Graph Traversals

CSCI 241 Winter 2019

Lecture 19

Look, a graph!

Depth-First Search

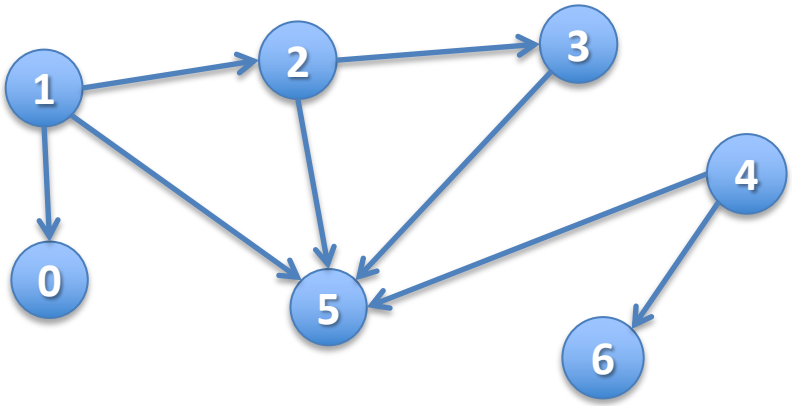
```
/** Visit all nodes that are explorable
from u. Precondition: u is unvisited. */
public static void dfs(int u) {

}
```

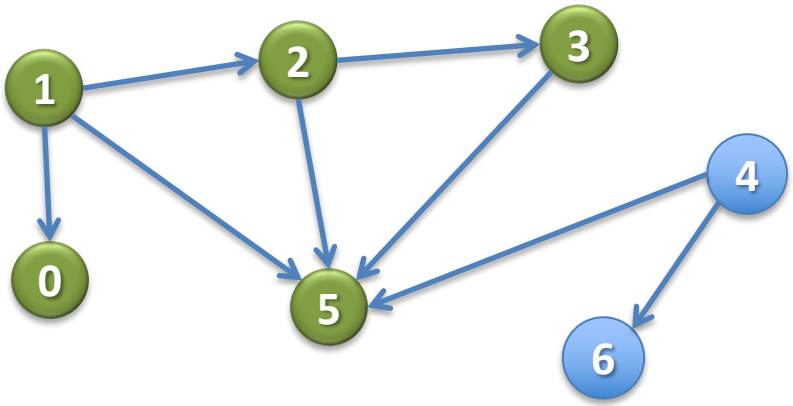
Let u be **1**

The nodes **explorable** from 1 are 1, 0, 2, 3, 5

Start



End



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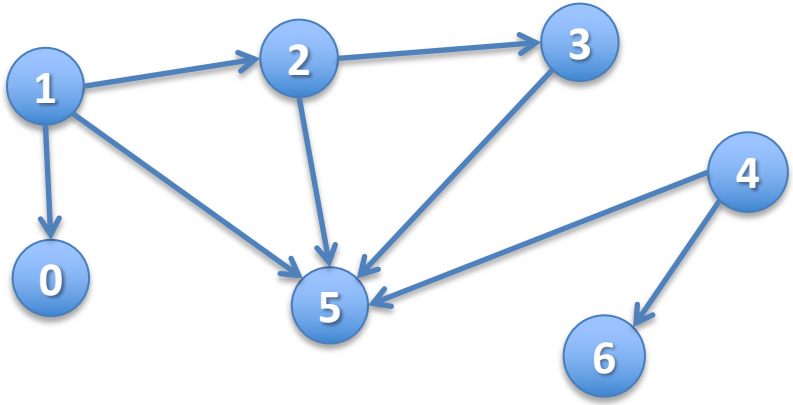

Depth-First Search

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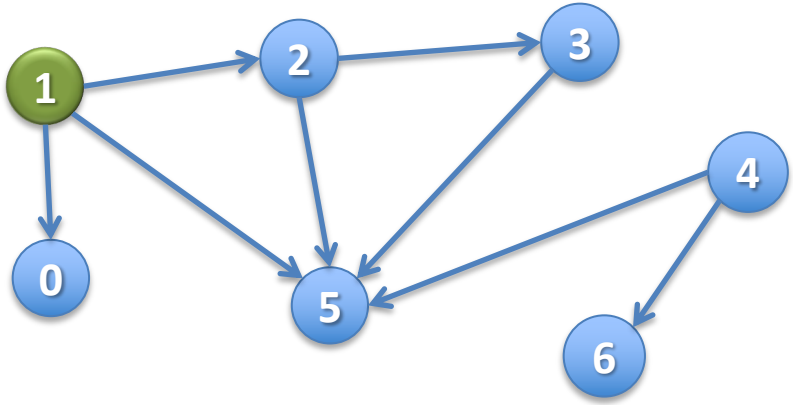

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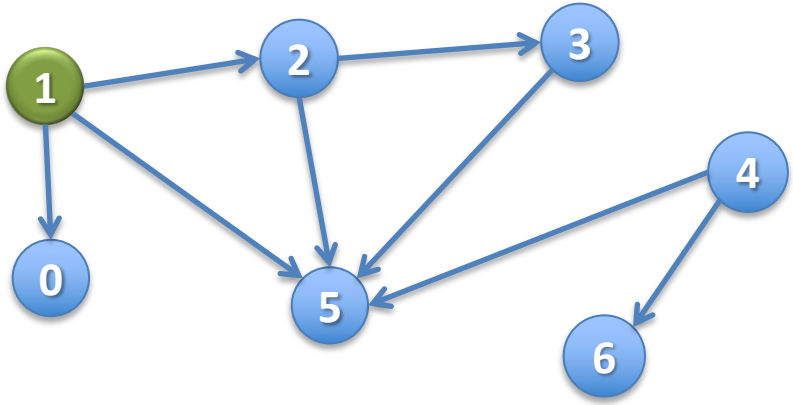
Let u be **1** (visited)

The nodes to be visited are 0, 2, 3, 5



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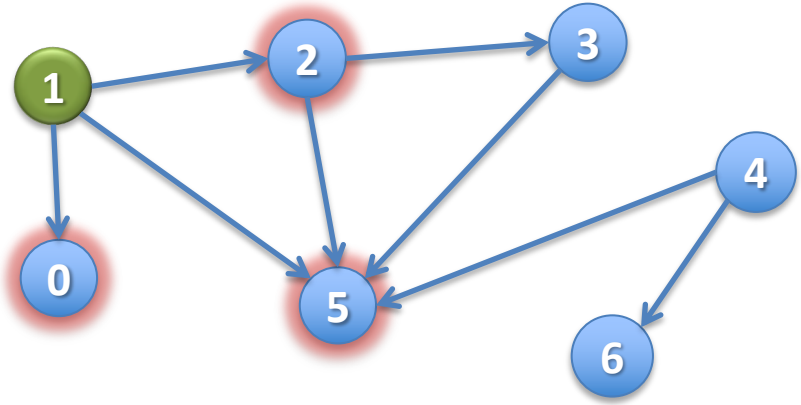
Let u be **1** (visited)

The nodes to be visited are 0, 2, 3, 5

Have to do DFS on all unvisited neighbors of u!



