CSCI 241

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Comparison-based sorting
Radix Sort

Goals

Know the meaning of a comparison sort.

Be able to execute LSD radix sort on paper.

Be prepared to implement LSD radix sort using bucket sort in the inner loop.

Comparison Sorts

(or "comparison-based sorting algorithms")

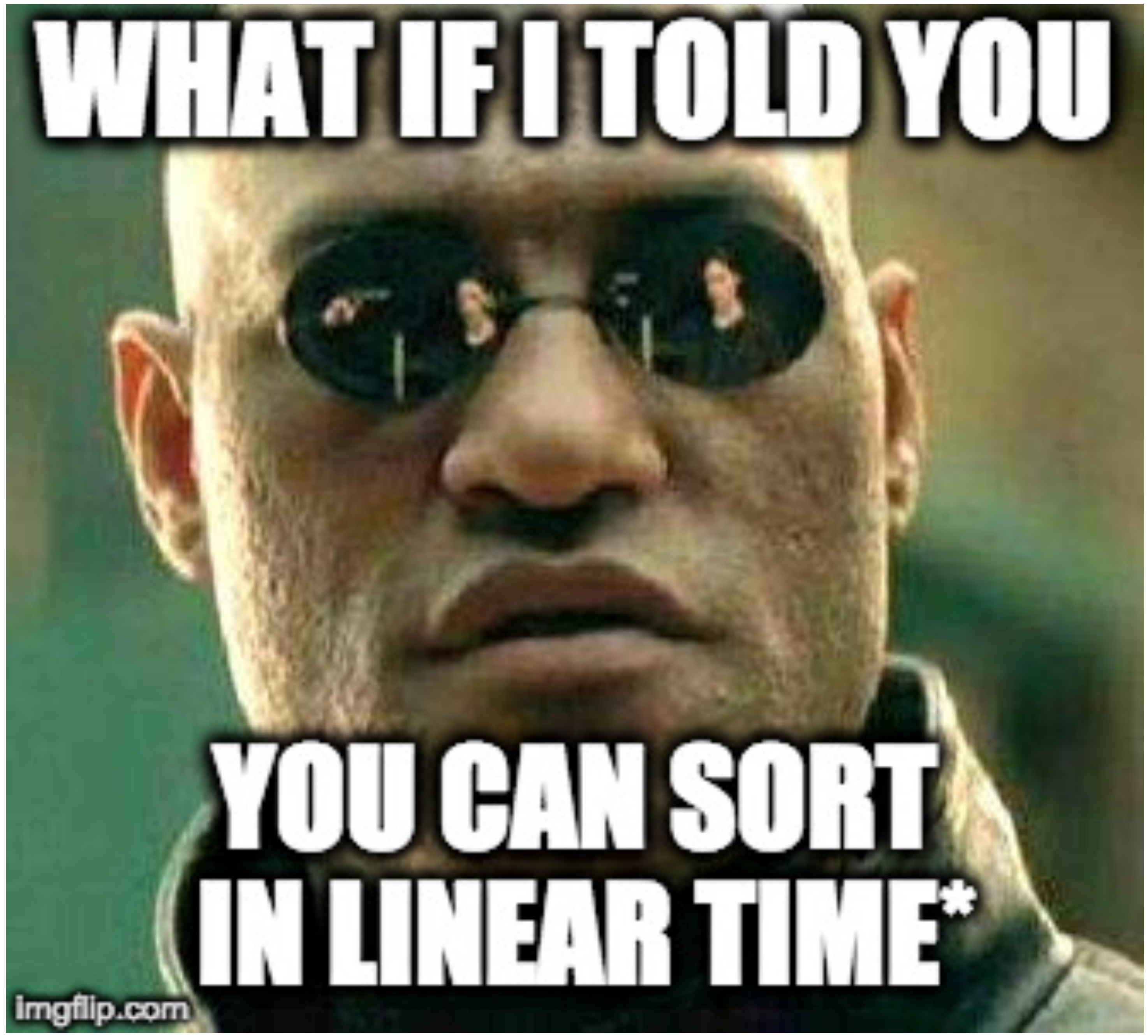
A comparison sort sorts values by comparing pairs of elements.

For example: **all** the sorts we've covered so far!

Fact:

- O(n log n) is the *best possible* worst-case runtime for a comparison-based sorting algorithm.
- It's *mathematically* impossible to do better!

...but is there any other way to do it?



