CSCI 241

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Graphs: Terminology

Goals

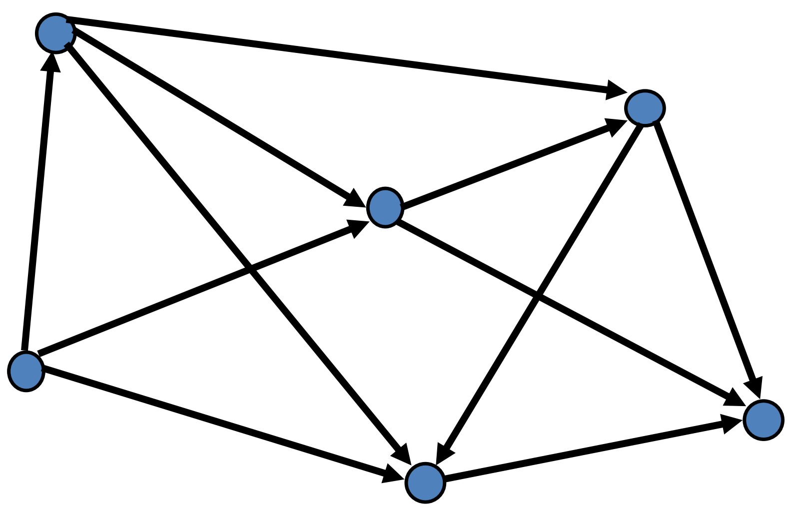
Know the definition of a graph and its basic associated terminology:

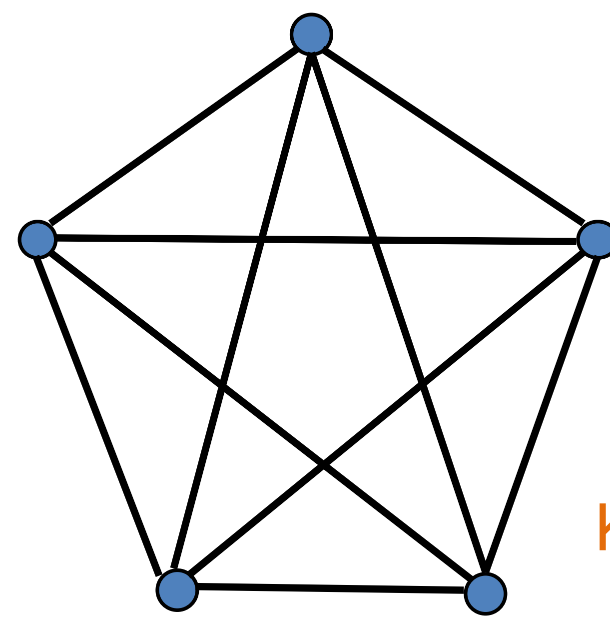
- node/vertex
- edge/arc
- directed, undirected
- adjacent, (in-/out-)degree
- path, cycle

Graphs: The Abstract View

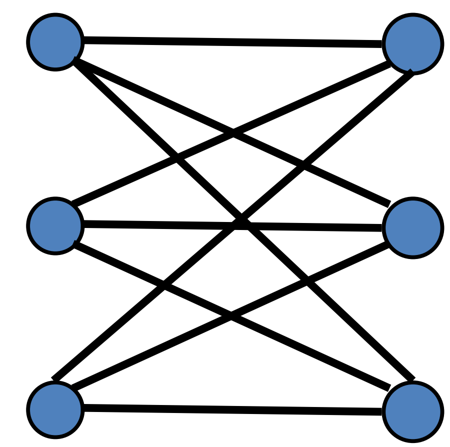
Graph: a bunch of points connected by lines.

The lines may have directions, or not.

