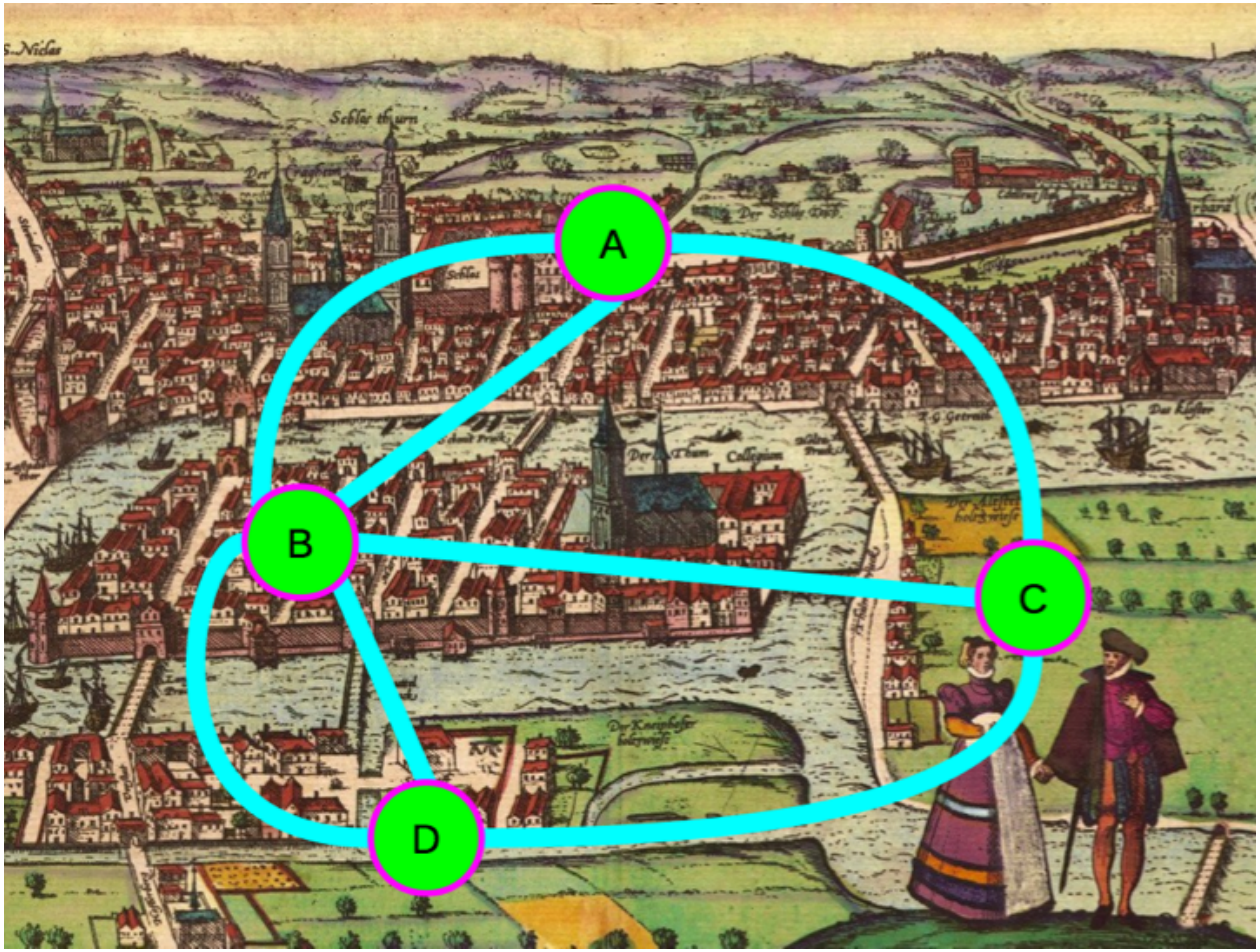

CSCI 241

Lecture 17

Some more hashing, Intro to Graphs

Announcements

Goals

- Know how to avoid using LinkedList buckets using **open addressing** with **linear** or **quadratic probing**.
- Understand the relationship between Java Object's **hashCode** and **equals** methods.
- Know the definition of a **graph** and basic associated terminology:
  - Node/vertex; edge/arc; directed, undirected; adjacent; (in/out-)degree; path; cycle;

Hash Functions:
Necessary Properties

Hash Functions: **Necessary** Properties

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- Equal objects hash to equal values:
  h(i) == *h*(j) if `i.equals(j)`

- **Collisions** are possible:
  If `!i.equals(j)` it **is possible** that *h*(i) == *h*(j)

  or: h(i) == h(j) does not imply `i.equals(j)`

Hash Functions: **Desirable Properties**

Hash Functions: Desirable Properties

We would ***like*** our hash functions distribute values evenly among buckets.

It’s hard to guarantee this without knowing keys ahead of time, but usually easy in practice using heuristics.

Hash Functions: Desirable Properties

A universally terrible hash function: h(k) = 0

Hash function quality often depends on the keys.
e.g., if keys are WWU CSCI course numbers:

- h(k) = k % 100 (1's place)
  - bad because many collisions (141, 241, 301, …)

- h(k) = k / 100 (100's place)
  - bad because this will only use buckets 0..6

One weird tip: make the table size prime so divisibility patterns in keys don't result in patterns in hash buckets.

Hashing Multiple Integers

- Various heuristic methods:
  - (a + b + c + d) % N
  - (ak^1 + bk^2 + ck^3 + dk^4) % N

Hashing Strings

- Interpret ASCII (or unicode) representation as an integer.
- Java String uses: `s[0]*31^(n-1) + s[1]*31^(n-2)+ … +s[n-1]`

Collision Resolution

Collision Resolution

- **Chaining** - use a LinkedList to store multiple elements per bucket.

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  - - Wastes space (linked list overhead)
  - - Wastes time (pointer lookups, cache locality)
- **Open Addressing** - use empty buckets to store things that belong in other buckets.
  - Need some scheme for deciding which buckets to look in.

Open Addressing with Linear Probing

- **Open Addressing** - use empty buckets to store things that belong in other buckets.

- Which empty bucket? Using the next empty one is called **Linear Probing**

put(1, “dog”);
put(11, “auk”);
put(10, “bear”);
put(14, “cat”);
put(24, “ape”);

0	
1	
2	
3	
4	

```
put(key):
    h = hash(key);
    while A[h] is full:
        h = (h+1) % N
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Open Addressing with Linear Probing

- Problem with linear probing:
  - Hashing clustered values (e.g., 1, 1, 3, 2, 3, 4, 6, 4, 5) will result in a lot of searching.

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put(10, “bear”);
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Open Addressing with Quadratic Probing

- **Quadratic Probing:** Jump further ahead to avoid clustering of full buckets.

Linear probing looks at H, H+1, H+2, H+3, H+4, …

Quadratic probing looks at H, H+1, H+4, H+9, H+16, …

put(1, “dog”);
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put(10, “bear”);
put(14, “cat”);
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```
put(key):
   H = hash(key);
   i = 0;
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      h = (H + i²) % N
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Open Addressing with Quadratic Probing

- **Quadratic Probing:** Jump further ahead to avoid clustering of full buckets.

Exercise: Which buckets are full after the following insertions into an array size of 10 using quadratic probing?

put(0, "ape");
put(1, "dog");
put(20, "elf");
put(21, "auk");
put(40, "bear");
put(41, "cat");
put(60, "elk");
put(61, "imp");

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put(key):
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        i++;
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Open Addressing with Quadratic Probing

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Exercise: Which buckets are full after the following insertions into an array size of 10 using quadratic probing?

put(0, “ape”);	0
put(1, “dog”);	1
put(20, “elf”);	0, 1, 4
put(21, “auk”);	1, 2
put(40, “bear”);	0, 1, 4, 9
put(41, “cat”);	1, 2, 5
put(60, “elk”);	0, 1, 4, 9, 6
put(61, “imp”);	1, 2, 5, 10, 7

```
put(key):
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    while A[h] is full:
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        i++;
    A[h] = value
```

Open Addressing: Runtime

- May be faster, but may not be. Depends on keys.
- There's no free lunch: worst-case is always O(n).
- In practice, average-case is O(1) if you make good design decisions and insertions are not done by an adversary.

Hashing in Java

- Object has a hashCode method.

  By default, this returns the object's address in memory.

- **Scenario 1: You are using a class** that someone else wrote.
  - All Java objects (i.e., non-primitive types) inherit from Object.
  - If you want to put an instance of the class in a hash table, you don't need to know how to hash it!
  - Just call its `hashCode` method.

Hashing in Java

- Object has a hashCode method.

  By default, this returns the object's address in memory.

- **Scenario 2: You are writing a class.**
  - Its hashCode method needs to have the properties of a hash function!

  1. Deterministic: always returns the same value for the same object.
  2. **Equal** objects have equal hash codes.
     - In Java, "equal" means whatever the `equals` method says it means.

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2. **Equal** objects have equal hash codes.
   - In Java, "equal" means whatever the `equals` method says it means.

Consequence: if you change the definition of `equals` (e.g., by overriding it), you may have to override `hashCode` make sure that equal objects have equal hash codes!

Hashing in Java

Consequence: if you override `equals`, you may have to override `hashCode` to match.

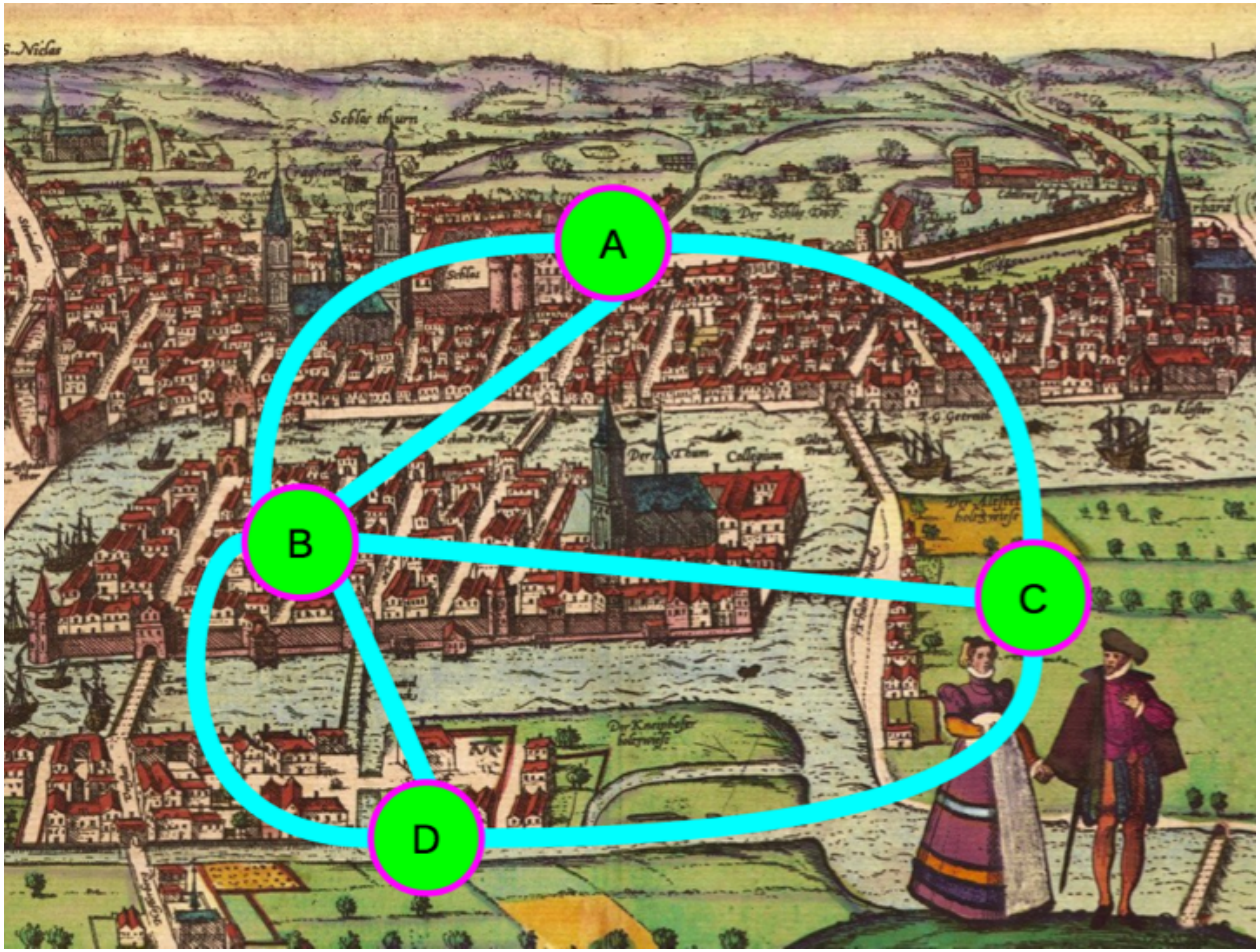
```
class Person {
  String firstName;
  String lastName;