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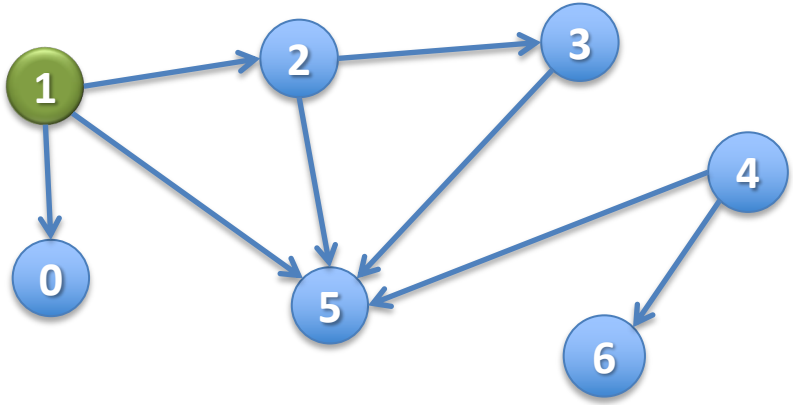
Suppose the **for** loop visits neighbors in numerical order. Then **dfs(1)** visits the nodes in this order: 1 …



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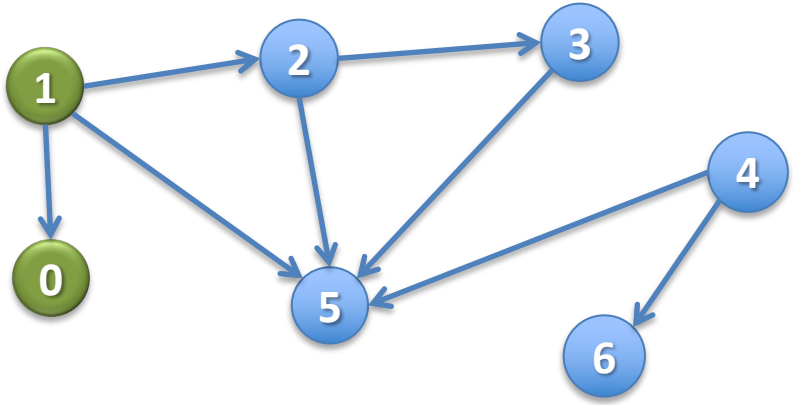

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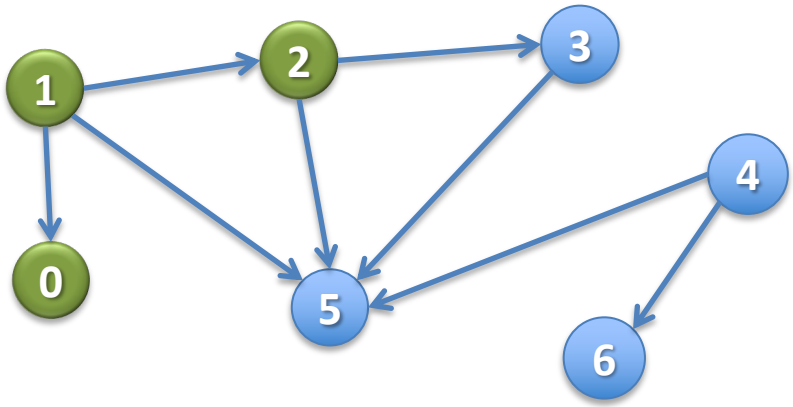

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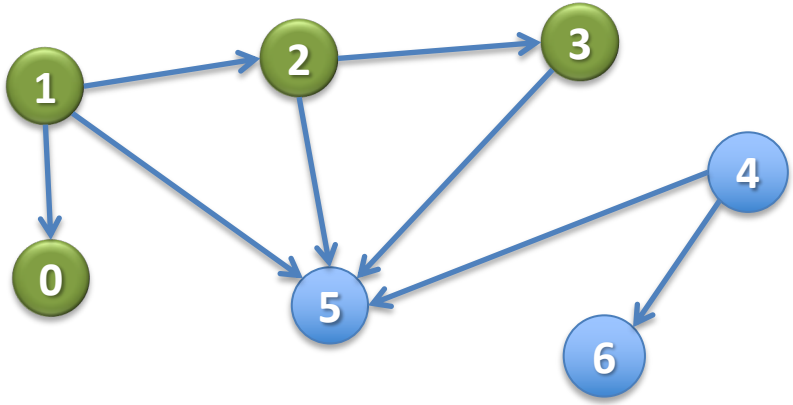

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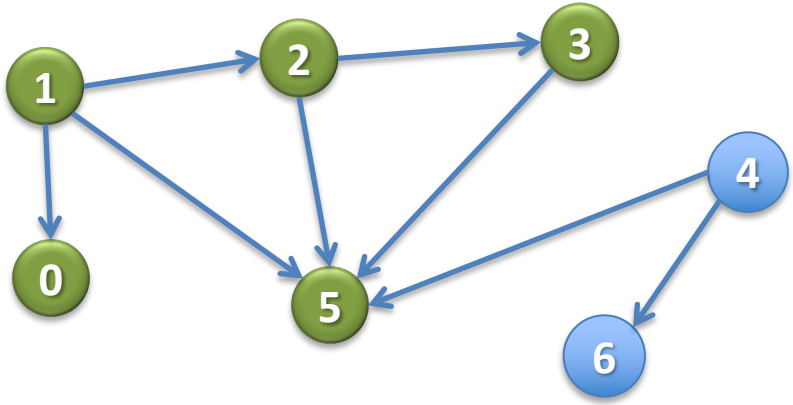

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Suppose n nodes are explorable along e edges (in total). What is

- Worst-case runtime?
- Worst-case space?

Depth-First Search

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```

Suppose n nodes are explorable along e edges (in total). What is

- Worst-case runtime? O(n+e)
- Worst-case space? O(n)

Depth-First Search

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public static void dfs(int u) {
    visited[u] = true;
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```

That's all there is to basic DFS. You may have to change it to fit a particular situation.

If you don't have this spec and you do something different, it's probably wrong.

Example: Use different way (other than array visited) to know whether a node has been visited

Example: We really haven't said what data structures are used to implement the graph

Depth-First Search in OO fashion

```
public class Node {
    boolean visited;
    List<Node> neighbors;

    /** Visit all nodes that are explorable
     * from u. Precondition: u is unvisited */
    public void dfs() {
        visited= true;
        for (Node n: neighbors) {
            if (!n.visited) n.dfs();
        }
    }
}
```

Each node of the graph is an object of type Node

No need for a parameter. The object is the node.

Depth-First Search written iteratively

```
/** Visit all nodes explorable from u. Pre: u is unvisited. */
public static void dfs(int nodeID) {
    Stack s = (nodeID);    // Not Java!
    // inv: all nodes that have to be visited are
    //        explorable from some node in s
    while ( s is not empty ) {
        u = s.pop();    // Remove top stack node, put in u
        if (u has not been visited) {
            visit u;
            for each edge (u, v) leaving u:
                s.push(v);
        }
    }
}
```

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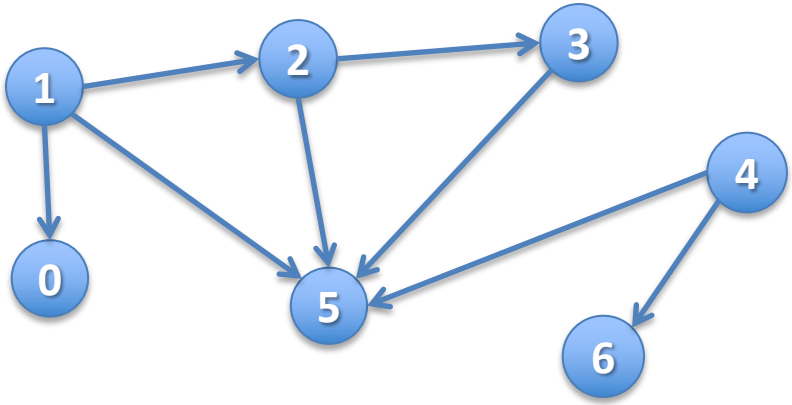
Call dfs(1)



