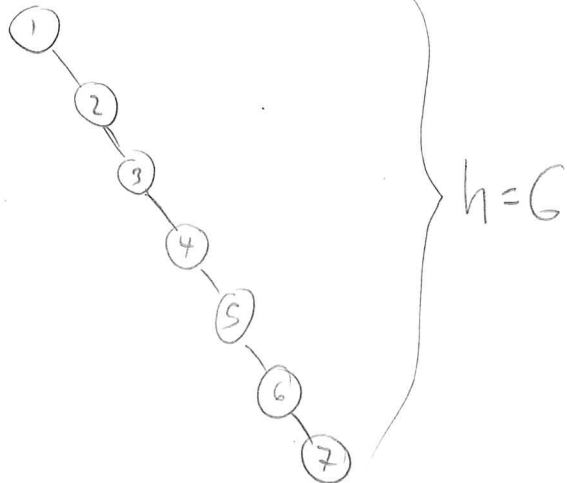


What's the height of a tree with n nodes?

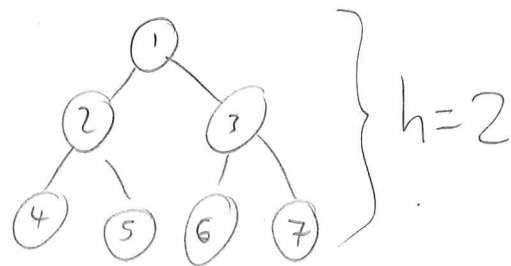
It depends! Consider an example: $n=7$

How tall can I make a tree with 7 nodes?



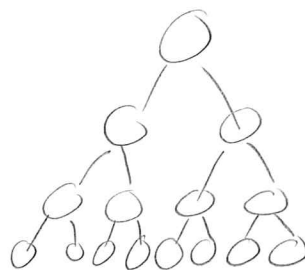
If runtime is $O(h)$, this is the worst case: $h=n-1$, which is $O(n)$.

How short can I make a tree with 7 nodes?



If runtime is $O(h)$, this is the best case. What is h in terms of n ?

Depth	# nodes @ d
0	1
1	2
2	4
3	8
\vdots	\vdots
d	2^d



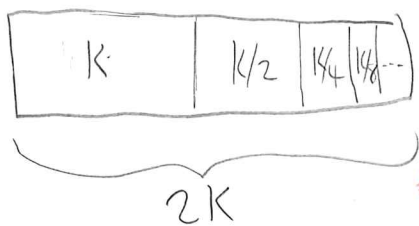
Suppose there are K nodes at depth h

Total nodes in the tree:

$$K + K/2 + K/4 + \dots + 1$$

Math fact: $K + K/2 + K/4 + \dots < 2K$

Proof:



nodes in lowest level

So total nodes in tree $< 2K$

At lowest level (depth h), at most 2^h nodes.

Total nodes in tree $< 2 \cdot 2^h = 2^{h+1}$

$$n = 2^{h+1}$$

$$\log_2 n = h+1$$

$$\log_2 n - 1 = h$$

h is $O(\log n)$.