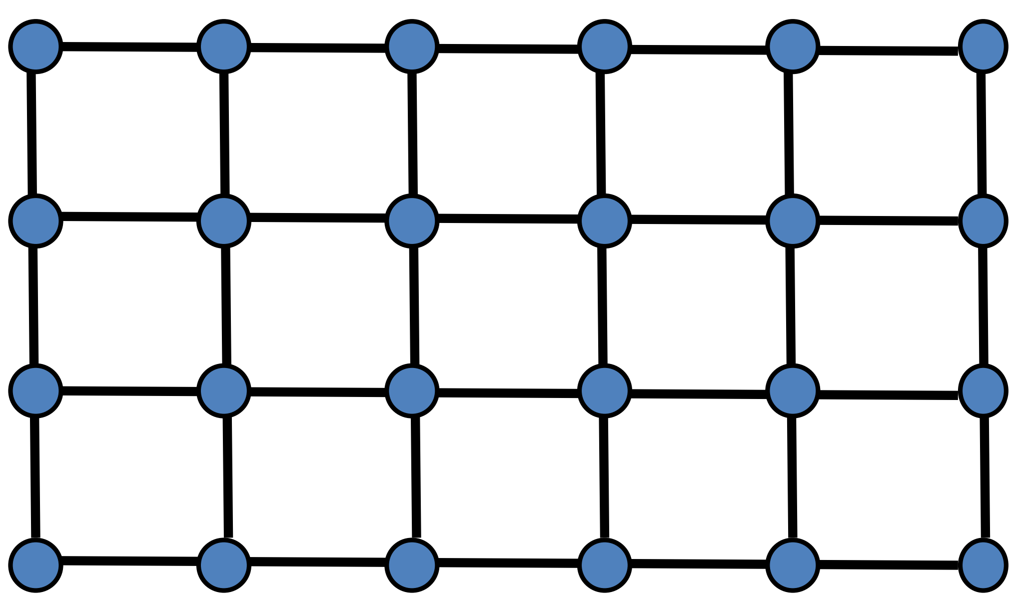


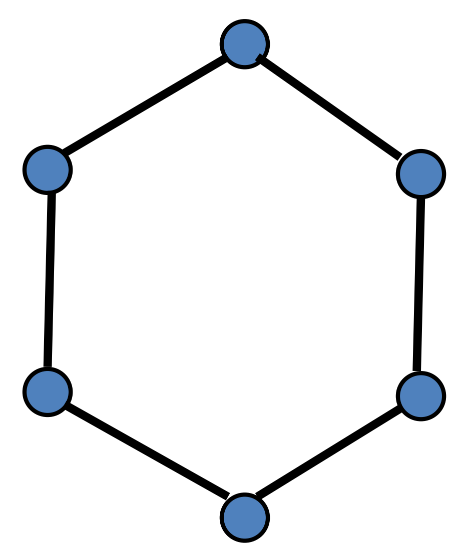


K_5



$K_{3,3}$





Graphs, Formally

A directed graph (digraph) is a pair (**V**, **E**) where:

- **V** is a (finite) set
- **E** is a set of **ordered** pairs (u, v) where u, v are in **V**
- Often (not always): u ≠ v
  (i.e., no edges from a vertex to itself)

An element in **V** is called a vertex or node

Elements in **E** are called edges or arcs

|**V**| = size of **V** (traditionally called n or v)

|**E**| = size of **E** (traditionally called m or e)