1

Stack s

Depth-First Search written iteratively

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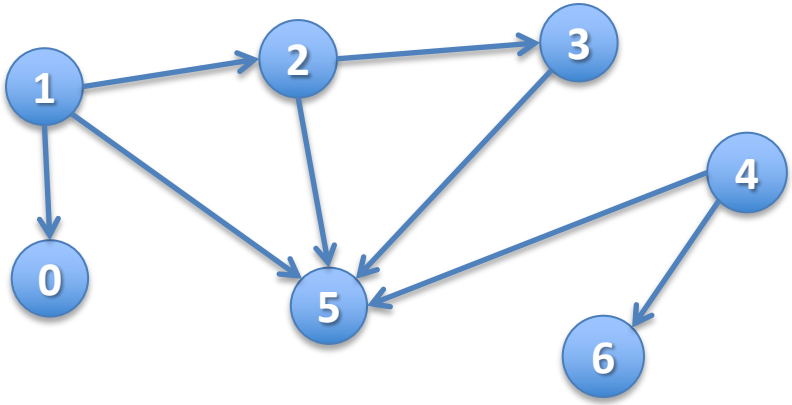
Iteration 0



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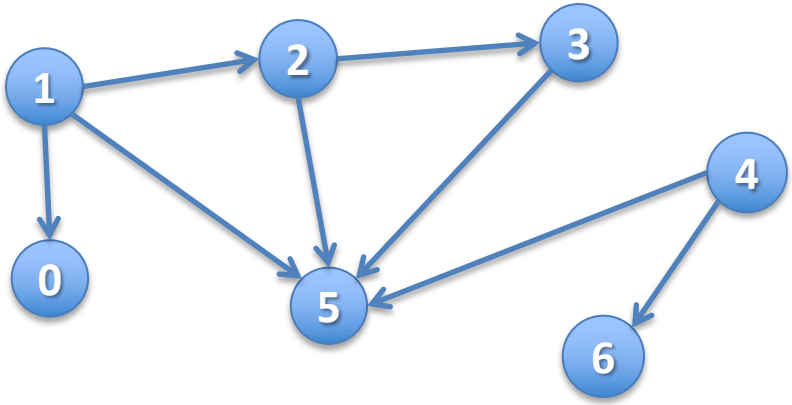

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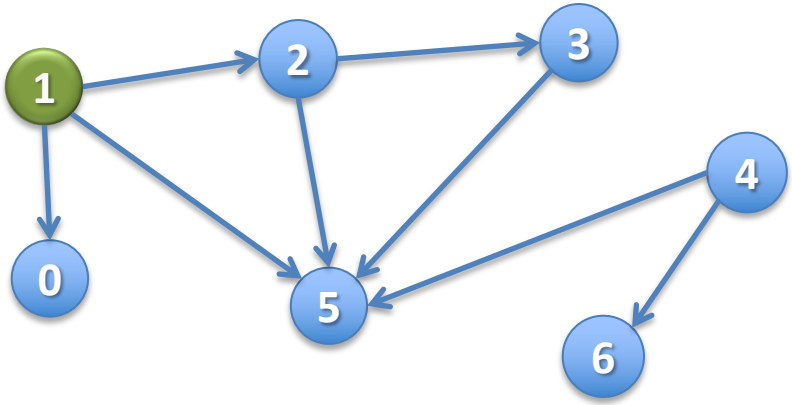

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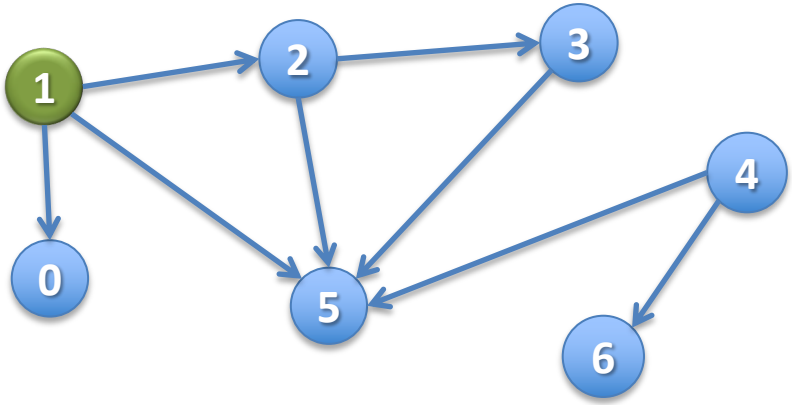

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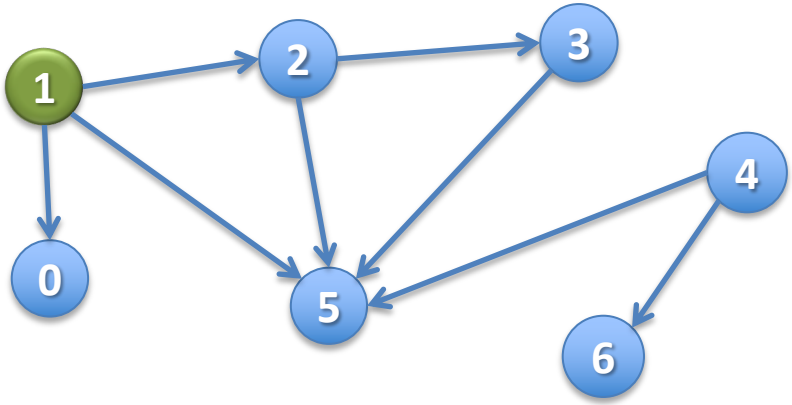

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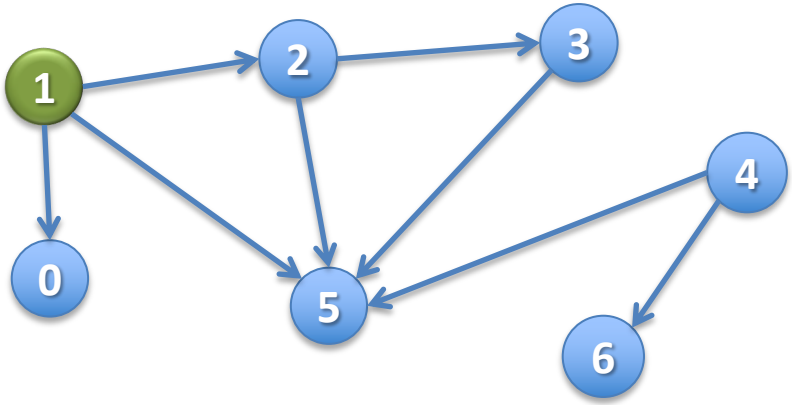

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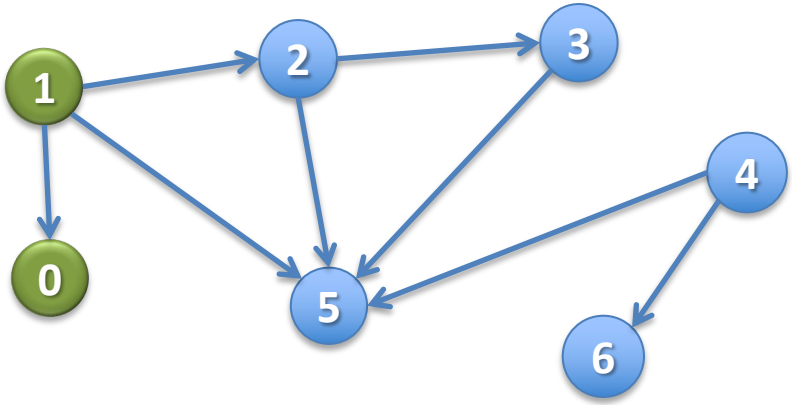

