(A[j], A[j-1])
      j--
    i++
```

[1 4 8 2 6]

i

1 4 8 2 6

1 4 2 8 6 (1)

1 2 4 8 6 (2)

1 2 4 6 8 (3)

Selection Sort: Exercise

```
selectionSort(A):
  i = 0;
  while i < A.length:
    // find min of A[i..A.length]
    // swap it with A[i]
    // increment i
```

Sort the following array using **selection** sort:

[1 4 8 2 6]

How many times did you swap two distinct elements?

A. 2
B. 3
C. 4
D. 5

Invariant: A | sorted, <= A[i..n] | ? |

i

```
selectionSort(A):
  i = 0;
  while i < A.length:
    // find min of A[i..A.length]
    // swap it with A[i]
    // increment i
```

[1 4 8 2 6]

1 4 8 2 6

1 2 8 4 6 (1)

1 2 4 8 6 (2)

1 2 4 6 8 (3)

Practice Problems

1. Write code for Selection Sort

2. Consider the array:

$$[\ 8\ \ 4\ \ 6\ \ 10\ \ 7\ \ 1\ \ 2\ ]$$

Write the state of the array at the conclusion of the loop iteration in which i == 4 (don't forget arrays are 0-indexed!).

InsertionSort:

SelectionSort:

Which sort should we use?

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- Which one takes less time?

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Measuring Runtime

Question: How could we measure how "fast" an algorithm runs?

```java
public int findMax(int[] a) {
  int currentMax = a[0];
  for (int i = 1; i < a.length; i++) {
    if (currentMax < a[i]) {
      currentMax = a[i];
    }
  }
  return currentMax;
}
```

How should we measure runtime?

How about metrics that are **invariant** to:

- Length of the array a?
- How fast your computer is?

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How about metrics that are **invariant** to:

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Approach: count the number of “operations” the computer needs to execute.

- Count it *in terms of* the input size
- “operations” may be faster or slower depending on the hardware

“Primitive” Operations

Things the computer can do in a “fixed” amount of time.

“fixed” - doesn’t depend on the input size (n)

A non-exhaustive list:

- Get or set the value of a variable or array location
- Evaluate a simple expression
- Return from a method

Strategies for counting primitive operations

Easiest case:

1. Identify all primitive operations

2. Identify how many time each one happens

3. Add them all up.

```
alg(A, n):                 set
1 [   sum = 0              set
n [ 1 for i = 1..n:        eval + set
n [     3 sum += A[i]      get
```

$$\begin{array}{r} 1 \\ +\ n \\ +\ 3n \\ \hline 4n+1 \end{array}$$

Strategies for counting primitive operations

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1. Identify all primitive operations
2. Identify how many time each one happens
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```
alg(A, n):
   sum = 0
   for i = 1..n:
      sum += A[i]
```