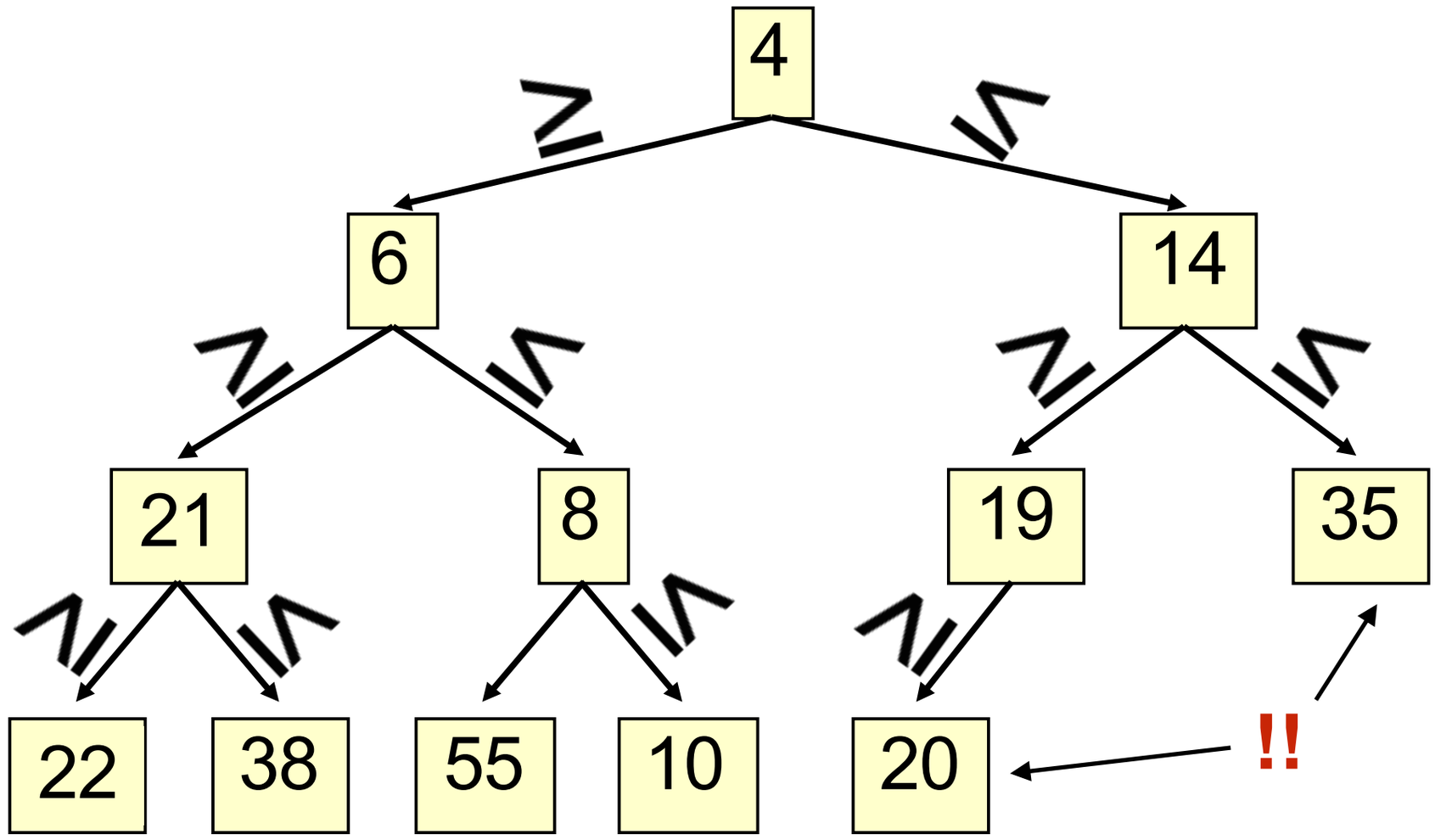
Preview: Heaps

A **heap** is a special binary tree with two additional properties.

A heap is a special binary tree.

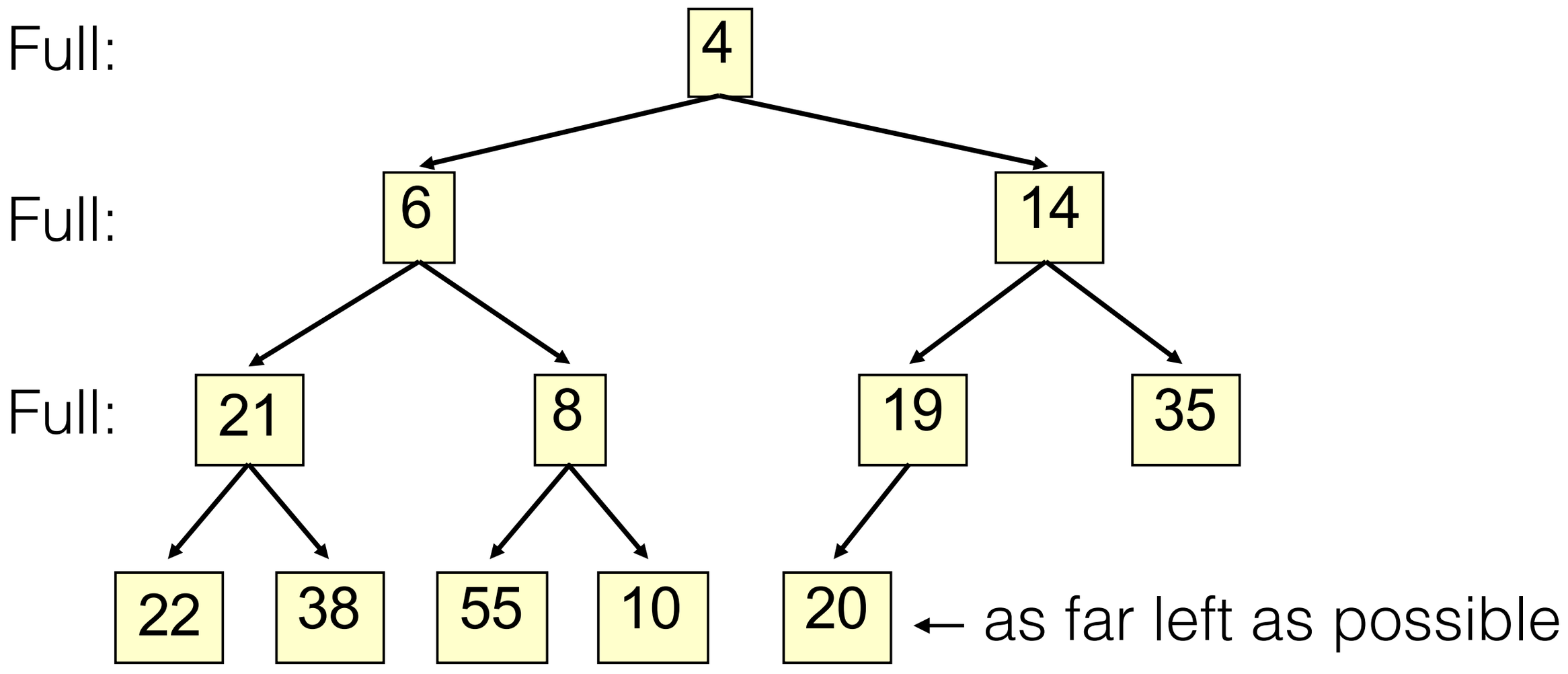
1. **Heap Order Invariant:**
Each element $\geq$ its parent.



A heap is a special binary tree.

2. **Complete:** no holes!

- All levels except the last are full.
- Nodes in last level are as far left as possible.



Heap operations

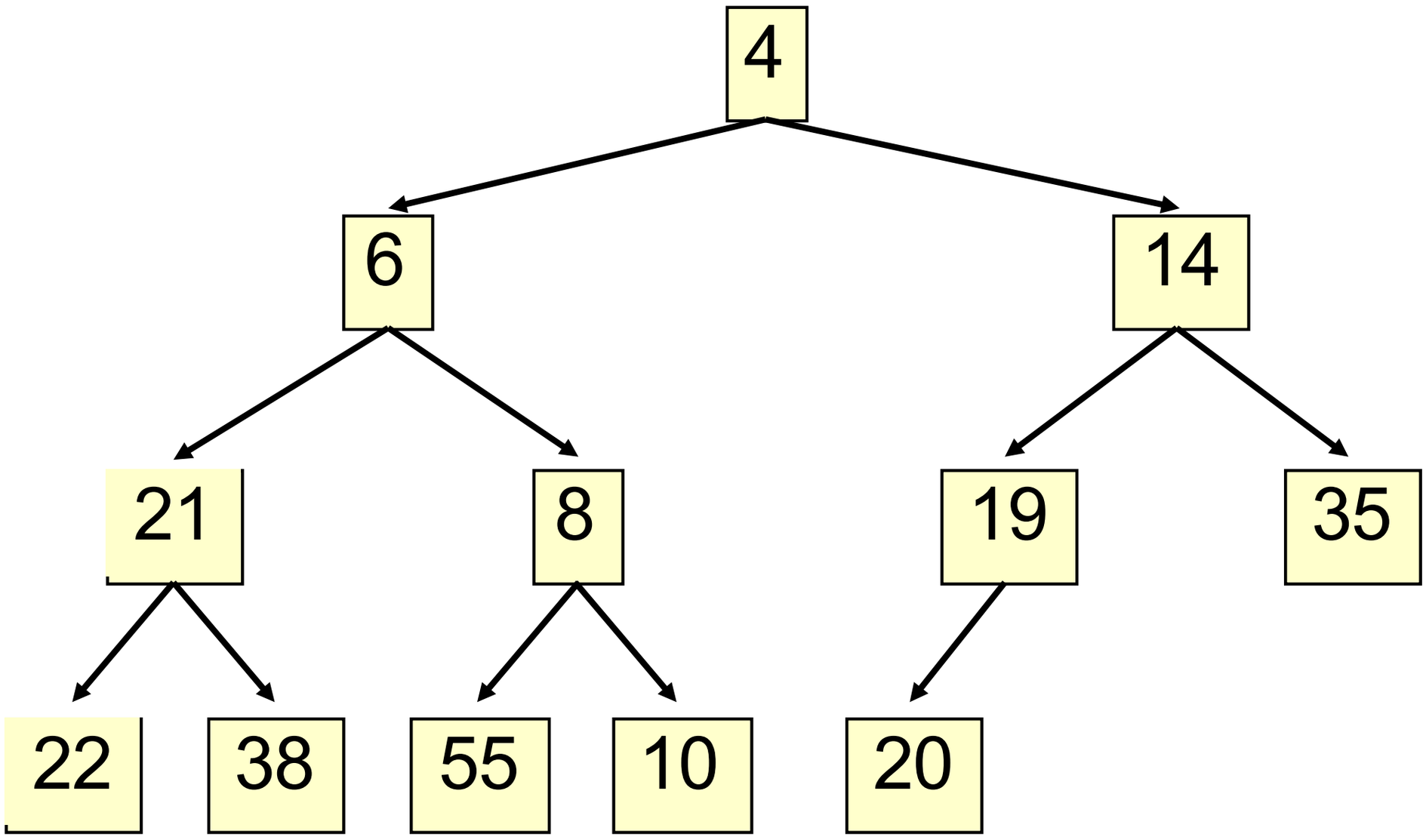
```
class Heap<E> {
  boolean add(E e); // insert e
  E peek(); // return min element
  E poll(); // remove/return min element
  void clear();
  boolean contains(E e);
  boolean remove(E e);