*if your values have a **constant** (O(1)) number of digits

LSD Radix Sort

(Least-Significant-Digit)

```
/** least significant digit radix sort A */
LSDRadixSort(A):
  max_digits = max # digits in any element of A
  for d in 0..max_digits:
    stably sort A on the dth least significant
    digit

// A is now sorted(!)
```

ones place, then
tens place, then
hundreds place,
and so on

Does this work?

```
/** least significant digit radix sort A */
LSDRadixSort(A):
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  for d in 0..max_digits:
    stably sort A on the dth least significant
    digit

// A is now sorted(!)
```

	[45, 26, 42, 32]
Sorted on ones:	[42, 32, 45, 26]
Sorted on tens:	[26, 32, 42, 45]

Why does this work?

```
/** least significant digit radix sort A */
LSDRadixSort(A):
  max_digits = max # digits in any element of A
  for d in 0..max_digits:
    stably sort A on the dth least significant
    digit

// A is now sorted(!)
```

Intuition: if we're sorting 3-digit numbers,

- sort on 100's place **last**
- 100's-place ties yield to the already-sorted 10's place
- Works because **stability** preserves orderings from (already sorted) less significant digits in case of ties.

That's well and good, but...

```
/** least significant digit radix sort A */
LSDRadixSort(A):
  max_digits = max # digits in any element of A
  for d in 0..max_digits:
    stably sort A on the dth least significant
    digit
```

...how do we do this part?

```
// A is now sorted(!)
```

Comparison sorts are O(n log n) at best.

To sort in O(n), we need something better...

How do you sort things without comparing them?

Suppose I asked you to sort 10 sticky notes with the digits 0 through 9.

What algorithm would you use?

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What algorithm would you use?

What algorithm would minimize the number of times you look at each sticky note?

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What algorithm would minimize the number of times you look at each sticky note?

What if there are duplicates?

Example: Radix sort this

`[ 7, 19, 61, 11, 14, 54,  1, 08]`

Buckets
on 1’s place:

0	1	2	3	4	5	6	7	8	9

Sorted on
1’s place:

Buckets
on 10’s place:

0	1	2	3	4	5	6	7	8	9

Sorted on
10’s place:

Example: Radix sort this

`[07, 19, 61, 11, 14, 54, 01, 08]`

Buckets
on 1's place:

0	1	2	3	4	5	6	7	8	9

Sorted on
1's place:

Buckets
on 10's place:

0	1	2	3	4	5	6	7	8	9

Sorted on
10's place:

Example: Radix sort this

`[07, 19, 61, 11, 14, 54, 01, 08]`

Buckets on 1's place:

0	1	2	3	4	5	6	7	8	9
	01								
	11			54					
	61			14			07	08	19

Sorted on 1's place: 61 11 01 14 54 07 08 19

Buckets on 10's place:

0	1	2	3	4	5	6	7	8	9
08	19								
07	14								
01	11				54	61			

Sorted on 10's place: 01 07 08 11 14 19 54 61

Try it out yourself: https://visualgo.net/en/sorting

Radix sort using bucket queues

```
Pseudocode adapted from visualgo.net:

LSDRadixSort(A):
 create a bucket (queue) for each digit (0 to 9)
 for each digit (least- to most-significant):
   for each element in A:
     enqueue element into its bucket based on digit
   for each bucket, starting from smallest digit
     while bucket is non-empty
       dequeue element into list
```

Counting Sort

Bucket sort is not in-place: requires O(n) storage

Counting sort is an in-place alternative: requires only O(d) extra storage.

Intuition:

http://www.cs.miami.edu/home/burt/learning/Csc517.091/workbook/countingsort.html

Pseudocode in CLRS (reproduced on the next slide).

Counting Sort - from CLRS

Notes:

- k is the base or radix (10 in our examples)
- B is filled with the sorted values from A.
- C maintains counts for each bucket.
- The final loop **must** go back-to-front to guarantee stability.

```
COUNTING-SORT(A, B, k)
let C[0..k] be a new array
for i = 0 to k
    C[i] = 0
for j = 1 to A.length
    C[A[j]] = C[A[j]] + 1
// C[i] now contains the number of elements equal to i.
for i = 1 to k
    C[i] = C[i] + C[i - 1]
// C[i] now contains the number of elements less than or equal to i.
for j = A.length downto 1
    B[C[A[j]]] = A[j]
    C[A[j]] = C[A[j]] - 1
```