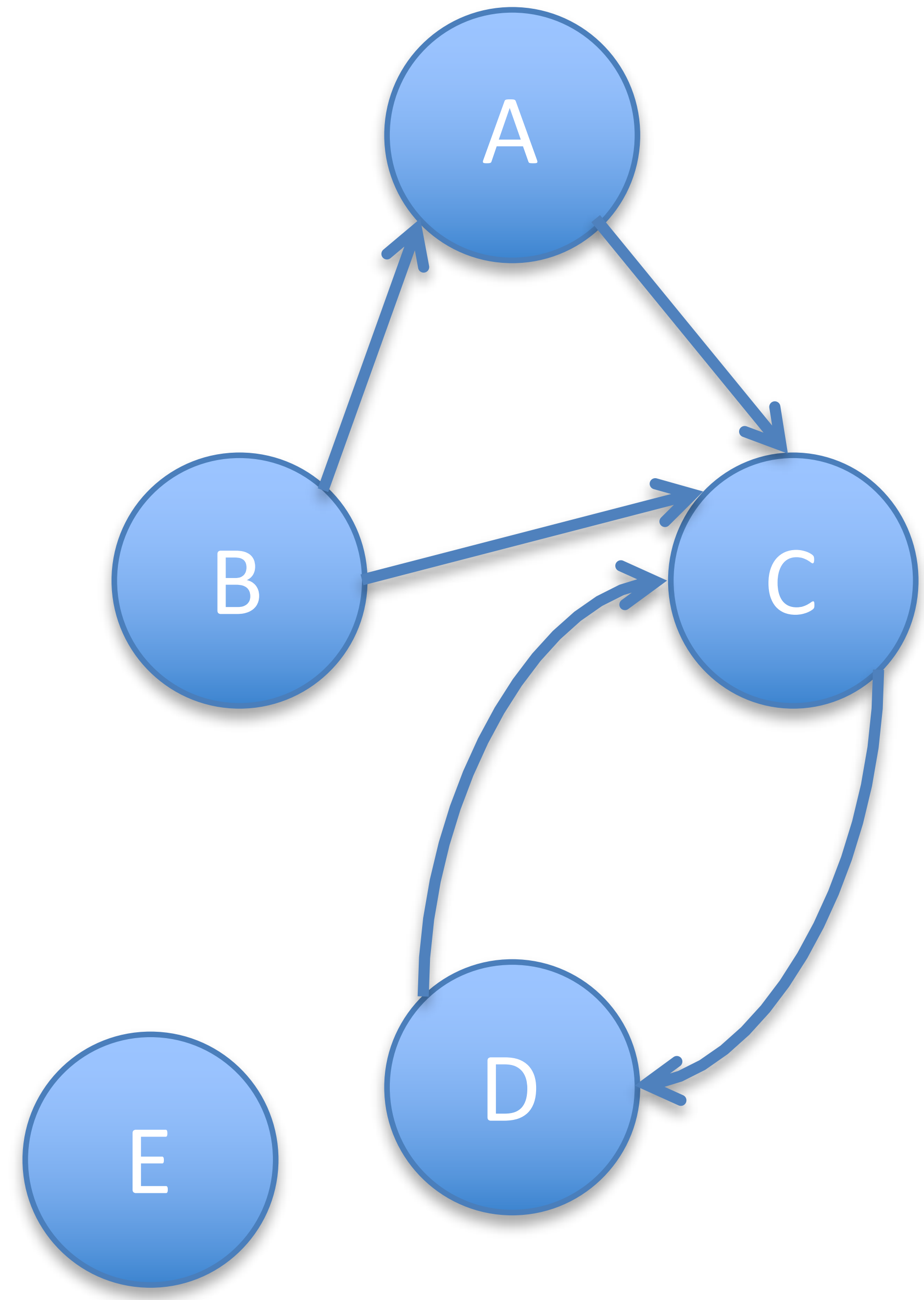
An example directed graph

V = {A, B, C, D, E)