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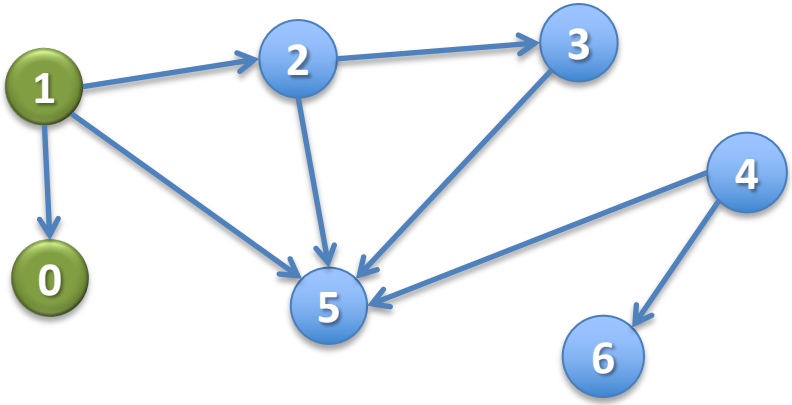
Iteration 2



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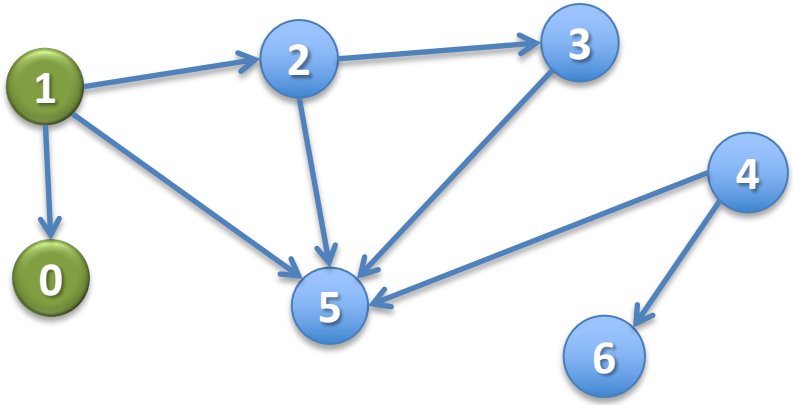

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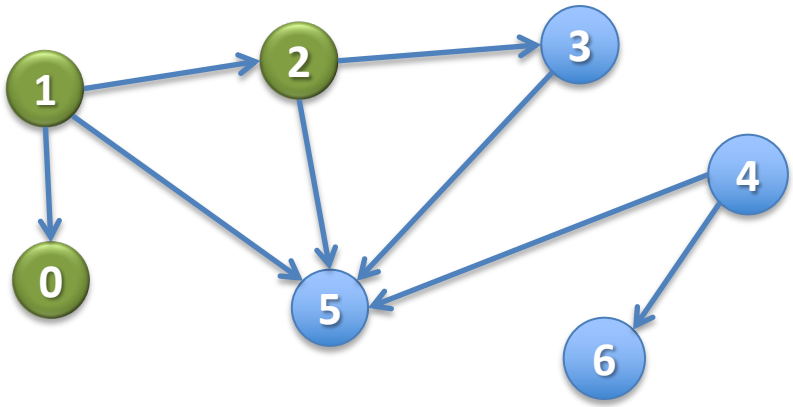

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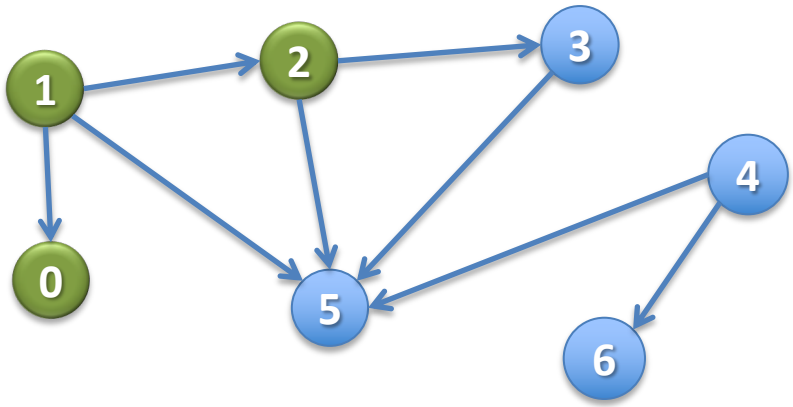
Call dfs(1)

Iteration 2

Yes, 5 is put on the stack twice, once for each edge to it. It will be visited only once.



3
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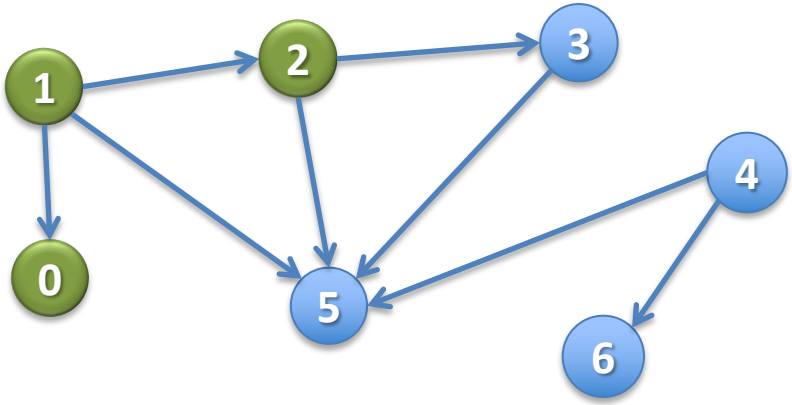

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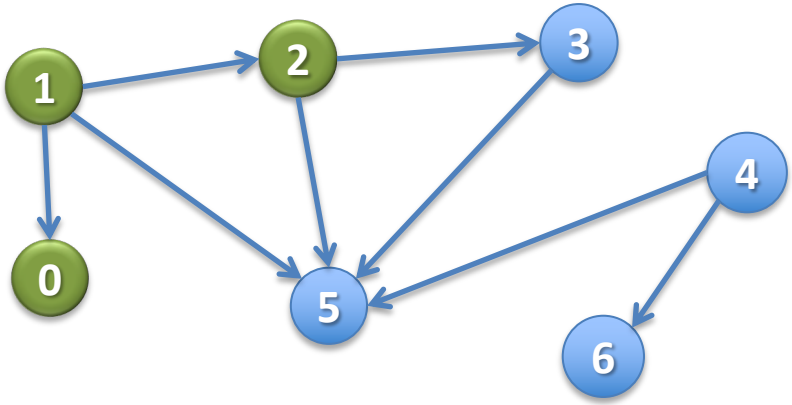

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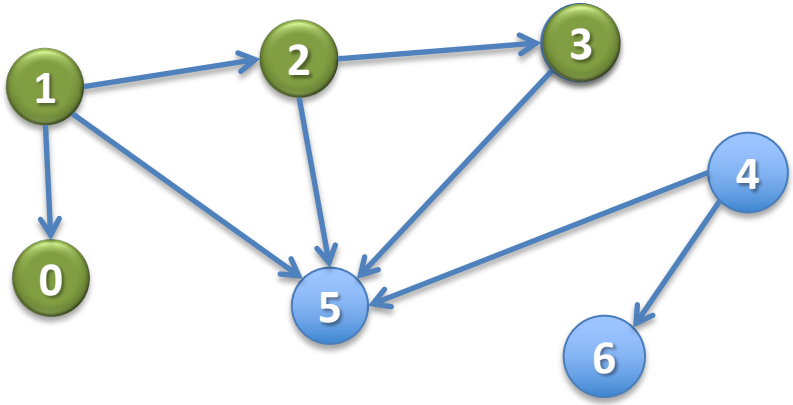
Iteration 3



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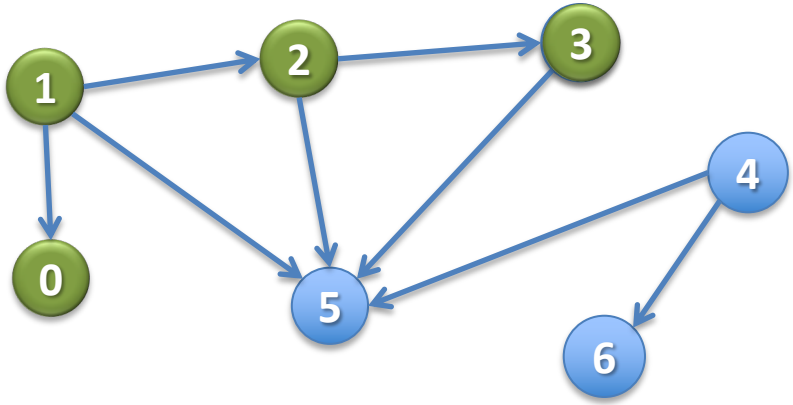

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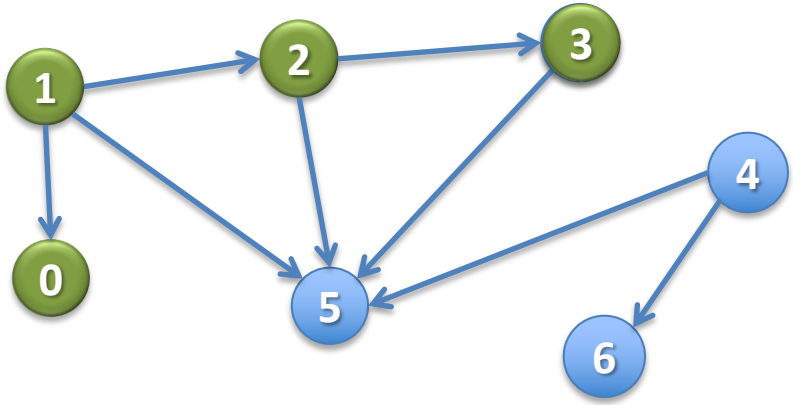
Call dfs(1)

Iteration 4



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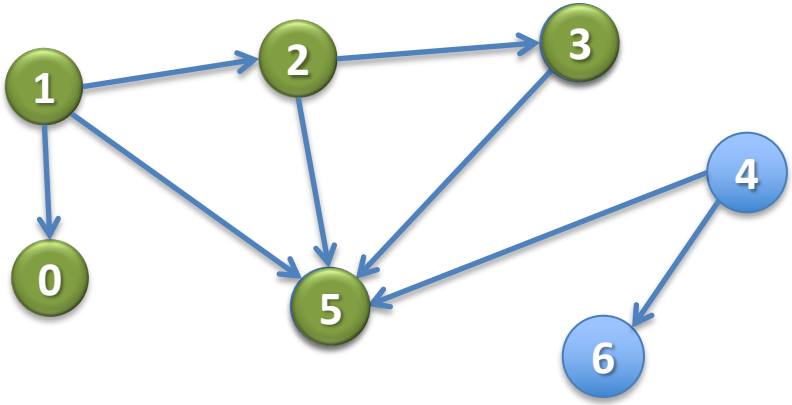
```

Call dfs(1)

Iteration 4



5

5

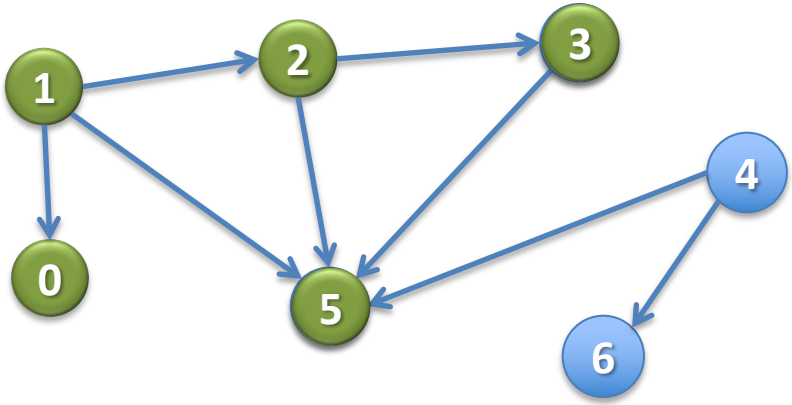
Stack s

Depth-First Search written iteratively

```
/** Visit all nodes explorable from u. Pre: u is unvisited. */
public static void dfs(int u) {
    Stack s= (u);
    while (s is not empty) {
        u= s.pop();
        if (u has not been visited) {
            visit u;
            for each edge (u, v) leaving u:
                s.push(v);
        }
    }
}
```

Call dfs(1)

Iteration 4



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            for each edge (u, v) leaving u:
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        }
    }
}
```
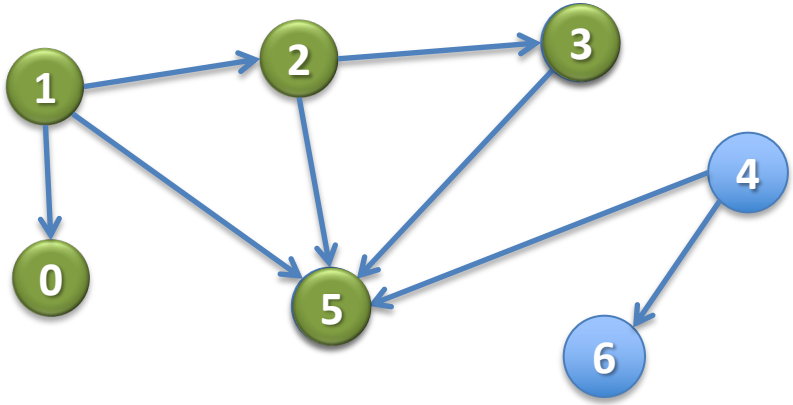

Call dfs(1)

Iteration 5



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Stack s

Depth-First Search written iteratively

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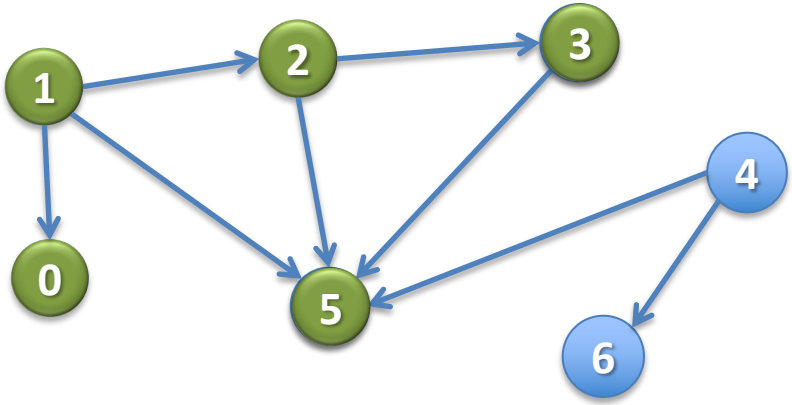
        u= s.pop();
        if (u has not been visited) {
            visit u;
            for each edge (u, v) leaving u:
                s.push(v);
        }
    }
}
```

Call dfs(1)

Iteration 6



Stack s

Depth-First Search written iteratively

```
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public static void dfs(int u) {
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        u= s.pop();
        if (u has not been visited) {
            visit u;
            for each edge (u, v) leaving u:
                s.push(v);
        }
    }
}
```

That's DFS!

```
/** Visit all nodes explorable from u. Pre: u is unvisited. */
public static void dfs(int u) {
    Stack s= (u);    // Not Java!
    // inv: all nodes that have to be visited are
    //      explorable from some node in s
    while ( s is not empty ) {
        u= s.pop();    // Remove top stack node, put in u
        if (u has not been visited) {
            visit u;
            for each edge (u, v) leaving u:
                s.push(v);
        }
    }
}
```

Want to see a magic trick?

Depth-First Search

```
/** Visit all nodes explorable from u. Pre: u is unvisited. */
public static void dfs(int u) {
    Stack s= (u);    // Not Java!
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            visit u;
            for each edge (u, v) leaving u:
                s.push(v);
        }
    }
}
```

Breadth-First Search

```
/** Visit all nodes explorable from u. Pre: u is unvisited. */
public static void bfs(int u) {
    Queue q= (u);    // Not Java!
    // inv: all nodes that have to be visited are
    //        explorable from some node in s
    while ( q is not empty ) {
        u= q.popFirst();    // Remove first node in queue, put in u
        if (u has not been visited) {
            visit u;
            for each edge (u, v) leaving u:
                q.append(v);    // Add to end of queue
        }
    }
}
```

Breadth-First Search

```
/** Visit all nodes explorable from u. Pre: u is unvisited. */
public static void bfs(int u) {
    Queue q= (u);    // Not Java!
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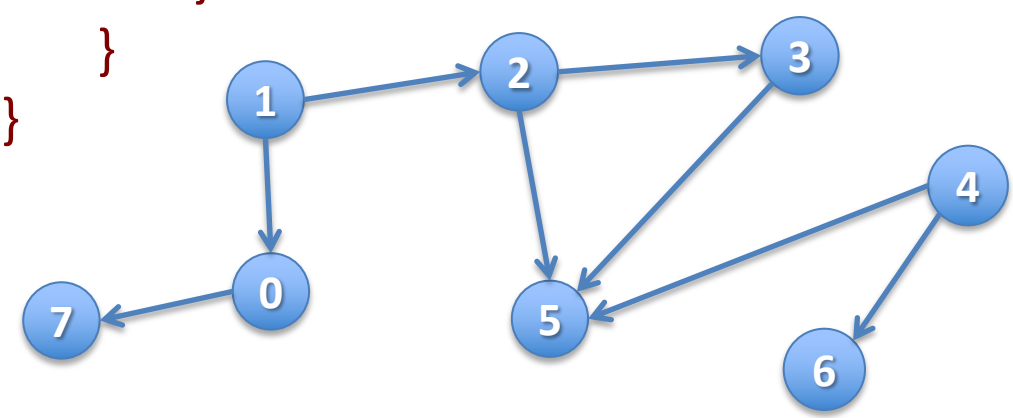
    }
}
```

Call bfs(1)



1

Queue q

Breadth-First Search

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}
```
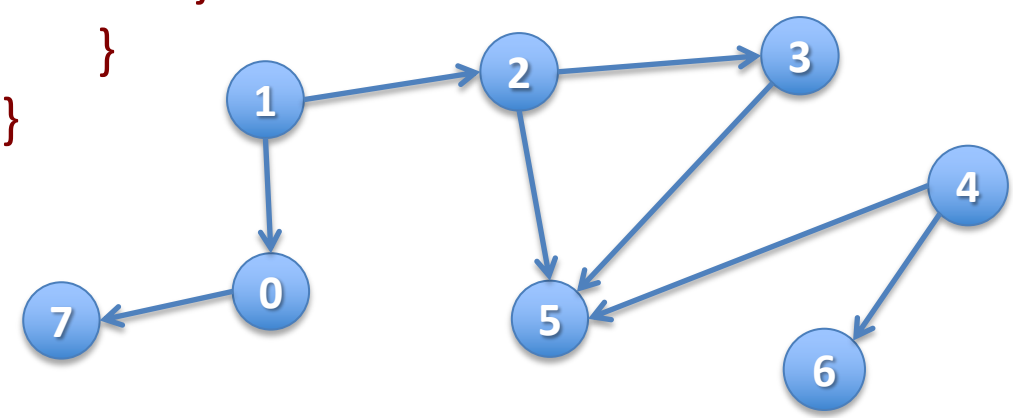

Call bfs(1)

Iteration 0



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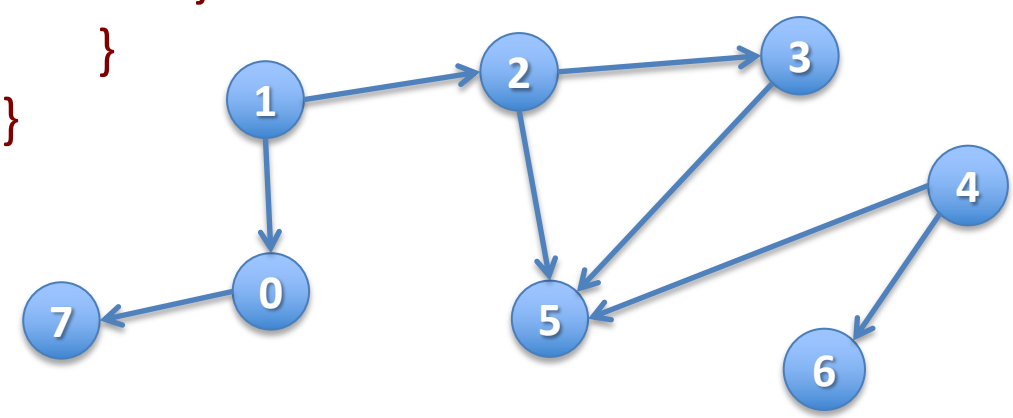
Call bfs(1)

Iteration 0



Queue q

Breadth-First Search

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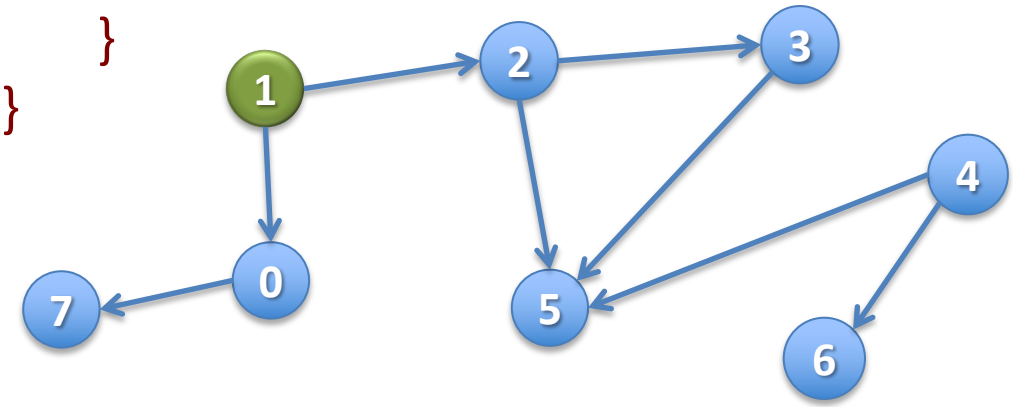
Call bfs(1)

Iteration 0



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Breadth-First Search

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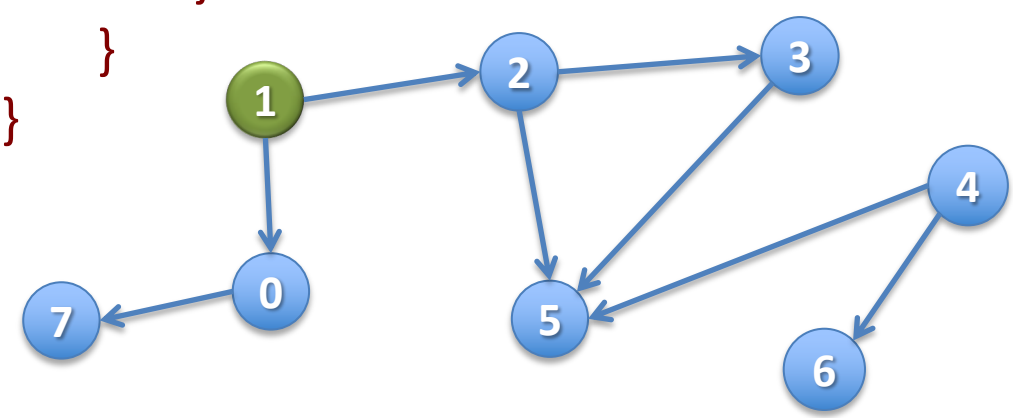
Call bfs(1)

Iteration 0



0 2

Queue q

Breadth-First Search

```
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}
```
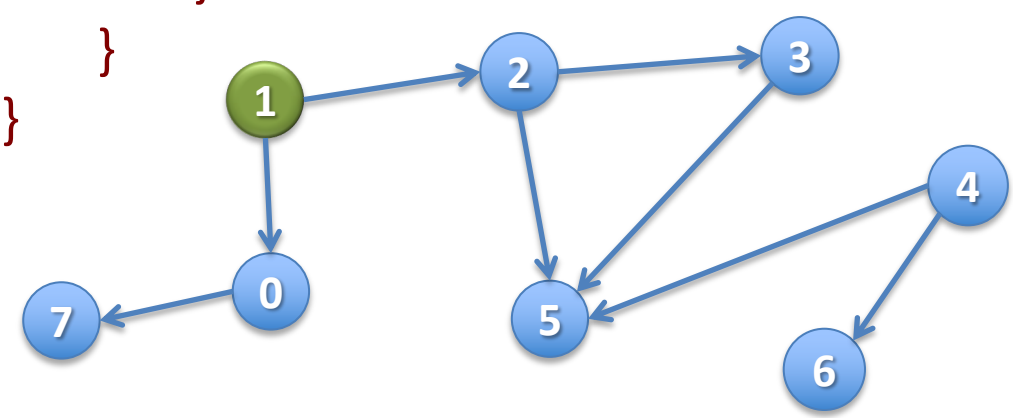

Call bfs(1)

Iteration 1



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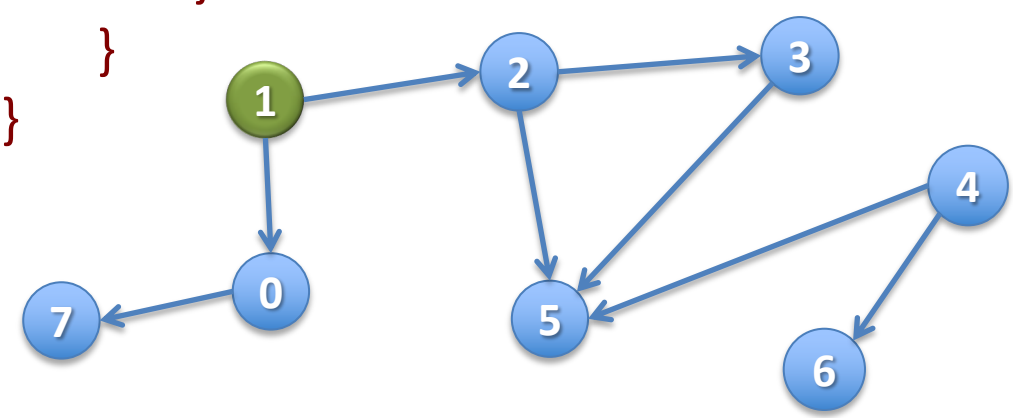
```

Call bfs(1)

Iteration 1



2

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Breadth-First Search

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```
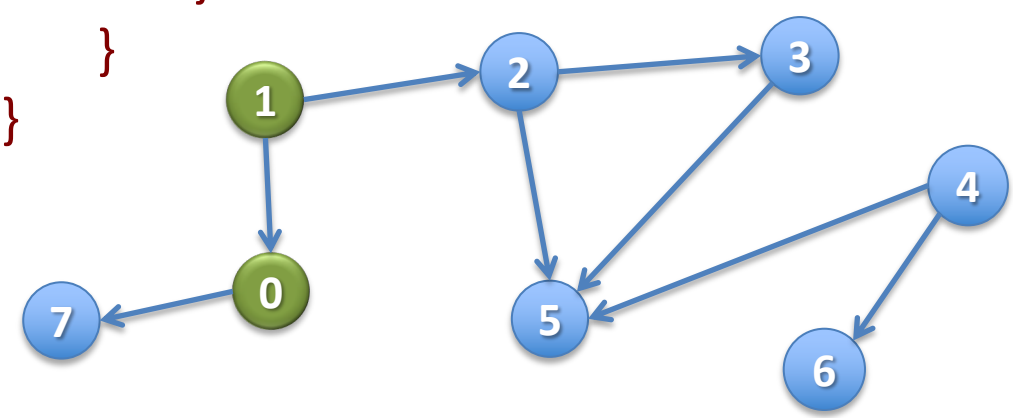

Call bfs(1)

Iteration 1



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Breadth-First Search

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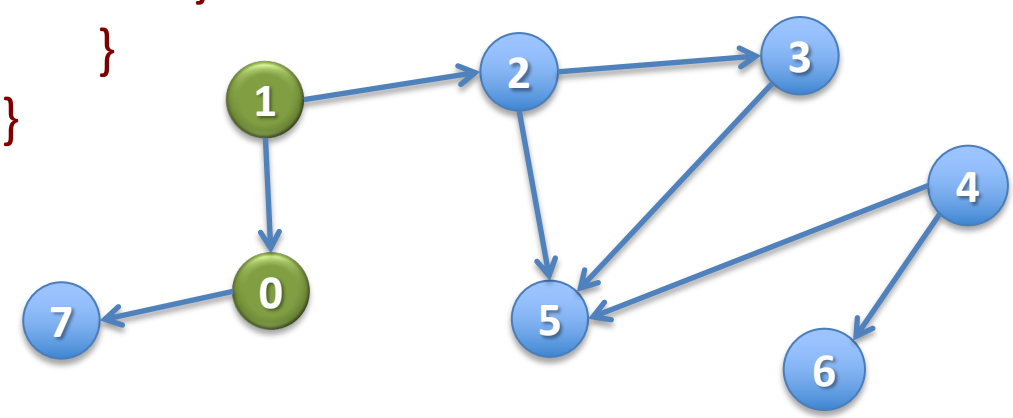
Call bfs(1)

Iteration 1



2 7

Queue q

Breadth-First Search

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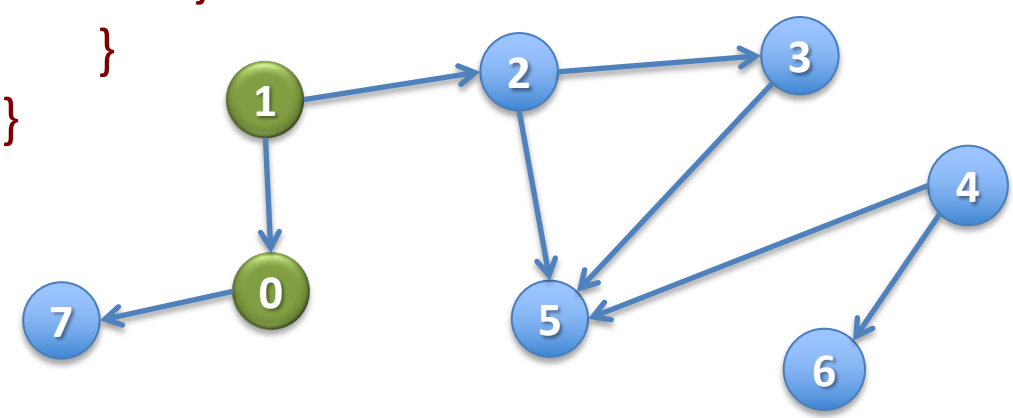
        }
    }
}
```

Call bfs(1)

Iteration 2



2 7

Queue q

Breadth-First Search

```
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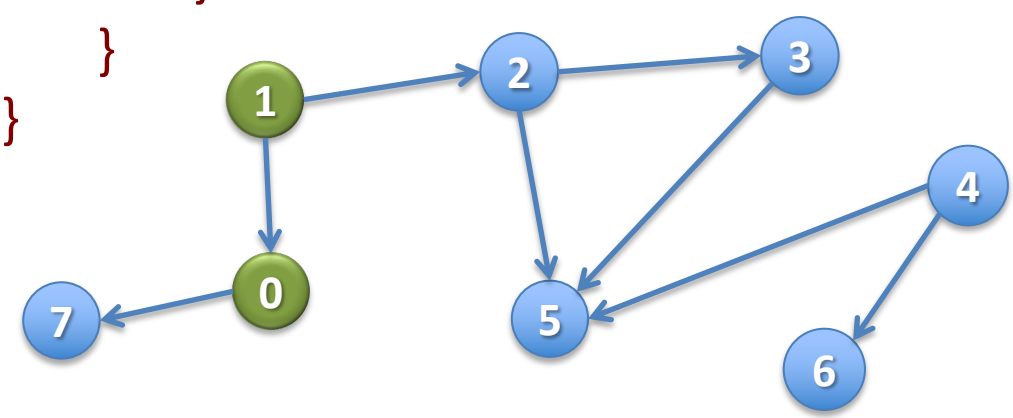
```

Call bfs(1)

Iteration 2



7

Queue q

Breadth-First Search

```
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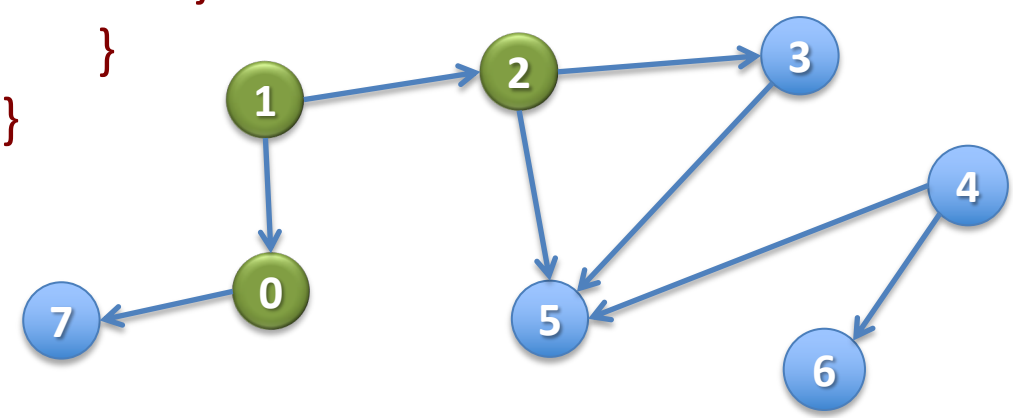
```

Call bfs(1)

Iteration 2



7

Queue q

Breadth-First Search

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                q.append(v);
        }
    }
}
```

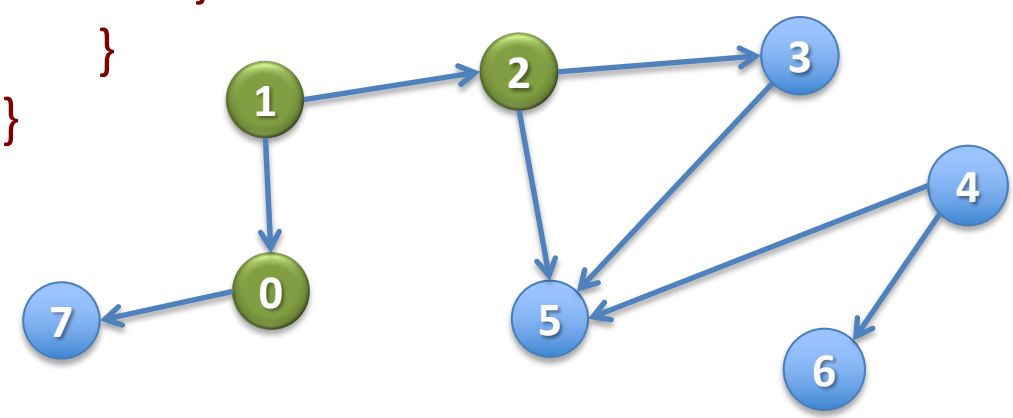
Call bfs(1)

Iteration 2

Breadth first:

(1) Node u
(2) All nodes 1 edge from u
(3) All nodes 2 edges from u
(4) All nodes 3 edges from u

…

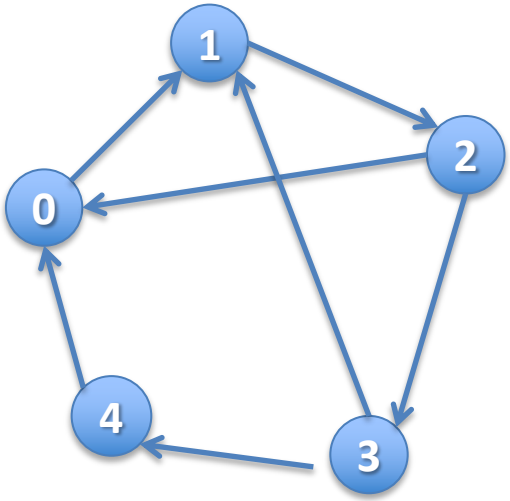


7 3 5

Queue q

Some working code for DFS

- https://codeboard.io/projects/97448
- Sample graph constructed by the code:



- Suggested exercises:
  - Run DFS by hand
  - Run BFS by hand
  - Code BFS

Questions to Ponder

- BFS(root) on a tree corresponds to which tree traversal?
- Write out the order nodes are visited in this undirected graph, when calling:
  - BFS(5)
  - DFS(5)
  - DFS(0)

(if there are ties, visit the lower # first)