  int size();
  Iterator<E> iterator();
}
```

```
boolean add(E e);
```

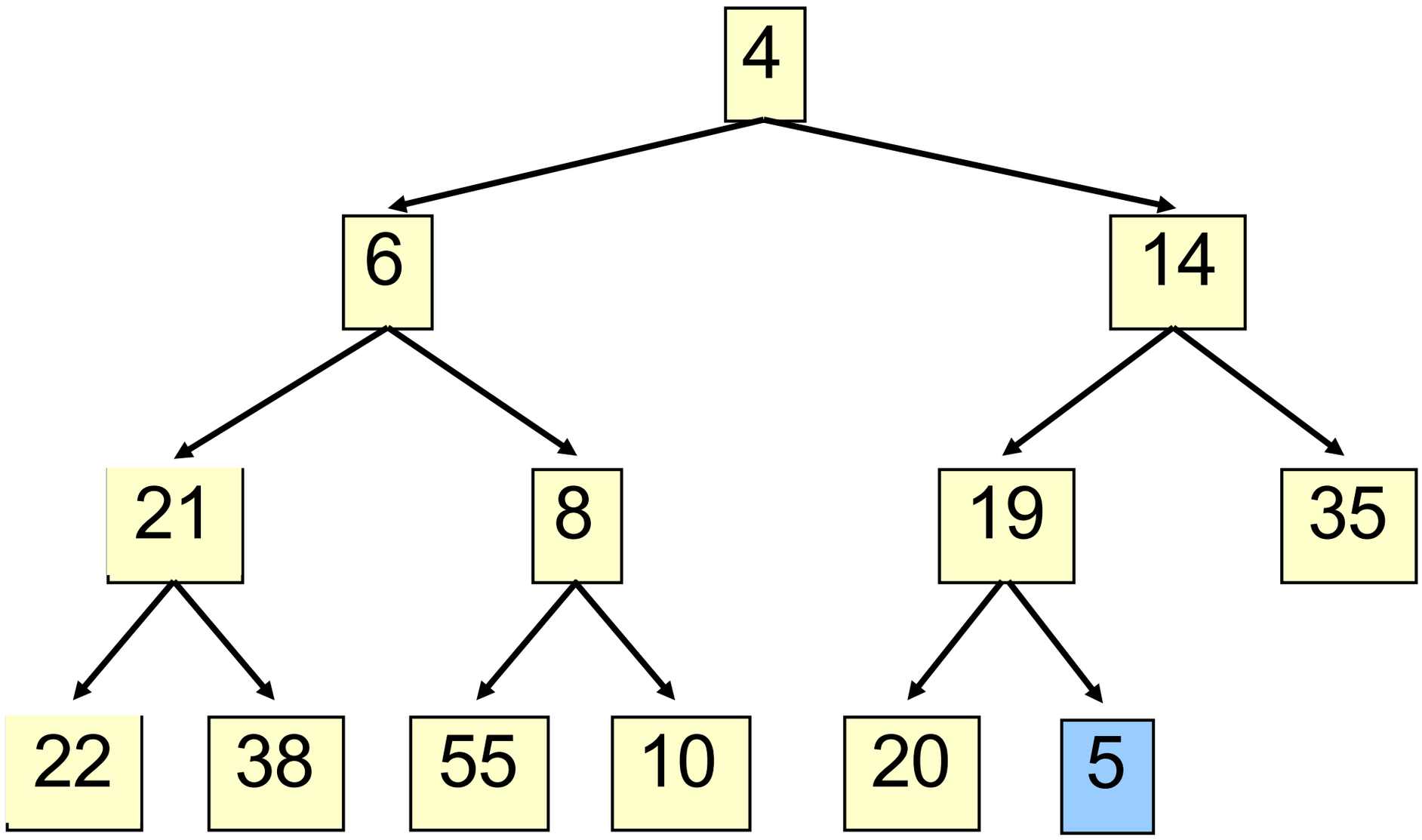
Algorithm:

- Add e in the wrong place
- While e is in the wrong place
  - move e towards the right place

```
boolean add(E e);
```



```
boolean add(E e);
```



```
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```

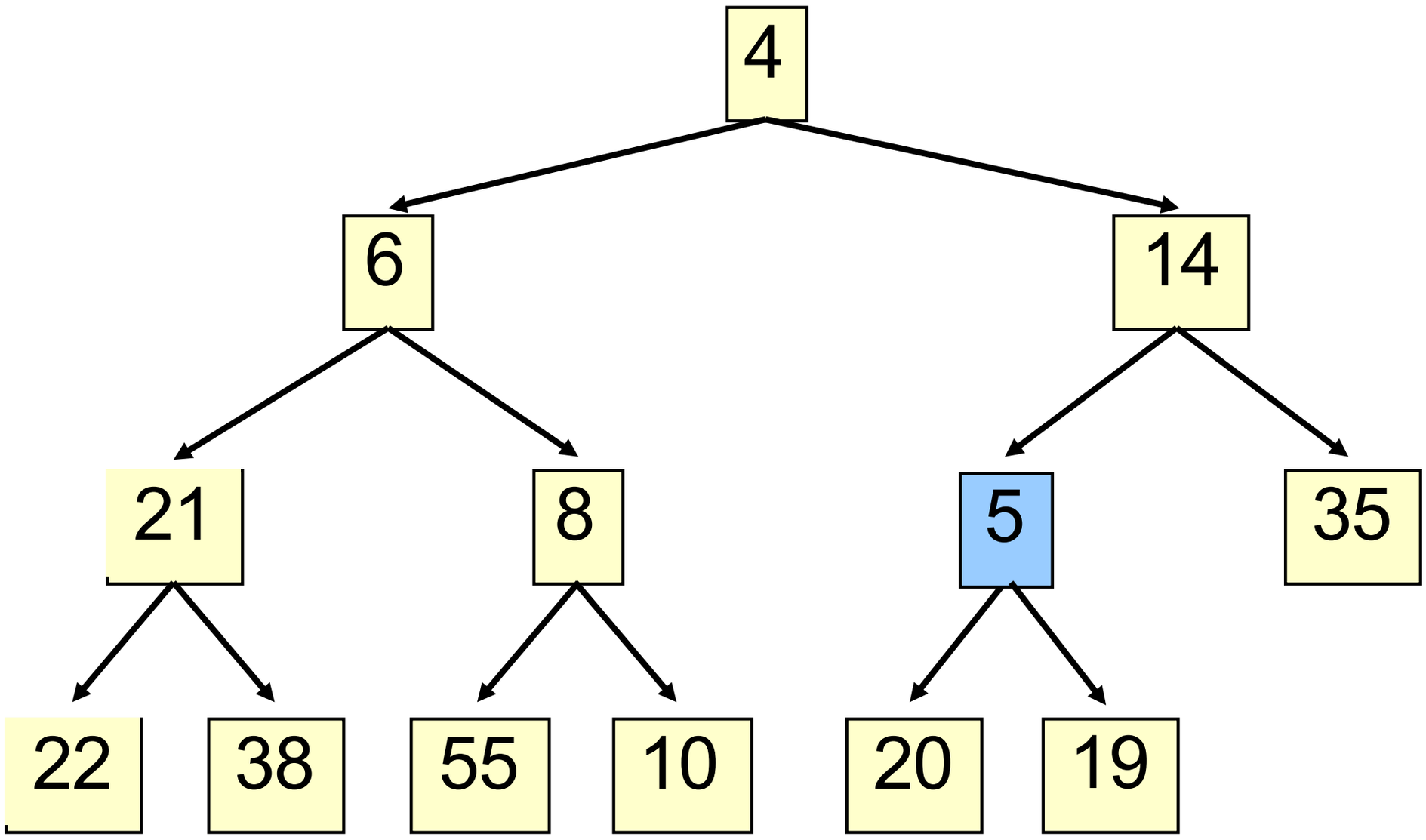

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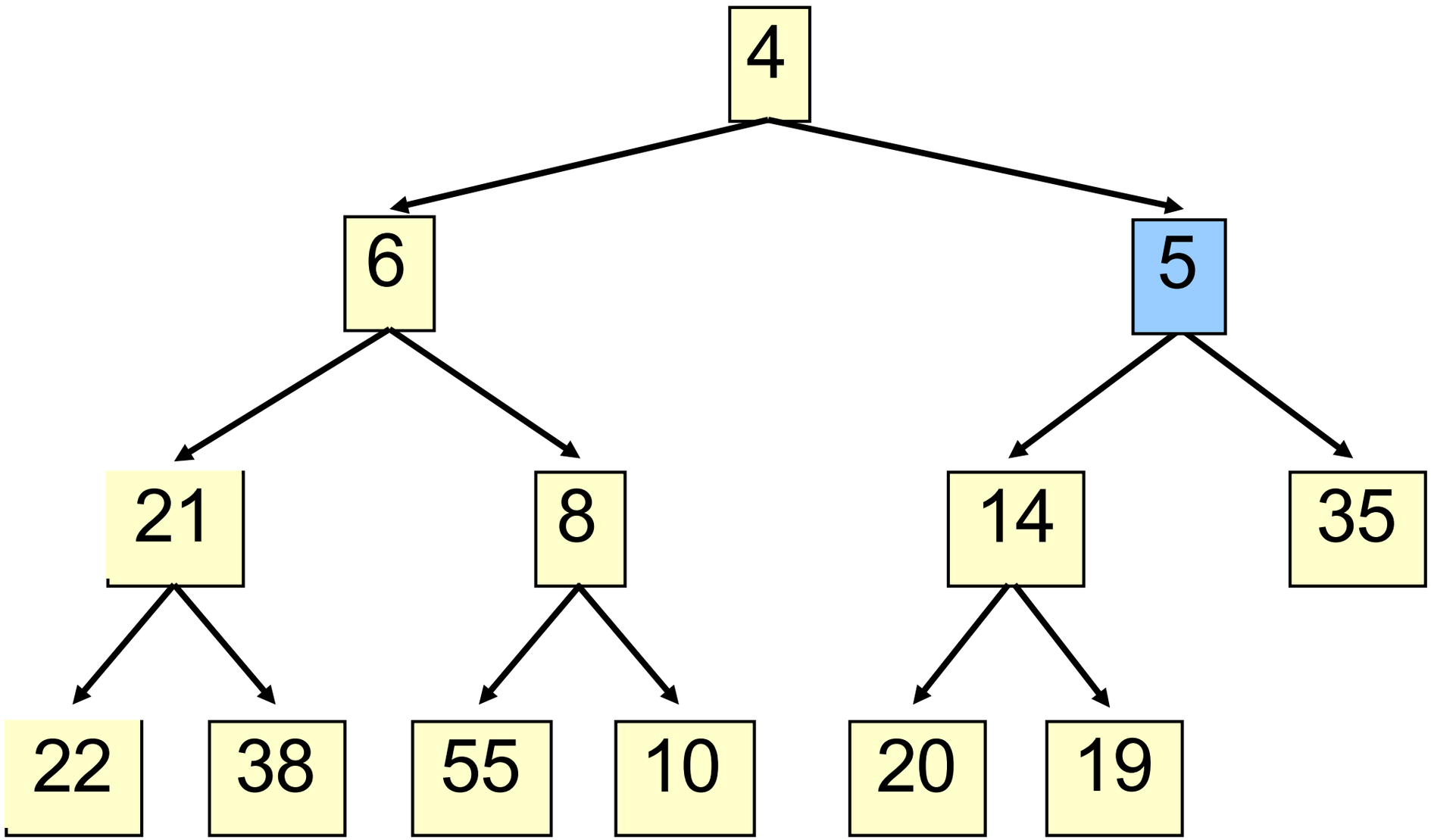

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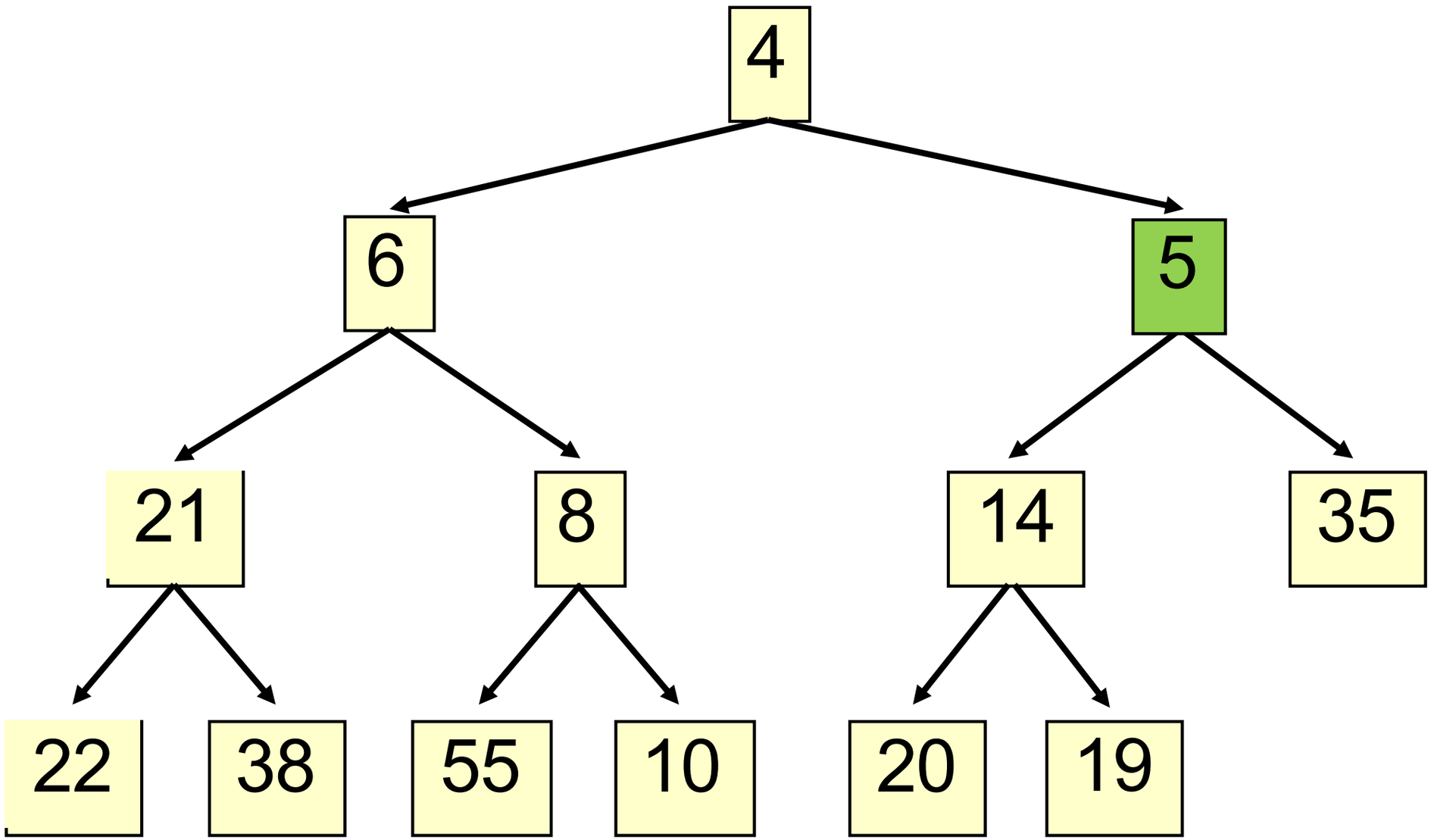

```
boolean add(E e);
```

Algorithm:

- Add e in the wrong place (the leftmost empty leaf)
- While e is in the wrong place (it is less than its parent)
  - move e towards the right place (swap with parent)

The heap invariant is maintained!

What's the runtime?

- O(number of swap/bubble operations)

= O(height of tree)

- Complete => balanced

    =>height is **O(log n)**

- Maximum number of swaps is O(log n)