E = {(A, C), (B, A), (B, C), (C, D), (D, C)}

|**V**| = 5

|**E**| = 5



Graphs, Formally

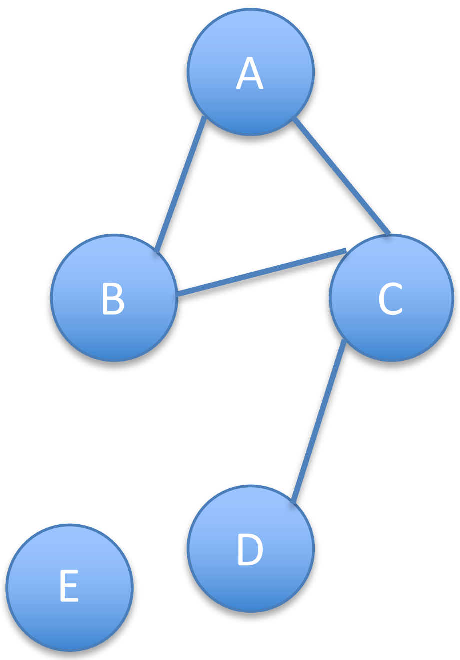
An **un**directed graph is a just like a digraph, but

- The pairs in **E** are **un**ordered

$$\mathbf{V} = \{A, B, C, D, E\}$$

$$\mathbf{E} = \{(A, C), (A, B), (B, C), (C, D)\}$$

$$|\mathbf{V}| = 5 \qquad |\mathbf{E}| = 4$$



Any **un**directed graph has an equivalent **directed** graph:

- Replace each undirected edge with two directed edges

A **directed** graph doesn't always have an equivalent **undirected** graph.

Graph Terminology: Adjacency

- Two vertices are adjacent if they are connected by an edge

  In graph G, B and C are *adjacent*.

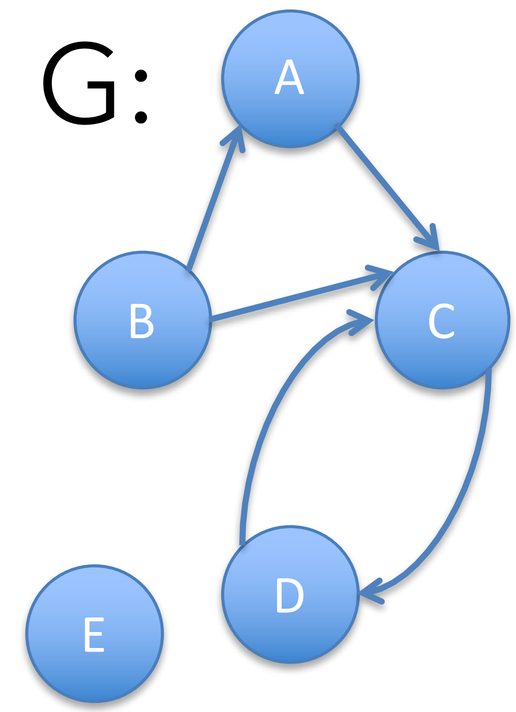
- Nodes u and v are called the source and sink of the **directed** edge (u, v)

  In graph G, B is the *source* and C is the *sink* on the edge from B to C.

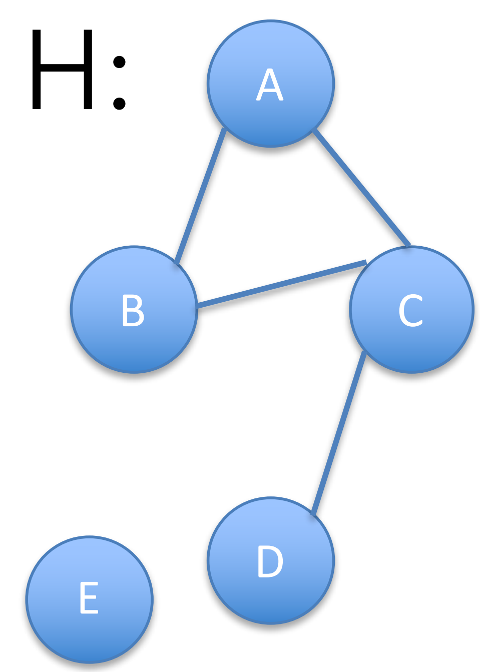
- Nodes u and v are endpoints of an edge (u, v) (directed or undirected)

  In graph H, C and D are *endpoints* of the edge between C and D.