  public boolean equals(Person p) {
    return firstName.equals(p.firstName)
        && lastName.equals(p.lastName);
  }

  public int hashCode() {
    return auxHash(firstName)
         + auxHash(lastName);
  }
}
```

Further Reading

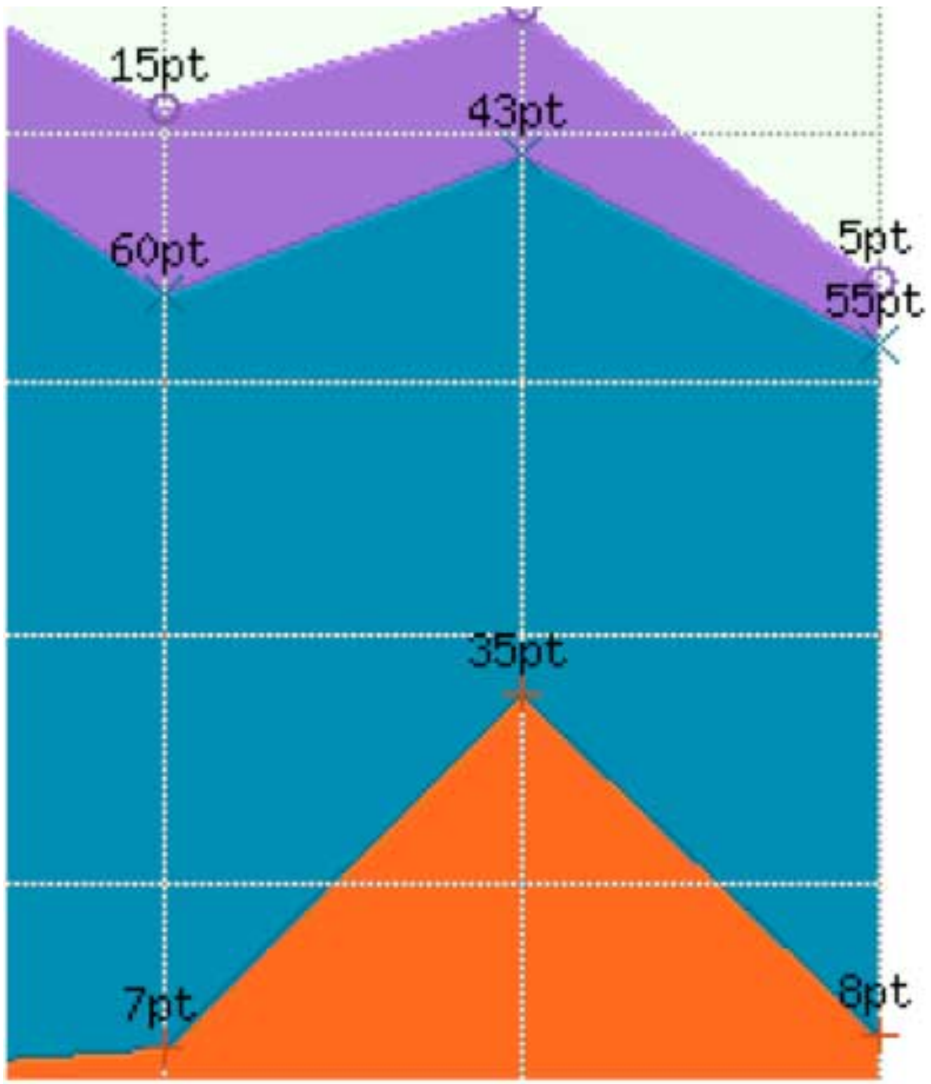
- CLRS 11.5: Perfect Hashing
    - You can guarantee O(1) lookups and insertions if the set of keys is fixed
- C++ implementations from Google:
    - sparse_hash_map - optimized for memory overhead
    - dense_hash_map - optimized for speed

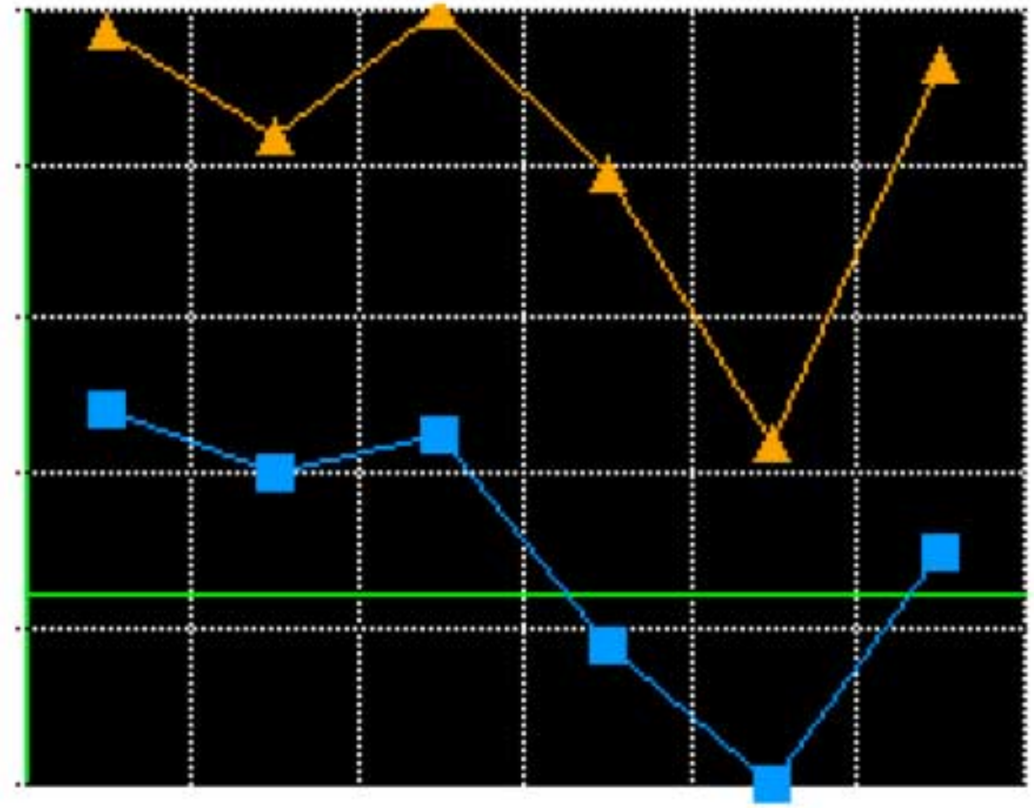


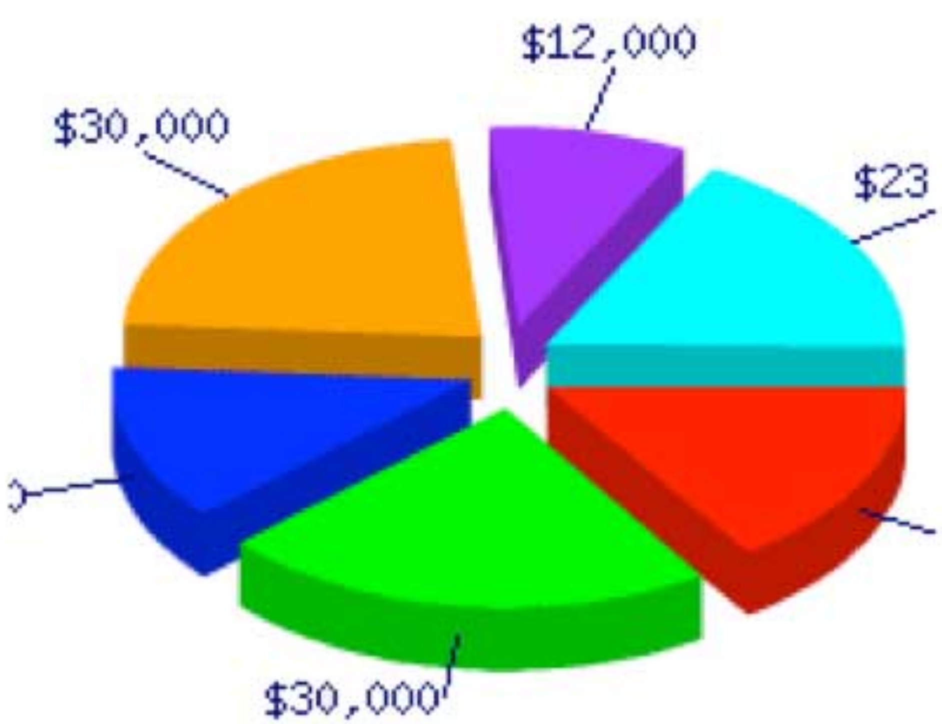
CSCI 241

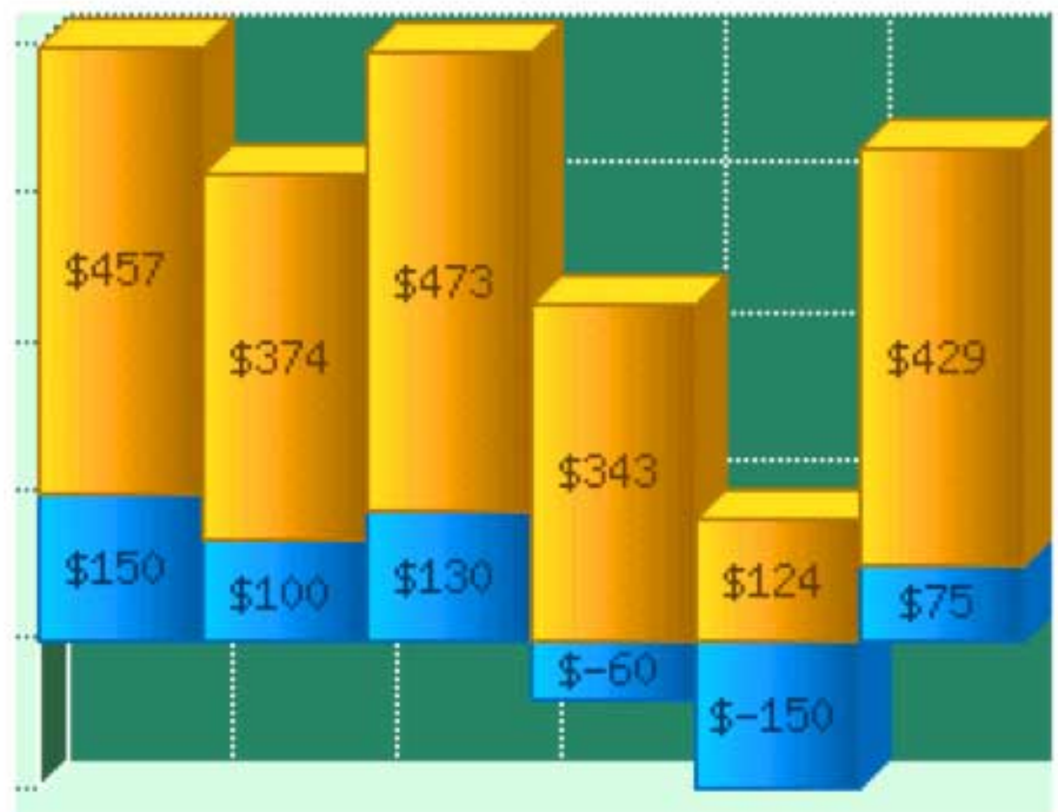
Lecture 17

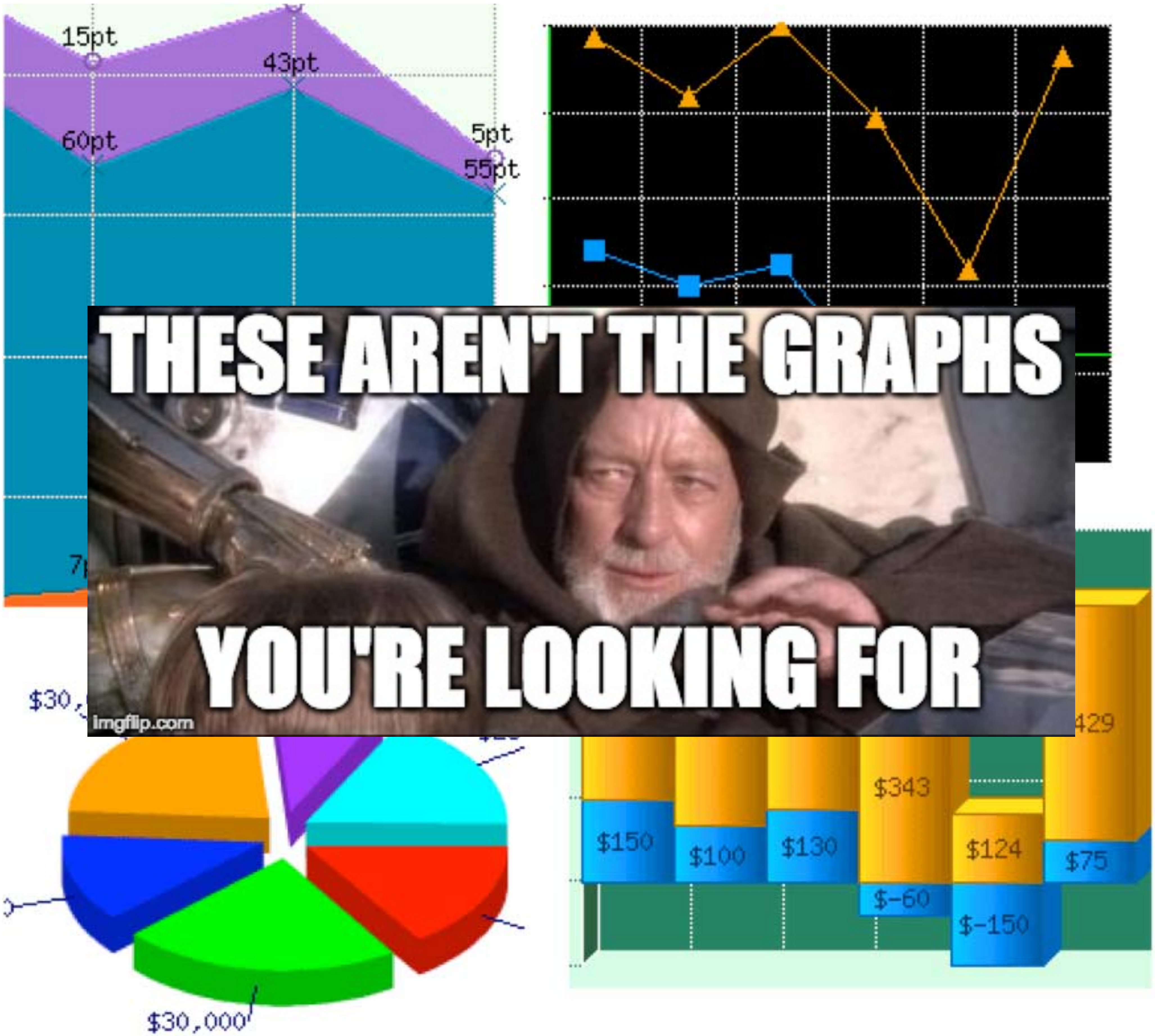
~~Some more hashing~~, Intro to Graphs


15pt
43pt
60pt
5pt
55pt
35pt
7pt
8pt

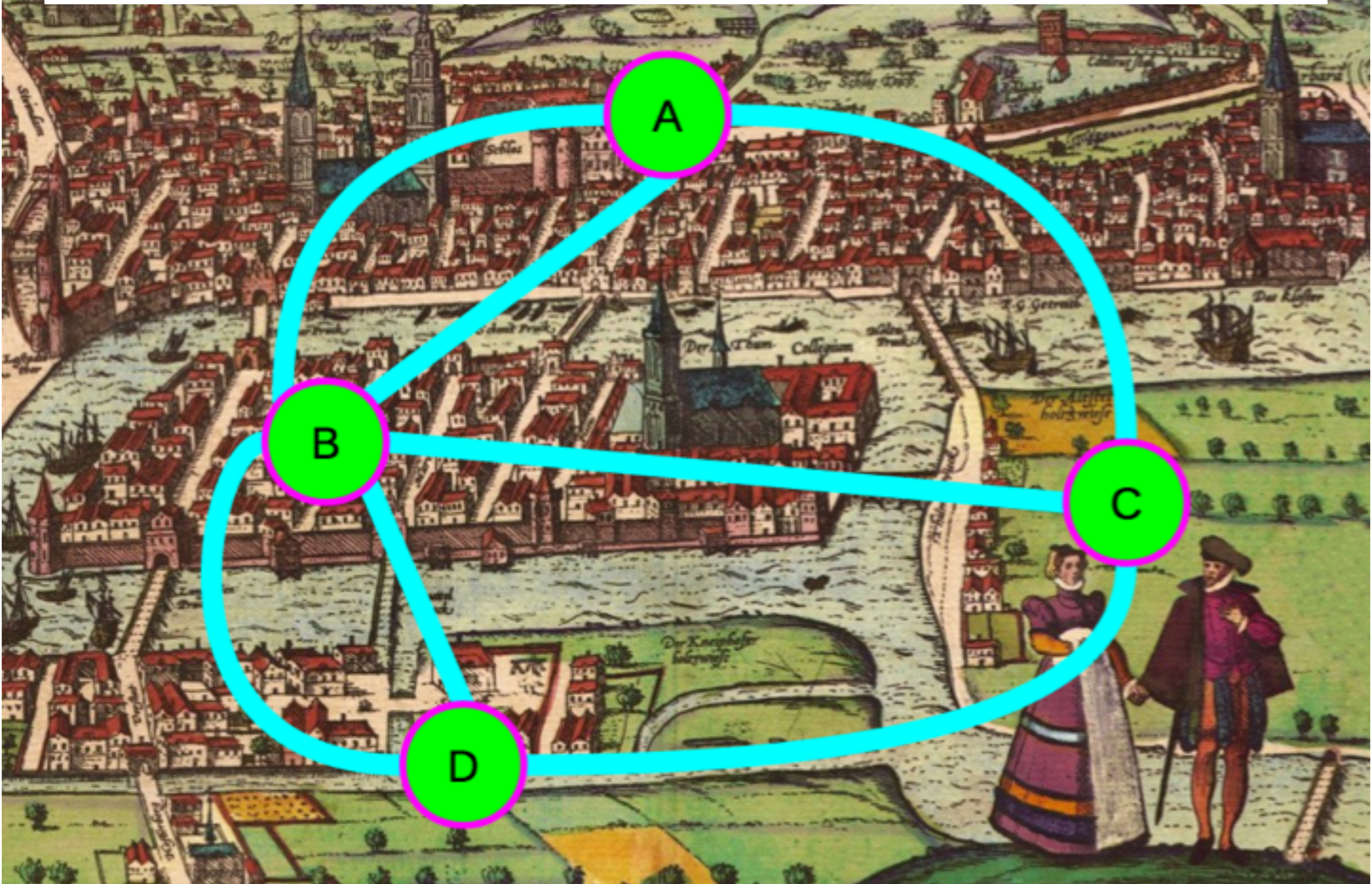



$12,000
$30,000
$23
$30,000

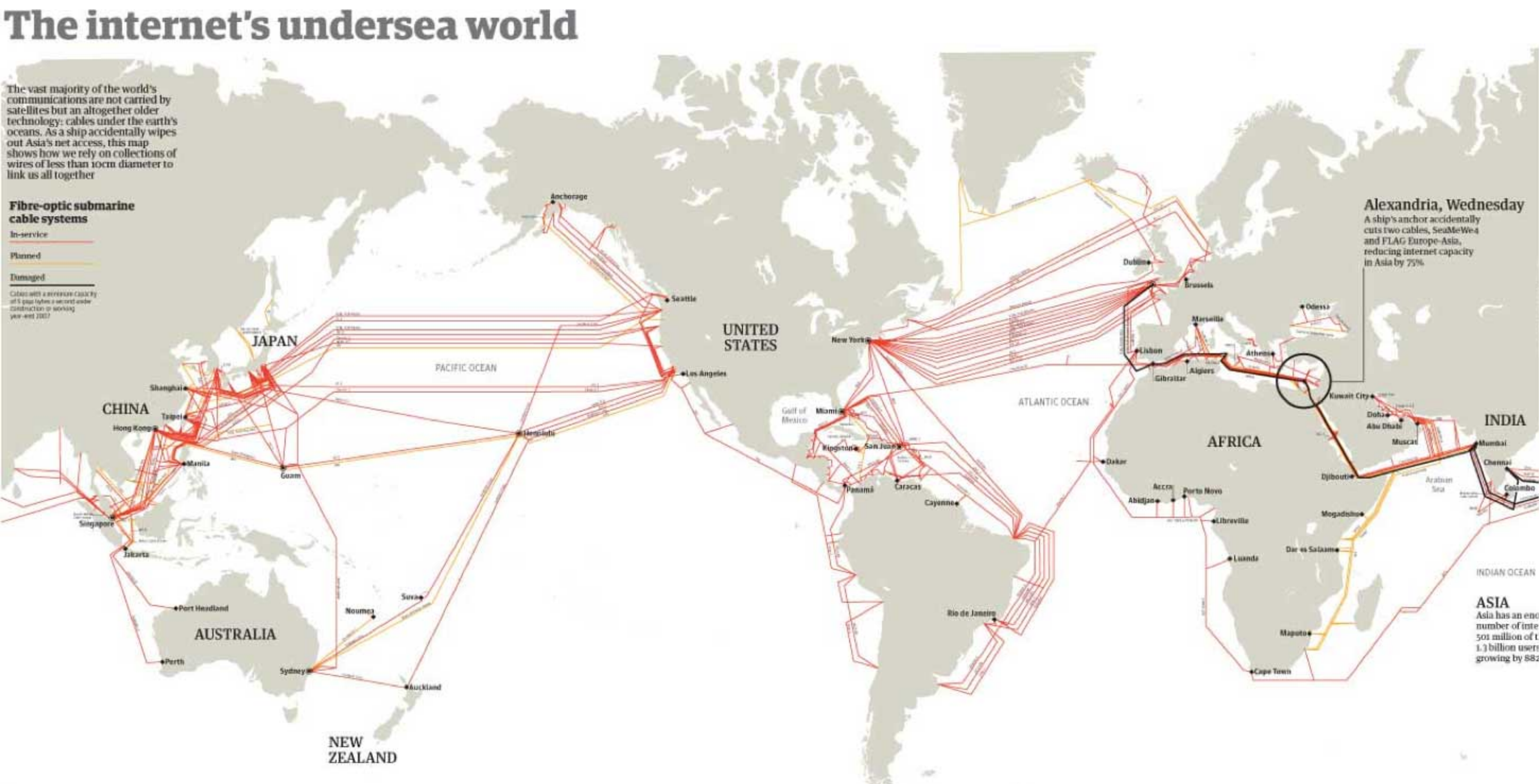

$457
$374
$473
$343
$124
$429
$150
$100
$130
$-60
$-150
$75


THESE AREN'T THE GRAPHS
YOU'RE LOOKING FOR
imgflip.com
15pt
43pt
60pt
5pt
55pt
$30,000
$343
$150
$100
$130
$124
$75
$-60
$-150

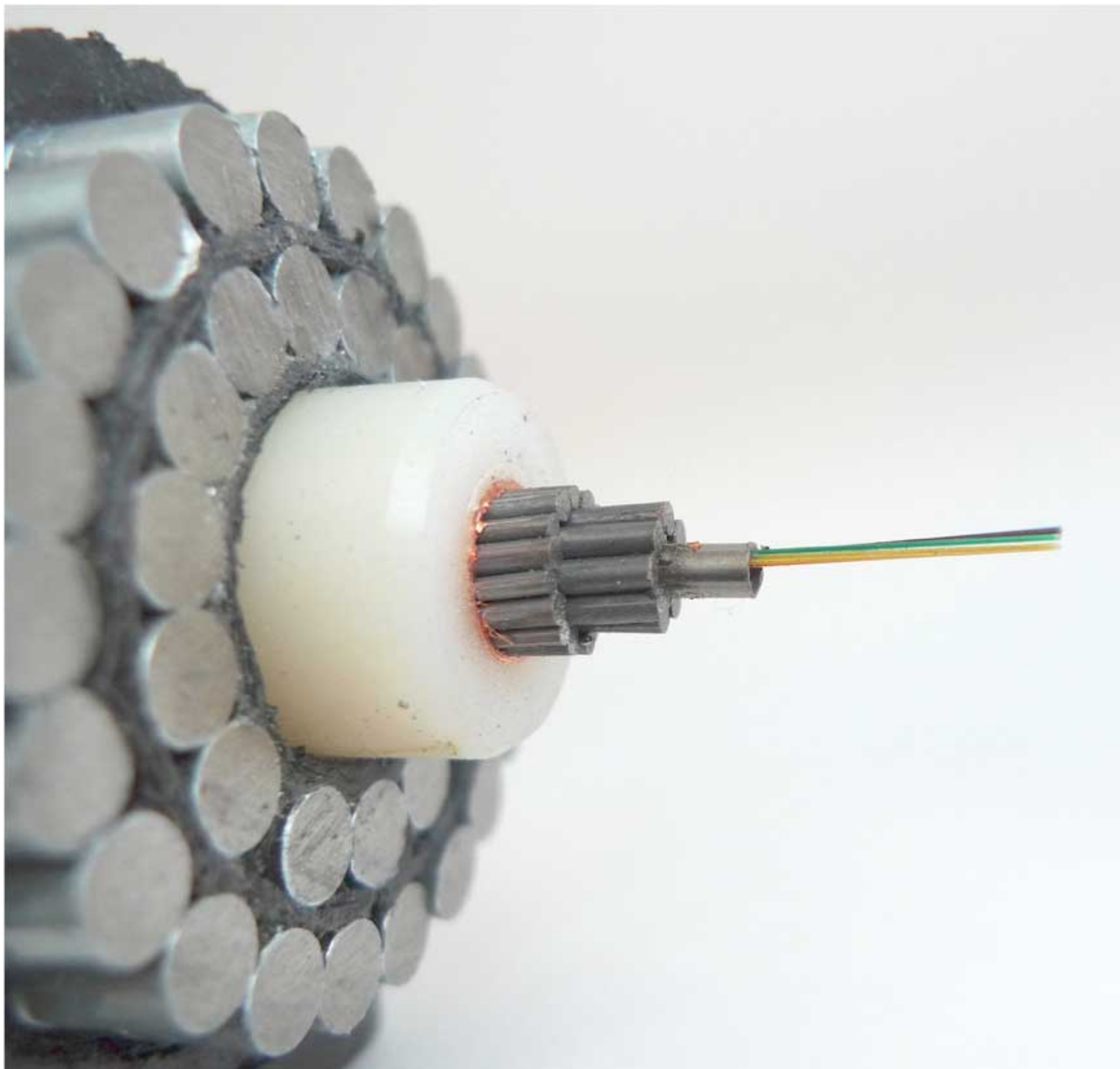
Graph: a bunch of points connected by lines. The lines may have directions, or not.



This is a graph:

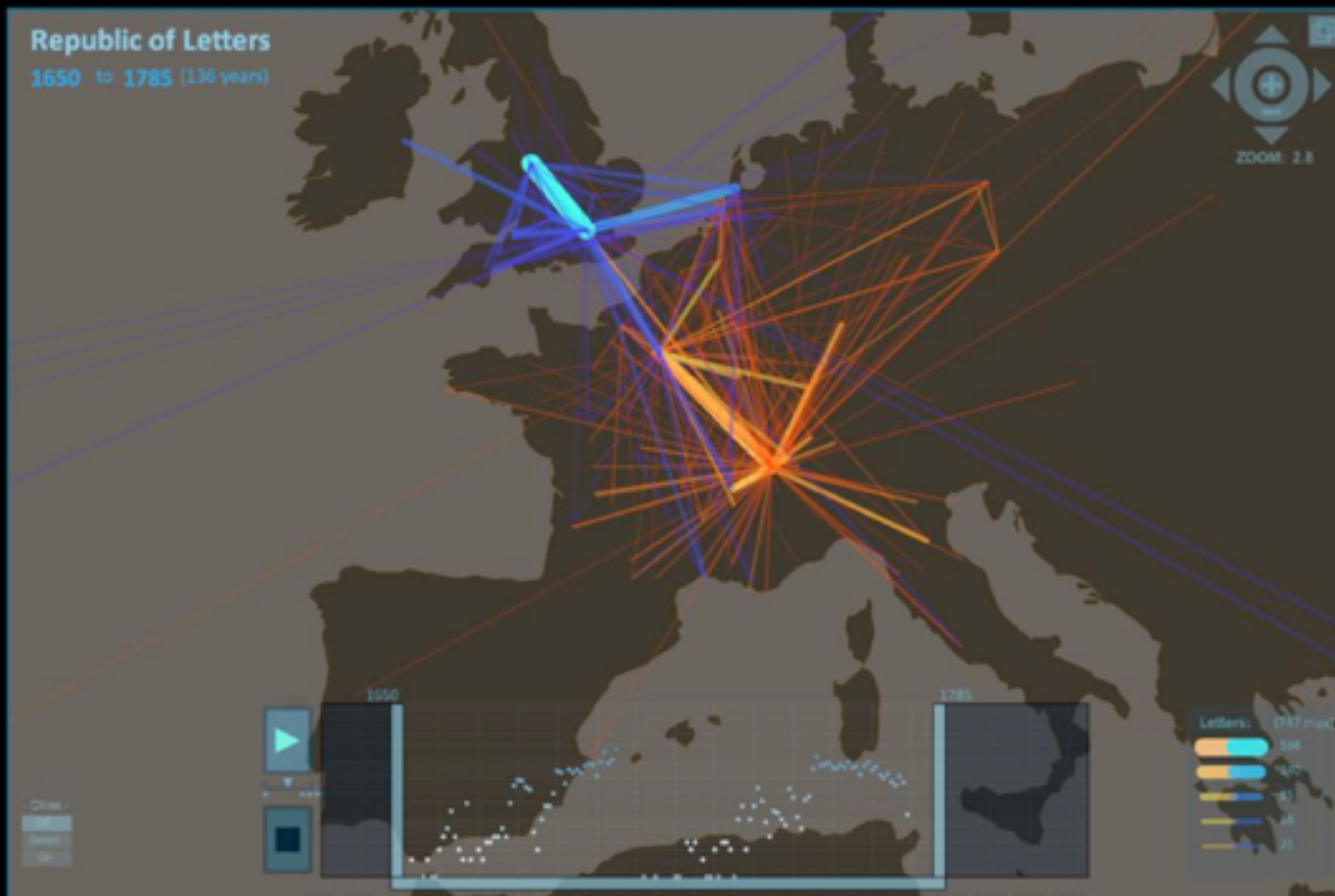


The edges are made of these:



Social Networks

(before they were cool)



Locke's (blue) and Voltaire's (yellow) correspondence.
Only letters for which complete location information is available are shown.
Data courtesy the Electronic Enlightenment Project, University of Oxford.

Social Networks

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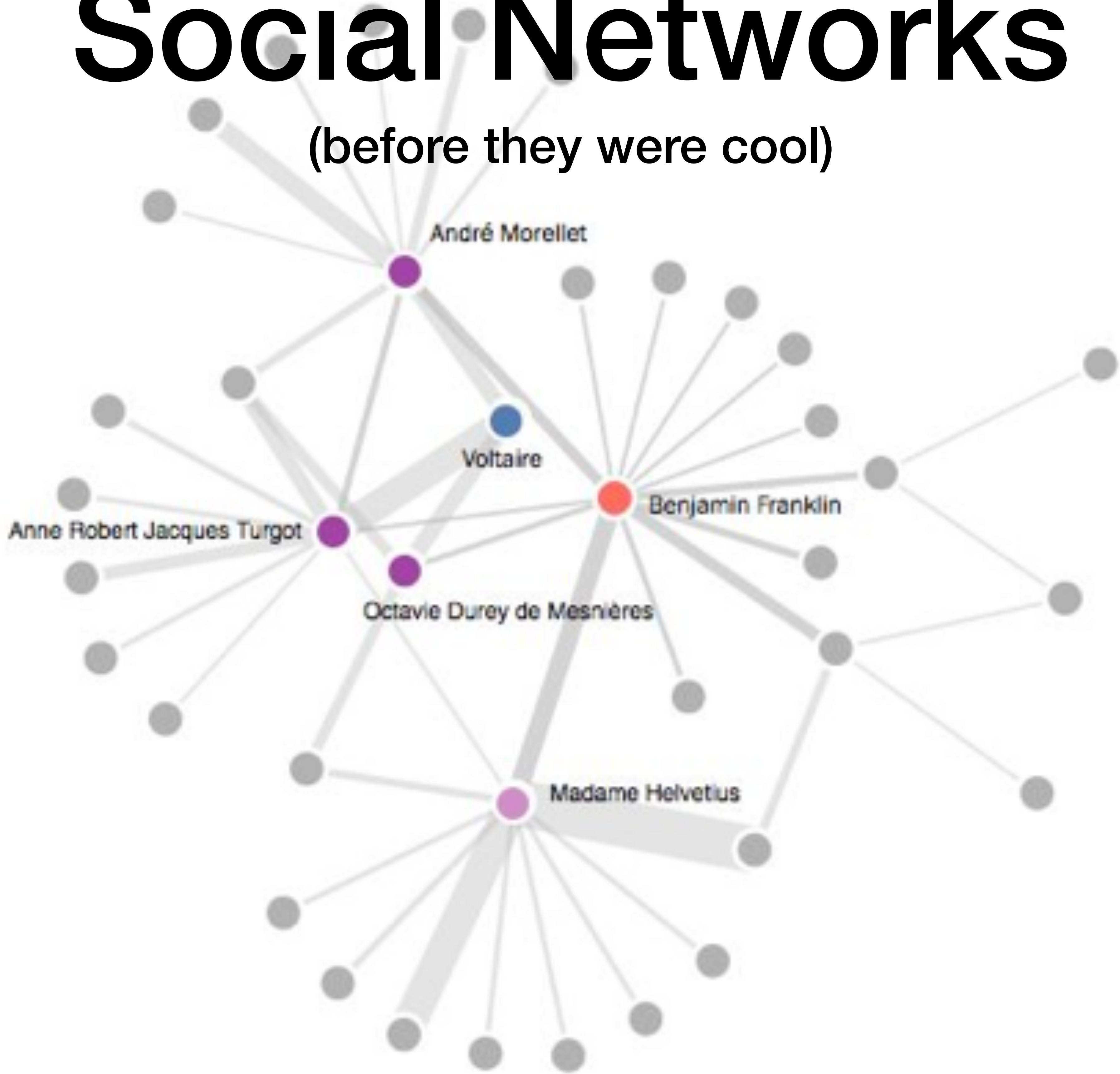
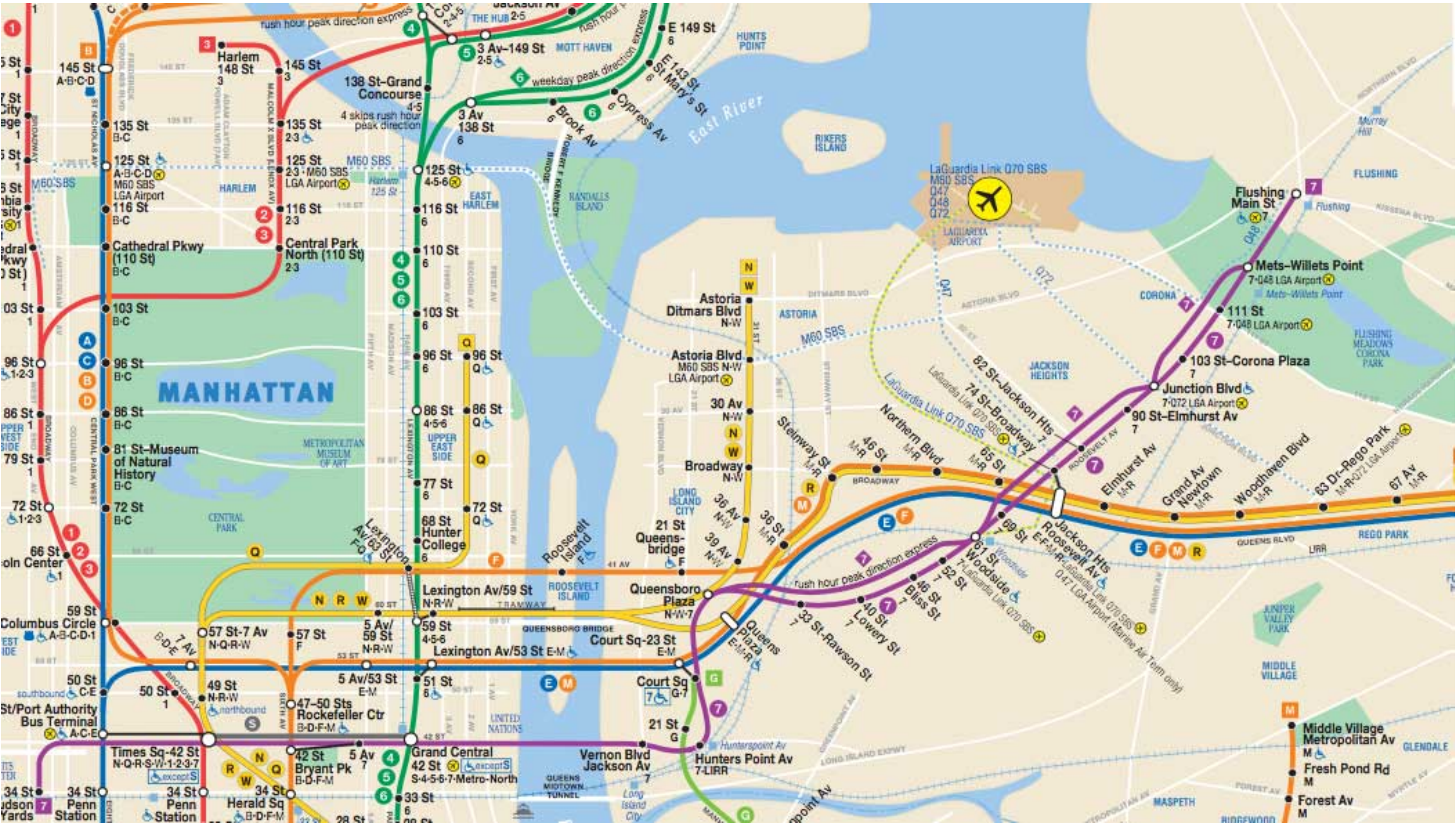
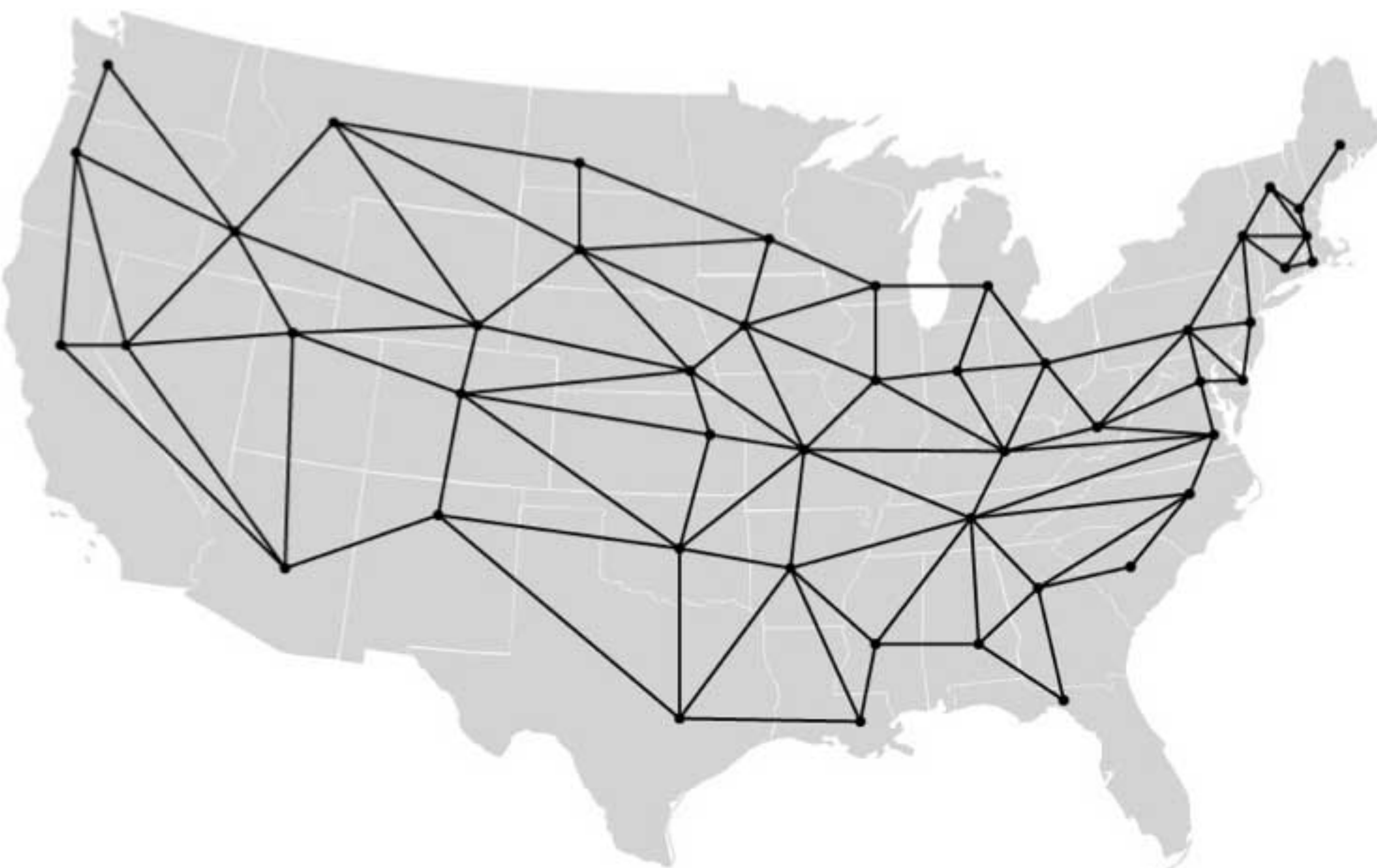


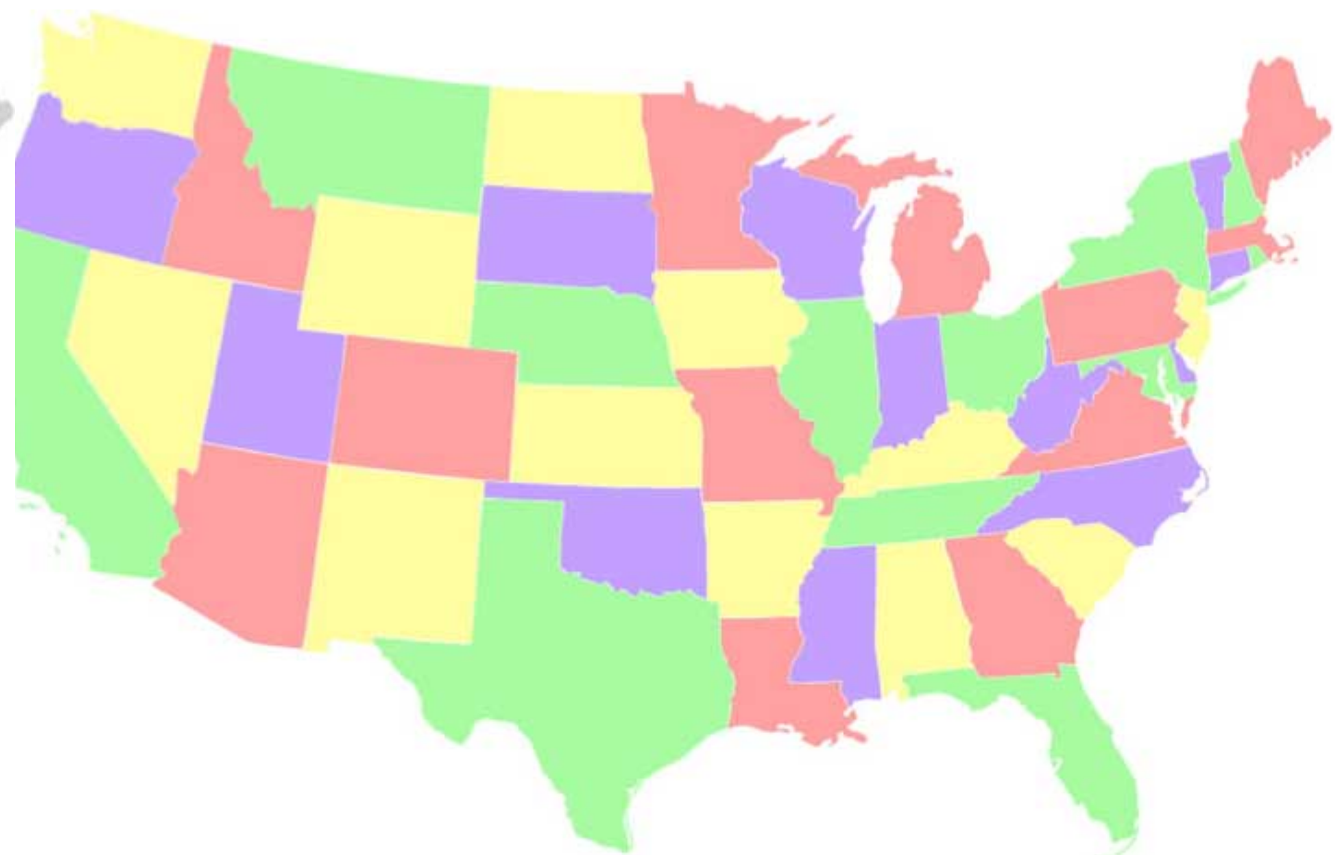


MANHATTAN
East River
RIKERS ISLAND
HUNTS POINT
MOTT HAVEN
HARLEM
EAST HARLEM
UPPER EAST SIDE
CENTRAL PARK
METROPOLITAN MUSEUM OF ART
RANDALLS ISLAND
ROOSEVELT ISLAND
UNITED NATIONS
LONG ISLAND CITY
ASTORIA
JACKSON HEIGHTS
CORONA
FLUSHING
FLUSHING MEADOWS CORONA PARK
REGO PARK
JUNIPER VALLEY PARK
MIDDLE VILLAGE
MASPETH
GLENDALE
LAGUARDIA AIRPORT
LaGuardia Link Q70 SBS
M60 SBS
Q47
Q48
Q72
Harlem 148 St
145 St
135 St
125 St
116 St
Cathedral Pkwy (110 St)
Central Park North (110 St)
103 St
96 St
86 St
81 St-Museum of Natural History
79 St
72 St
66 St Lincoln Center
59 St Columbus Circle
57 St-7 Av
57 St
50 St
49 St
47-50 Sts Rockefeller Ctr
42 St Port Authority Bus Terminal
Times Sq-42 St
42 St Bryant Pk
5 Av
Grand Central 42 St
34 St Penn Station
34 St Herald Sq
34 St Hudson Yards
33 St
28 St
138 St-Grand Concourse
3 Av-149 St
3 Av 138 St
Brook Av
Cypress Av
E 143 St St Mary's St
E 149 St
THE HUB
125 St
116 St
110 St
103 St
96 St
86 St
77 St
68 St Hunter College
Lexington Av/63 St
Lexington Av/59 St
59 St
5 Av/59 St
Lexington Av/53 St
5 Av/53 St
51 St
Roosevelt Island
TRAMWAY
QUEENSBORO BRIDGE
Astoria Ditmars Blvd
Astoria Blvd
30 Av
Broadway
36 Av
39 Av
Queensboro Plaza
Queens-bridge
21 St
Court Sq-23 St
Court Sq
21 St
Vernon Blvd Jackson Av
Hunters Point Av
Steinway St
46 St
Northern Blvd
65 St
36 St
Queens Plaza
33 St-Rawson St
40 St Lowery St
46 St Bliss St
52 St
61 St Woodside
69 St
74 St-Broadway
82 St-Jackson Hts
Jackson Hts Roosevelt Av
90 St-Elmhurst Av
Junction Blvd
103 St-Corona Plaza
111 St
Mets-Willets Point
Flushing Main St
Elmhurst Av
Grand Av Newtown
Woodhaven Blvd
63 Dr-Rego Park
67 Av
Middle Village Metropolitan Av
Fresh Pond Rd
Forest Av
rush hour peak direction express
weekday peak direction express
4 skips rush hour peak direction
Q47 LGA Airport (Marine Air Term only)
QUEENS MIDTOWN TUNNEL

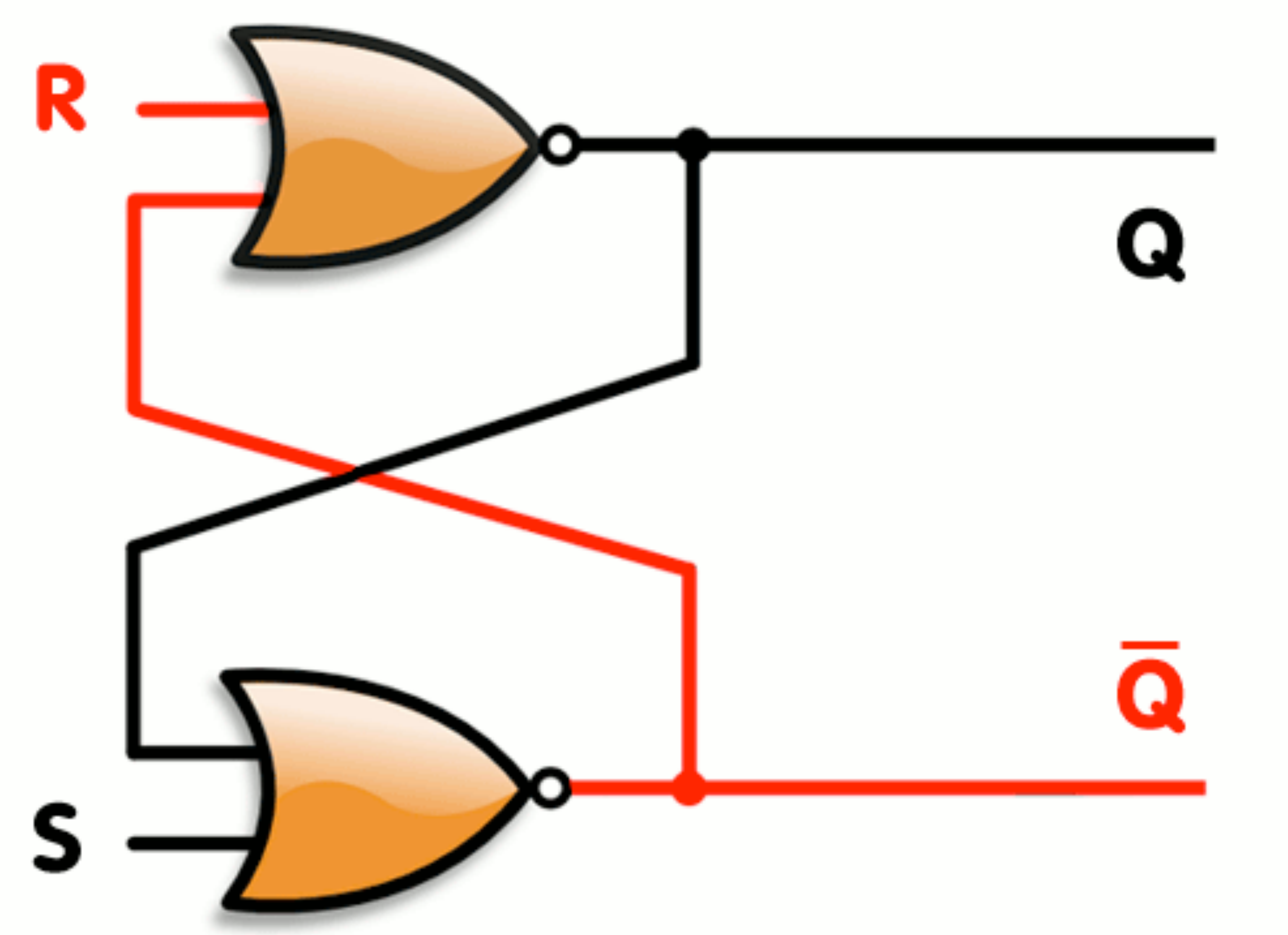
The USA as a graph:

- Neighboring states are connected by edges.

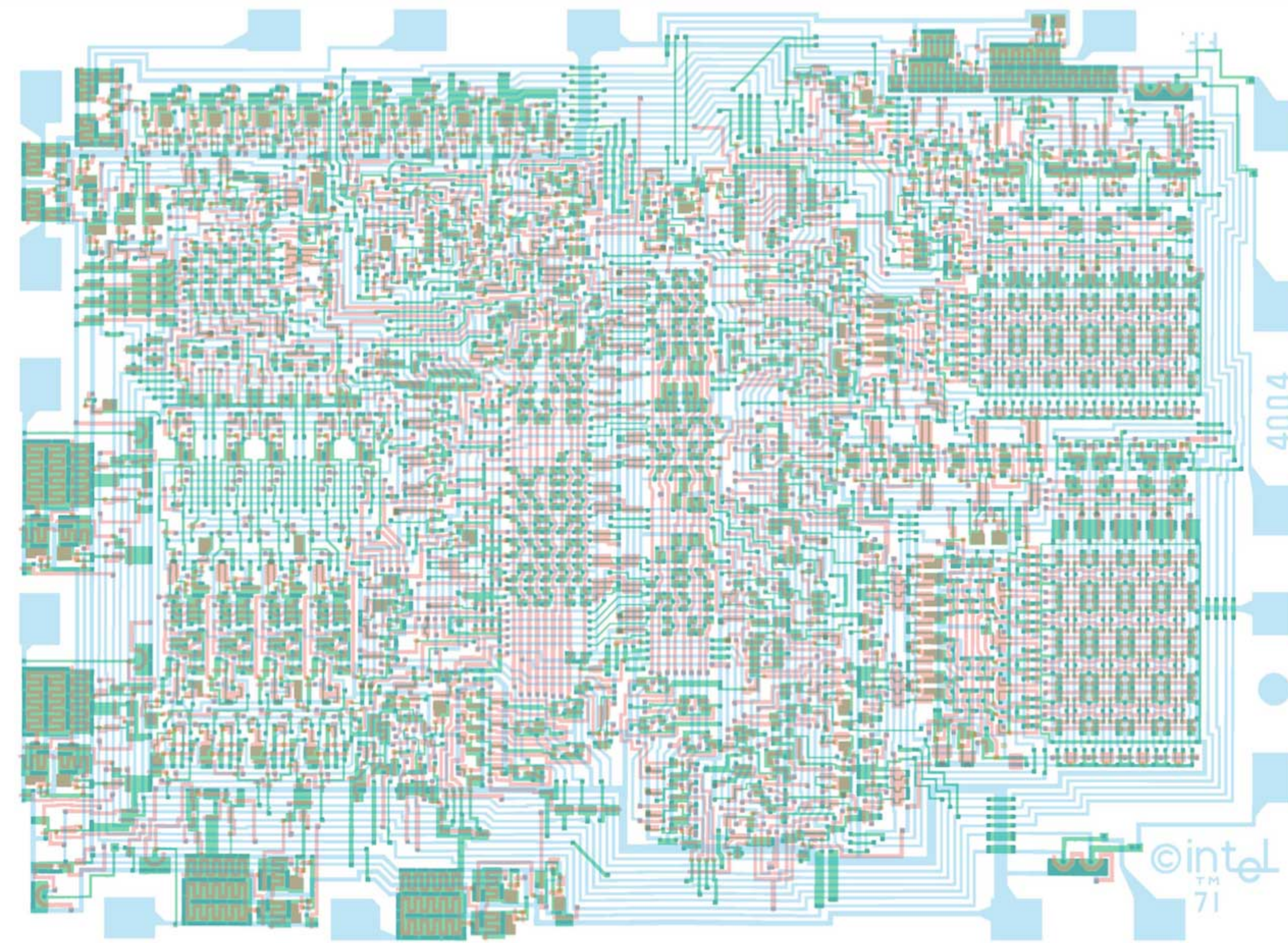




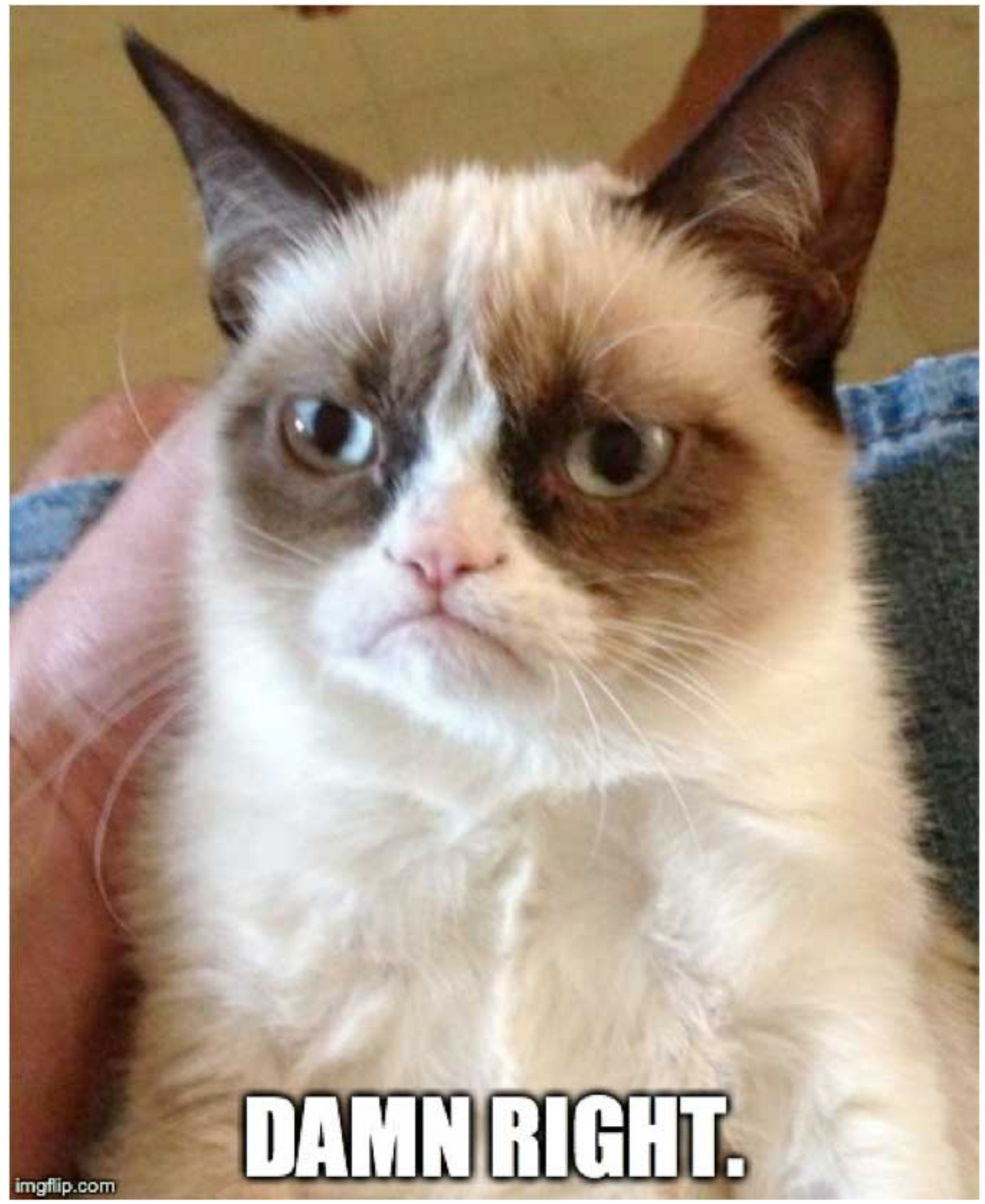
Electrical circuit



A bigger electrical circuit

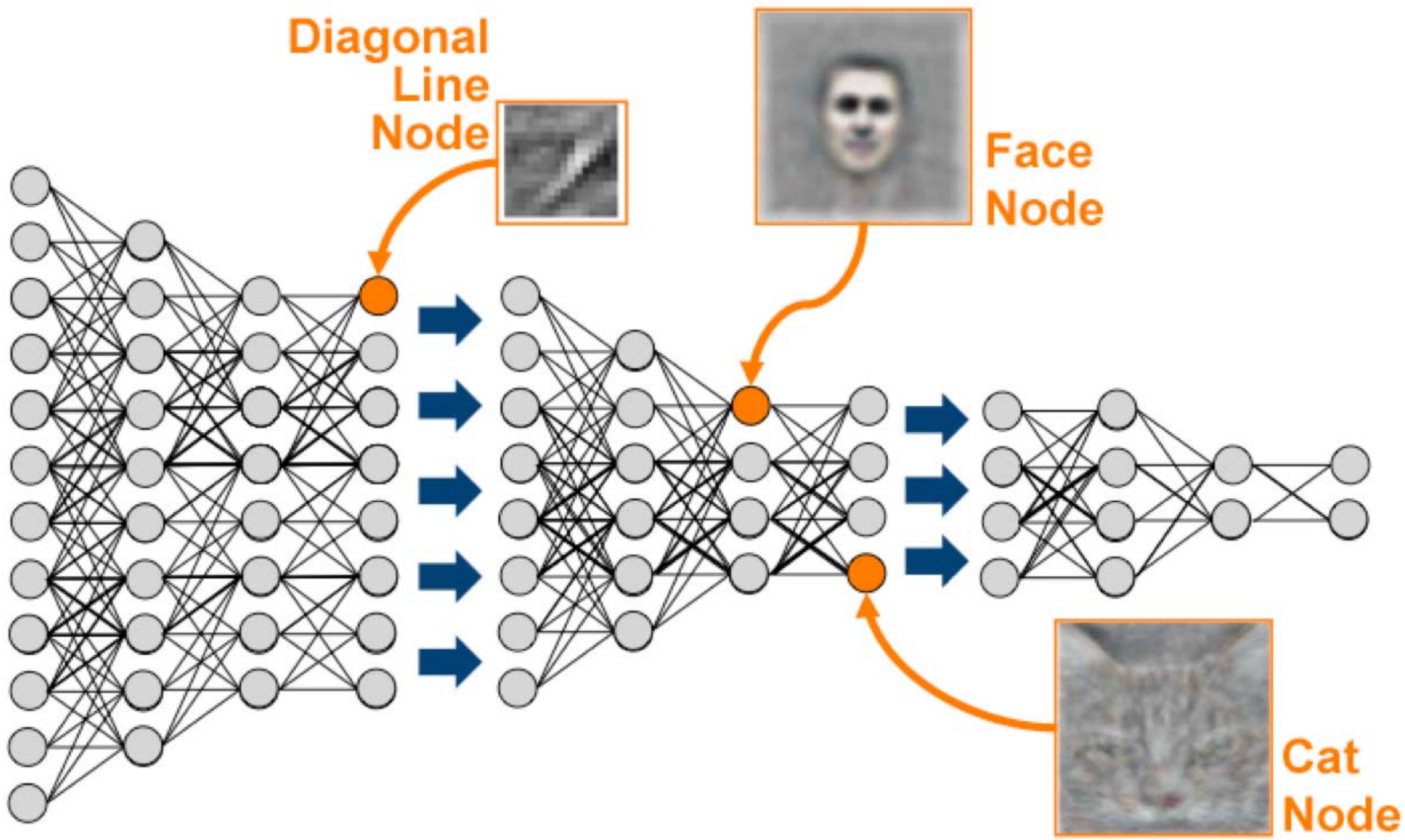


This is not a graph:



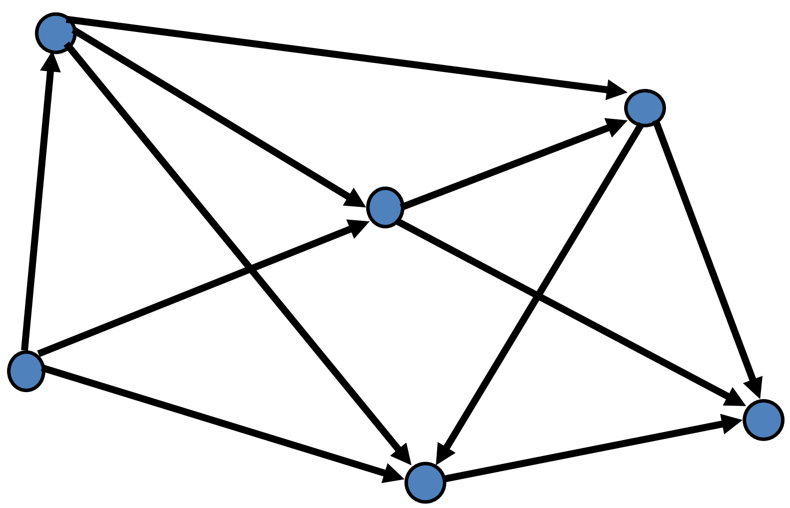
it is a cat.

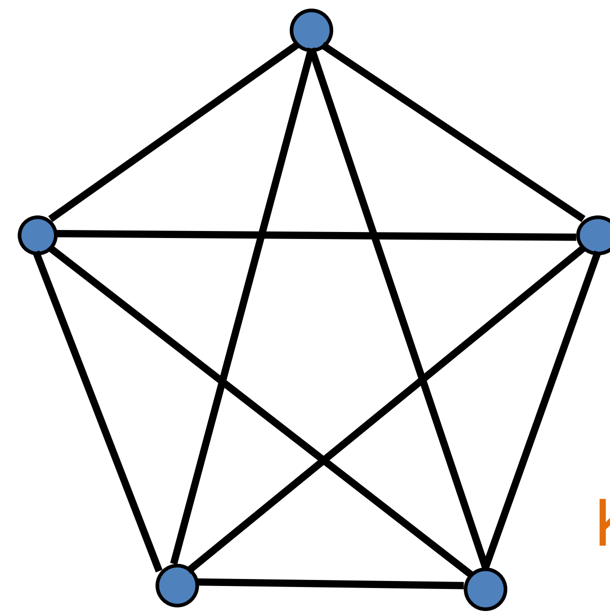
This is a graph



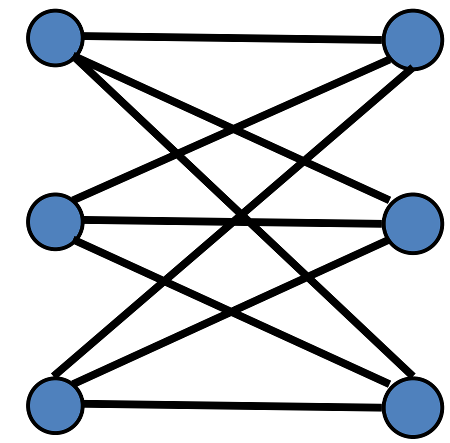
that can recognize cats.

Graphs: Abstract View

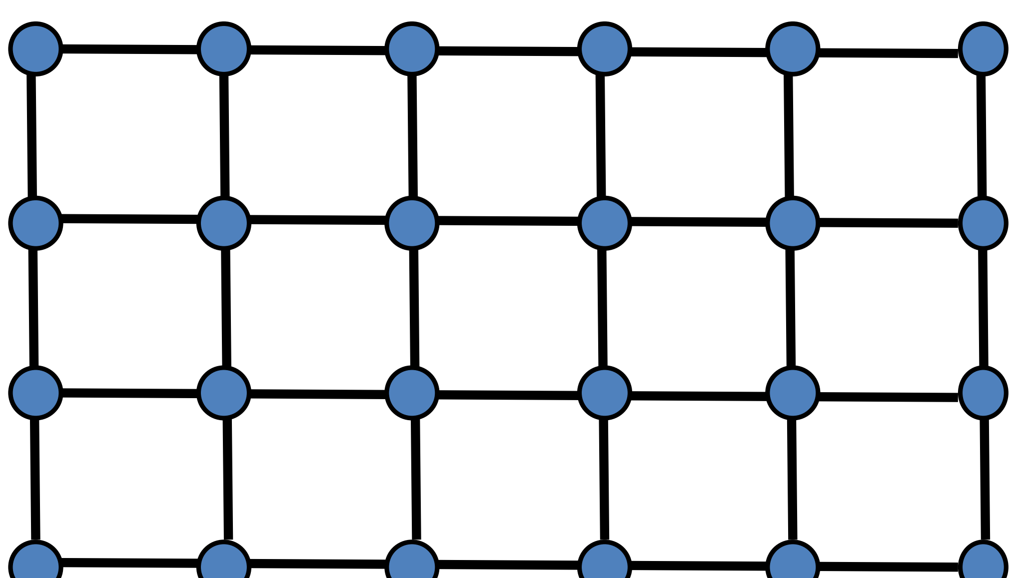


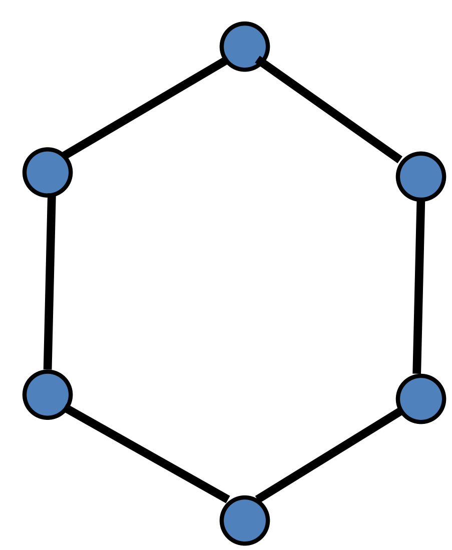


K_5



$K_{3,3}$





Graphs, Formally

- A directed graph (digraph) is a pair (V, E) where:
  - V is a (finite) set
  - E is a set of **ordered** pairs (u, v) where u, v are in V
  - Often (not always): u ≠ v (i.e. no edges from a vertex to itself)
- An element in V is called a vertex or node
- Elements in E are called edges or arcs
- |V| = size of V (traditionally called n)
- |E| = size of E (traditionally called m)

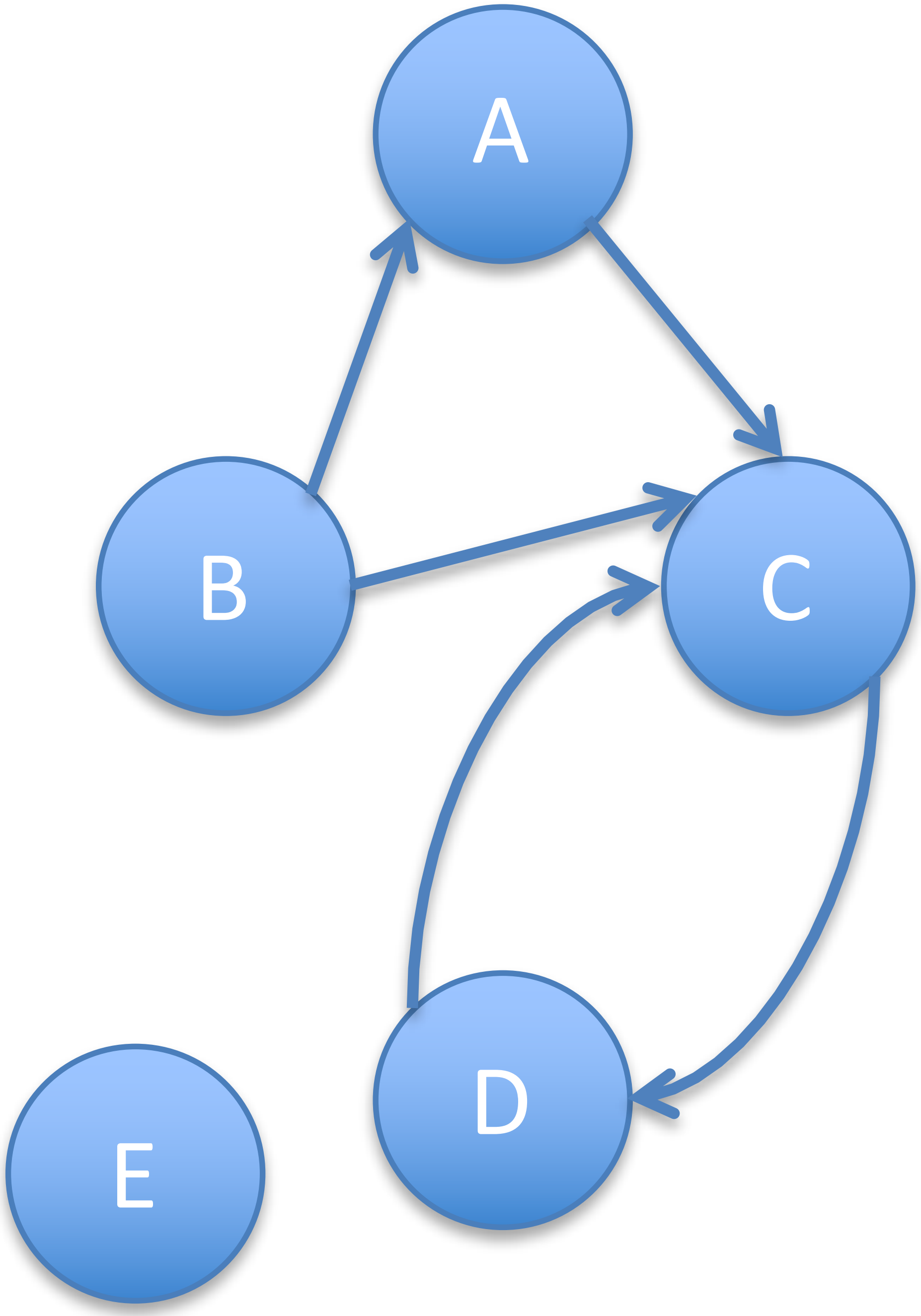
An example directed graph

$$V = \{A, B, C, D, E\}$$

$$E = \{(A, C), (B, A), (B, C), (C, D), (D, C)\}$$

$$|V| = 5$$

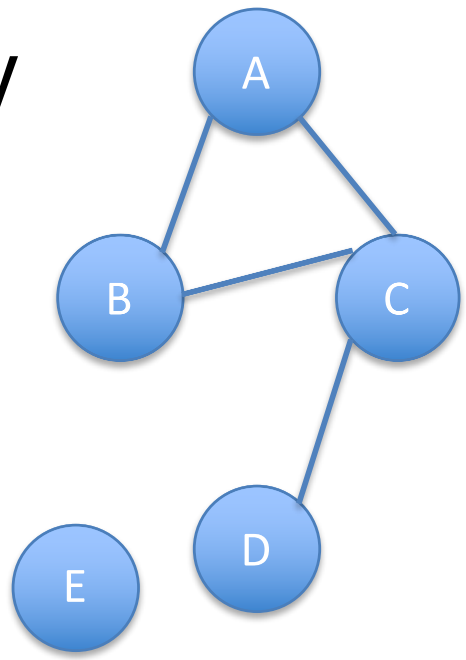
$$|E| = 5$$



Graphs, Formally

- An **undirected graph** is a just like a digraph, but
  - E is a set of **un**ordered pairs (u, v) where u, v are in V

$$V = \{A, B, C, D, E\}$$
$$E = \{\{A,C\}, \{B,A\}, \{B,C\}, \{C,D\}\}$$
$$|V| = 5$$
$$|E| = 4$$



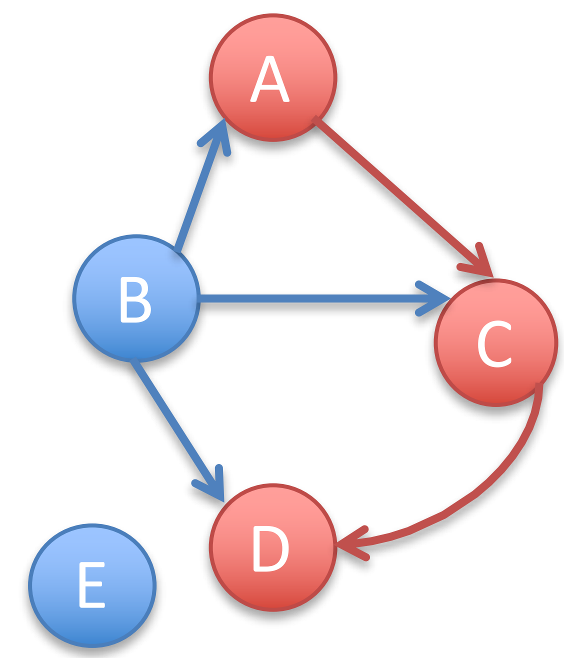
- An **un**directed graph can be converted to an equivalent **directed** graph:
  - Replace each undirected edge with two directed edges in opposite directions
- A **directed** graph can't always be converted to an **undirected** graph.

Graph Terminology: Adjacency, Degree

- Two vertices are adjacent if they are connected by an edge
- Nodes u and v are called the source and sink of the **directed** edge (u, v)
- Nodes u and v are endpoints of an edge (u, v) (directed or undirected)
- The outdegree of a vertex u in a **directed** graph is the number of edges for which u is the source
- The indegree of a vertex v in a **directed** graph is the number of edges for which v is the sink
- The degree of a vertex u in an **undirected** graph is the number of edges of which u is an endpoint

Graph Teminology: Paths, Cycles

- A path is a sequence of vertices where each consecutive pair are adjacent.
- In a directed graph, paths must follow the direction of the edges (nodes must be ordered source then sink).
- A cycle is a path that ends where it started, e.g.: x, y, z, x
- A graph is acyclic if it has no cycles.



Path A,C,D