G:



H:



Graph Terminology: Degree

- The outdegree of a vertex *u* in a **directed** graph is the number of edges for which *u* is the source

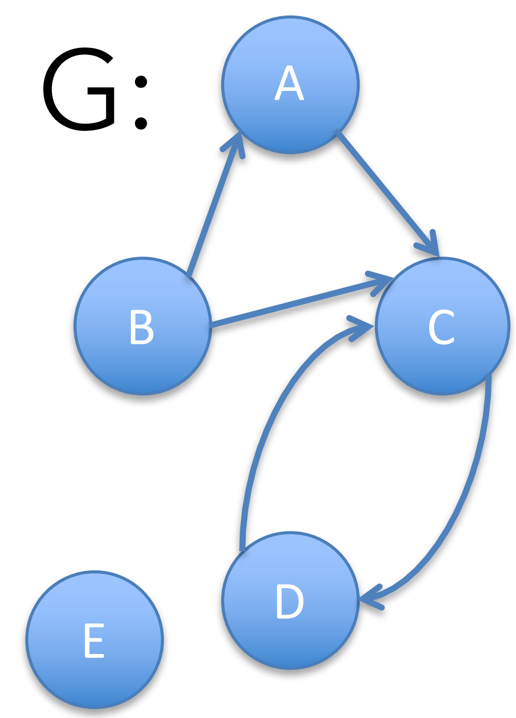
  In graph G, B has *outdegree* 2.

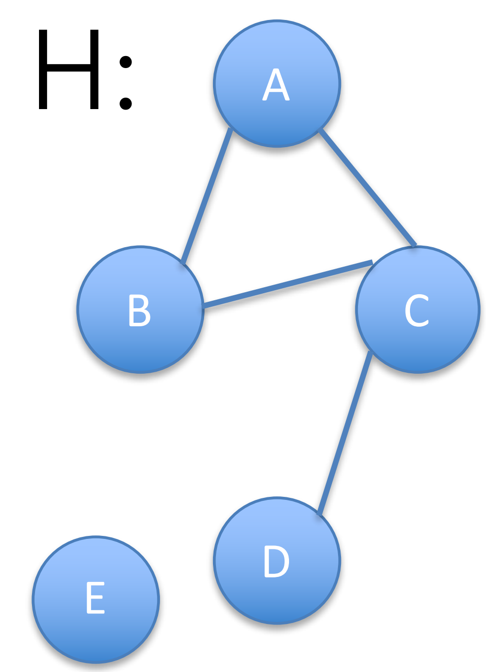
- The indegree of a vertex *v* in a **directed** graph is the number of edges for which *v* is the sink

  In graph G, B has *indegree* 0.

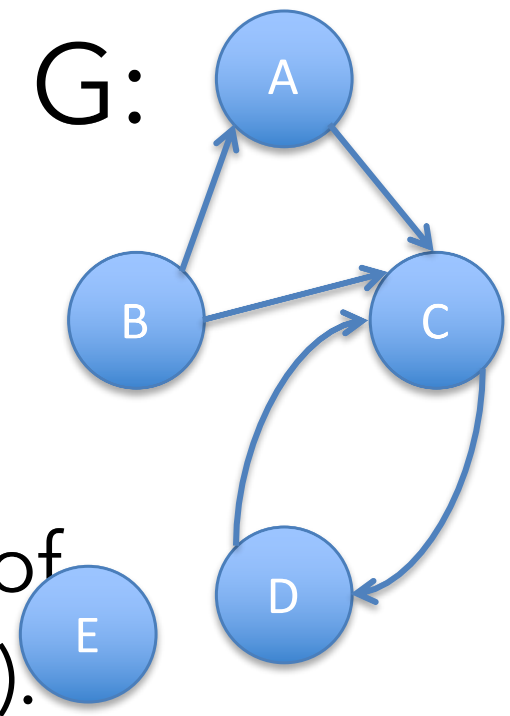
- The degree of a vertex *u* in an **undirected** graph is the number of edges of which *u* is an endpoint

  In graph H, A has *degree* 2.





Graph Terminology: Paths, Cycles

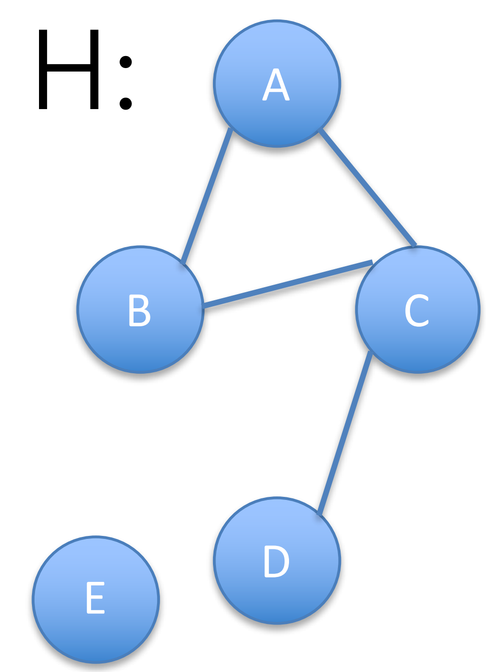


A path is a sequence of vertices in which each consecutive pair are adjacent.

A, C, D is a path in graph G.

In a directed graph, paths must follow the direction of the edges (nodes must be ordered source then sink).

A, B, C is not a path in graph G



A cycle is a path that ends where it started

A, B, C is a cycle in graph H

A graph is acyclic if it has no cycles.

Graph G is acyclic; Graph H is not.

Graph Terminology: Connectedness

A subgraph of a graph G is a graph whose node and edge sets are subsets of G's node and edge sets.

G' = V: {C, D}; E: {(C, D), (D, C)} is a subgraph of G.

An *undirected* graph is connected if there is a path between every pair of nodes in the graph.

H is not connected, but the subgraph excluding E is.

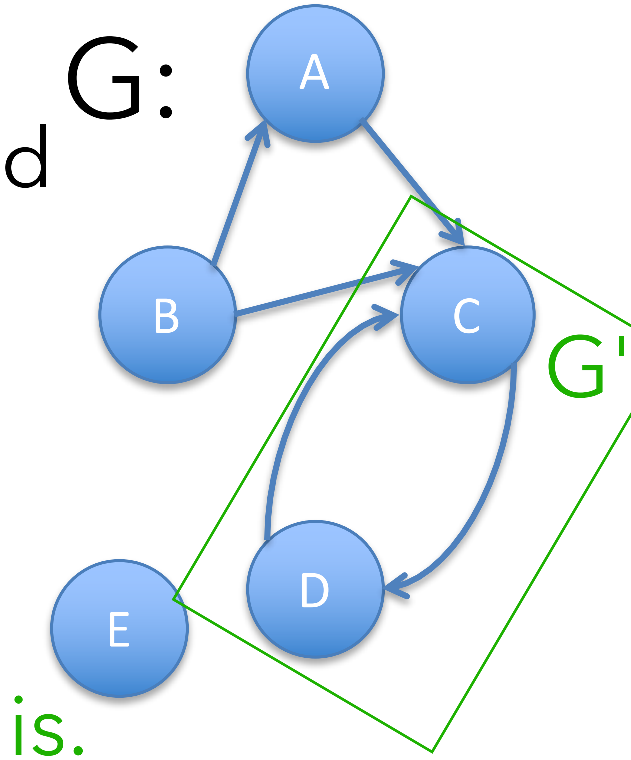
A *directed* graph is strongly connected if there is a path between every pair of nodes in the graph.

G is not strongly connected, but G' is

A *directed* graph is weakly connected if the graph would be connected if its edges were undirected.

The subgraph of G containing A, B, C is weakly connected.

G:



H: