CSCI 241

Lecture 10

Binary Search Trees: Removal, Balanced BSTs

Group Advising Session to Declare the CS Major!

3 - 4 pm on Thurs. Oct. 25 in CF 420

Course planning resources

Prepare for winter registration

Must have completed or be enrolled in CSCI 241, 247, and 301 in fall to be eligible

Major application deadline is Oct. 26 by 5 pm!

Email Mary.Hall@wwu.edu with any questions

For disability resources, contact 360-650-3083

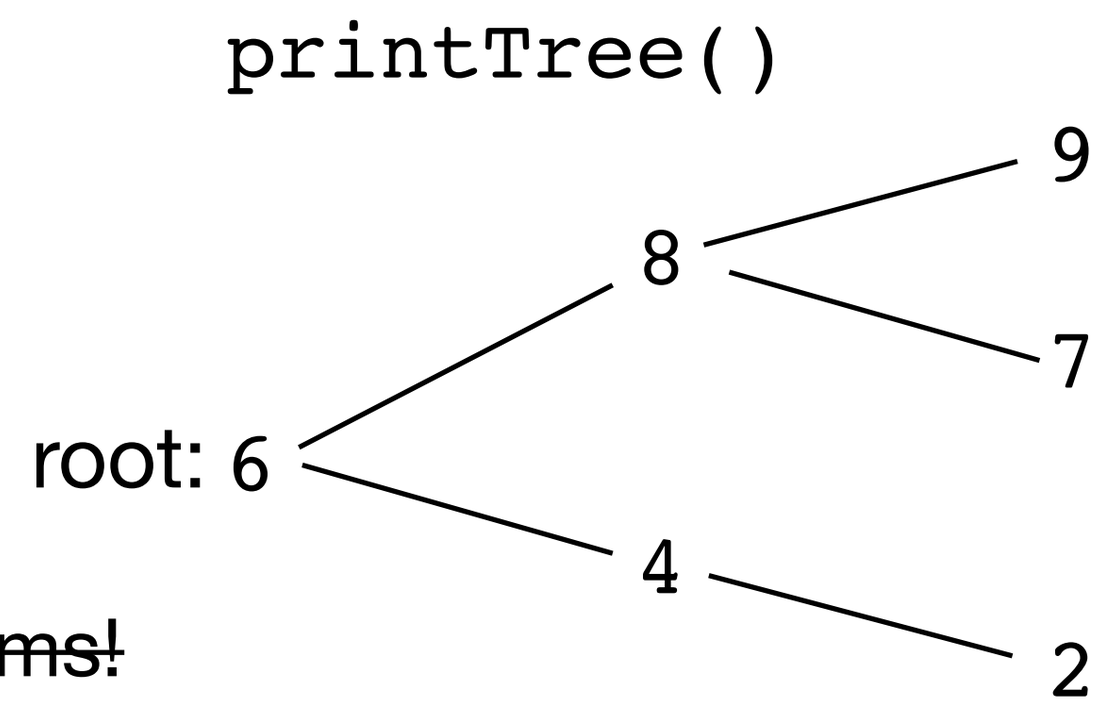
Announcements

- Quiz 2 grades: did you get the memo?

Announcements

Lab 3 printTree:

- up is right!
- left is down!
- ~~nothing is as it seems!~~



Printing a tree right-side up requires a **level-order** traversal (which isn't recursive and is implemented using a queue!)

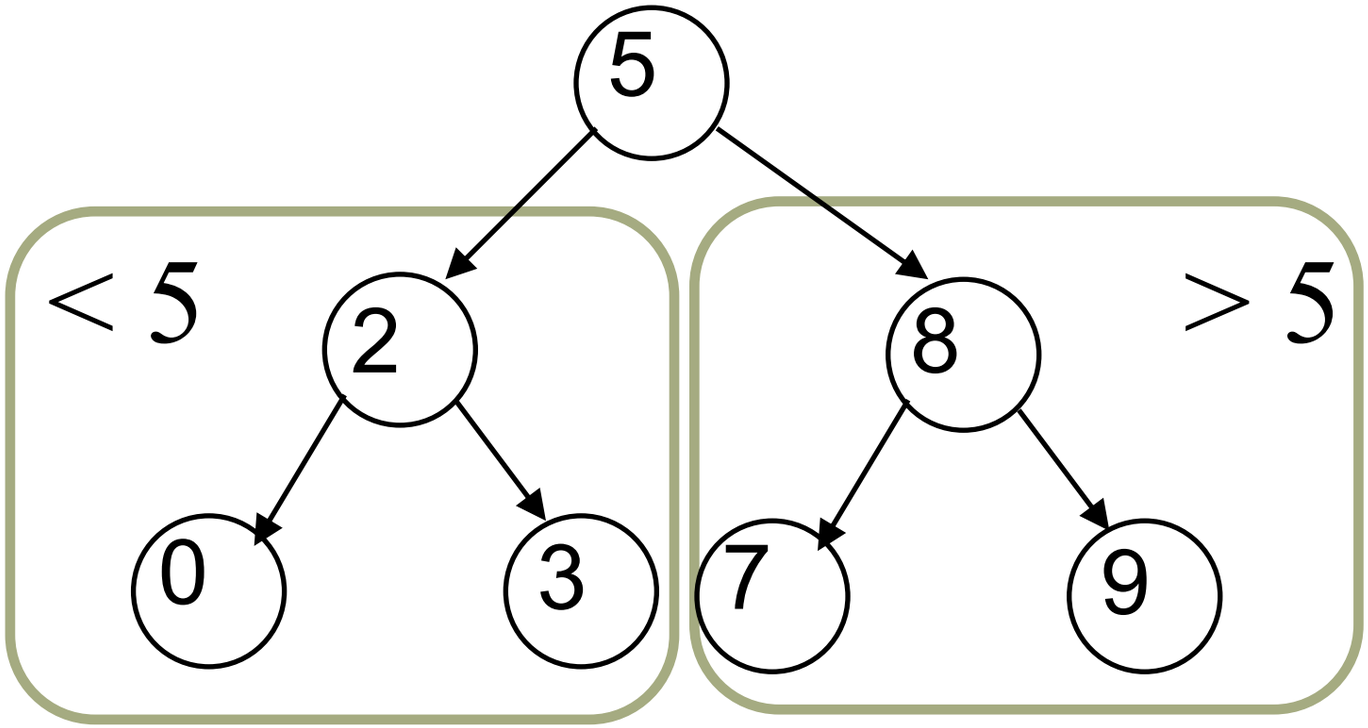
Goals (Monday and Today):

- Know the definition and uses of a binary search tree.
- Be prepared to implement, and know the runtime of, the following BST operations:
  - searching
  - inserting
  - deleting
- Know what a balanced BST is and why we want it.
- Be prepared to implement **rotations** in BSTs

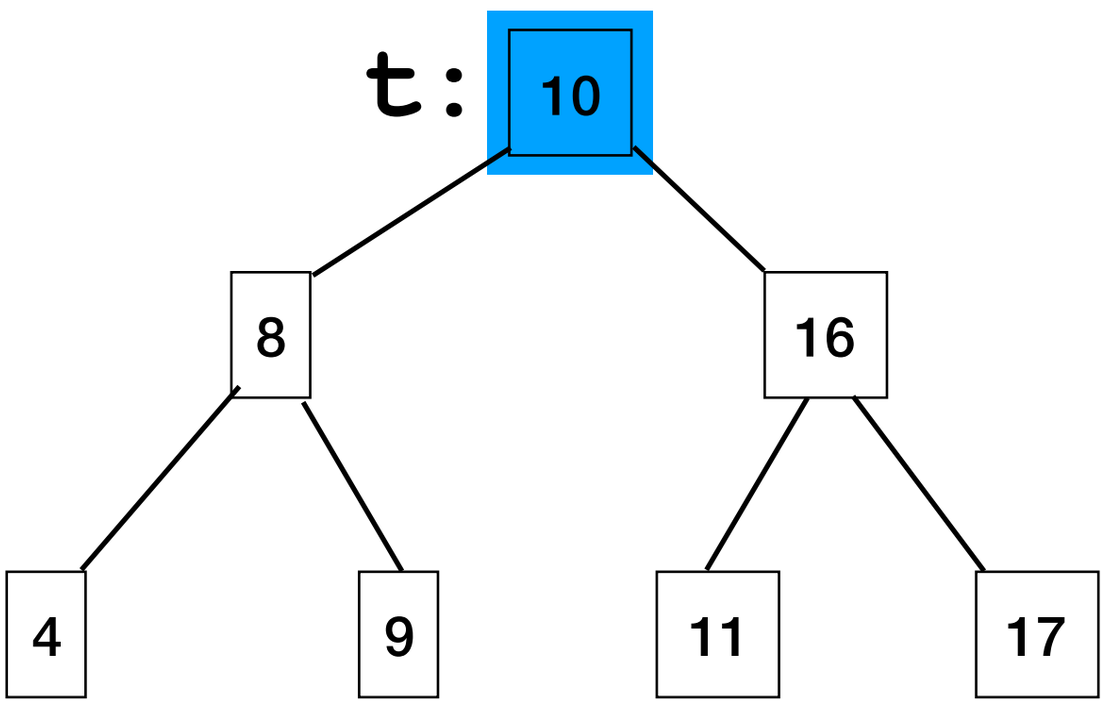
Binary Search Tree

```
/** BST: a binary tree, in which:
  * -all values in left are < value
  * -all values in right are > value
  * -left and right are BSTs */
public class BST {
  int value;
  BST parent;
  BST left;
  BST right;
}
```

Binary Search Tree



Searching a BST

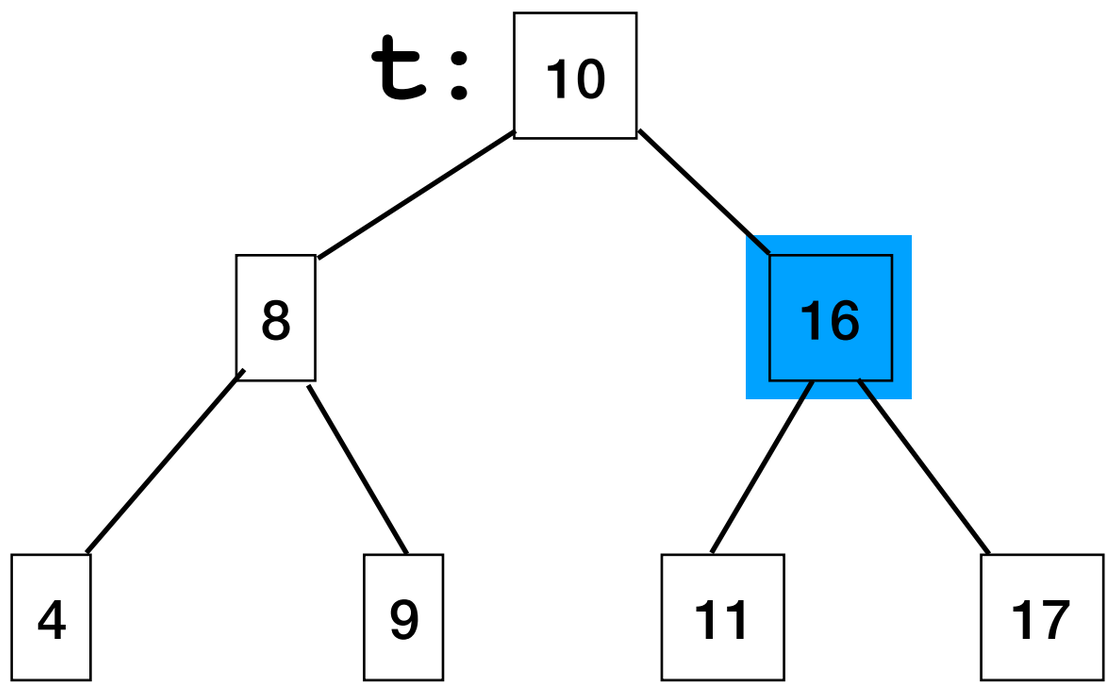



```
search(t, 11)
```

11 > 10

```
search(right, 11)
```

Searching a BST




```
search(t, 11)
```

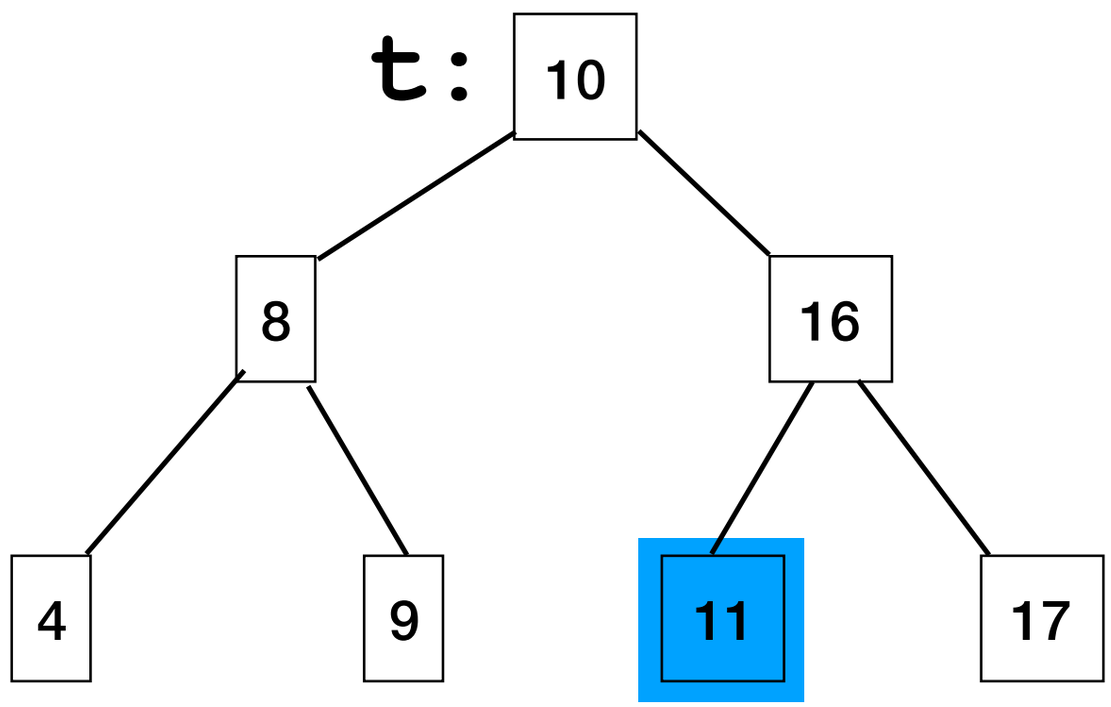
11 > 10

```
search(right, 11)
```

11 < 16

```
search(left, 11)
```

Searching a BST




```
search(t, 11)
```

11 > 10

```
search(right, 11)
```

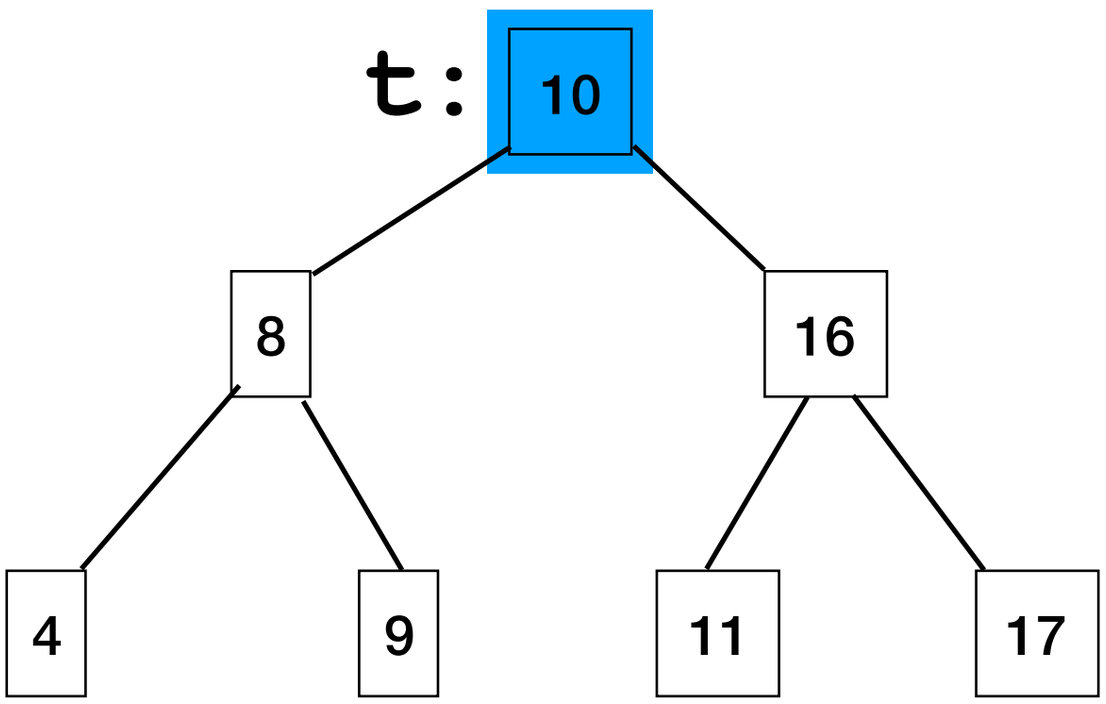
11 < 16

```
search(left, 11)
```

11 == 11

```
found it! return.
```

Searching a BST - the nonexistent case

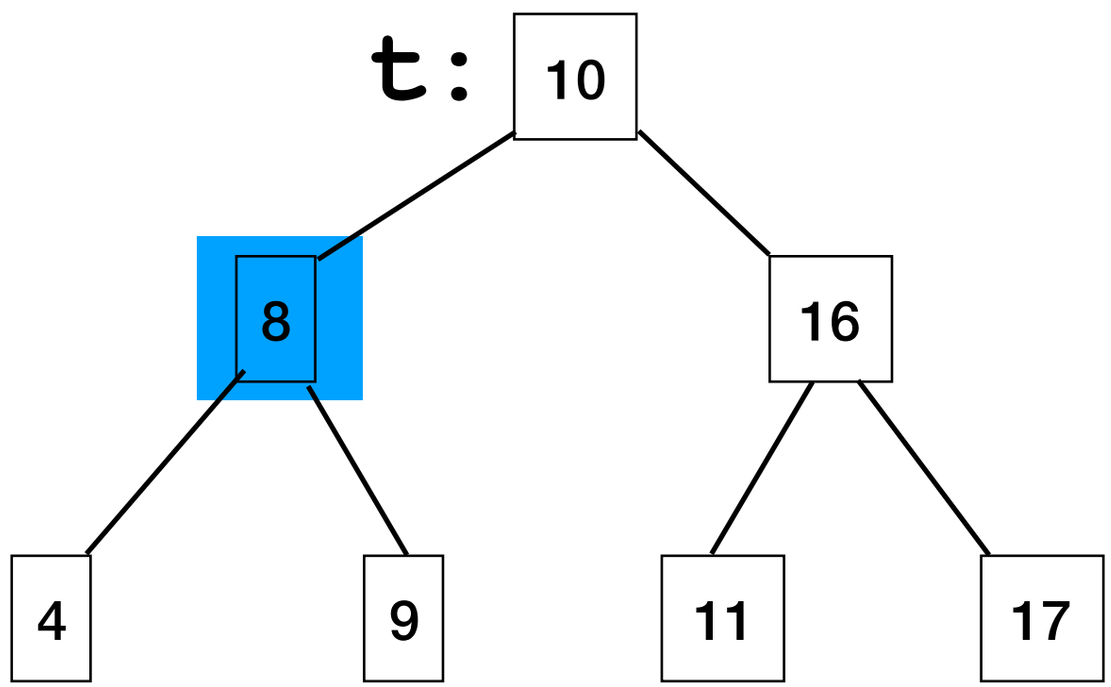



```
search(t, 5)
```

$5 < 10$

```
search(left, 5)
```

Searching a BST - the nonexistent case




```
search(t, 5)
```

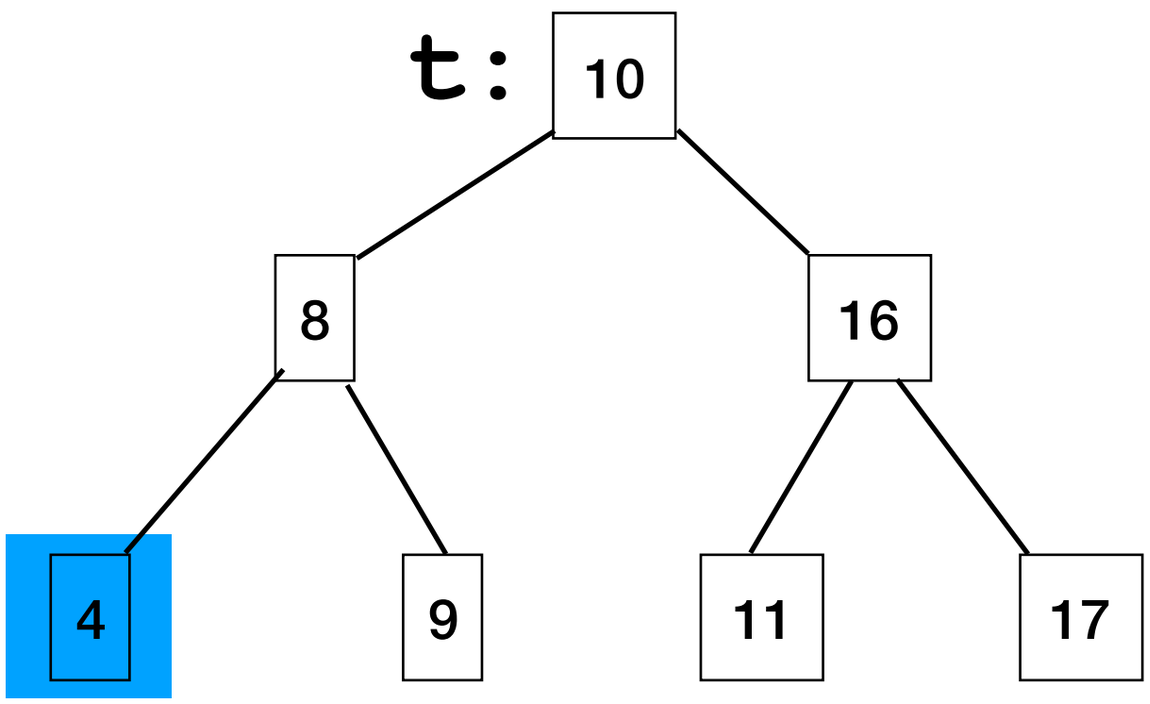
$5 < 10$

```
search(left, 5)
```

$5 < 8$

```
search(left, 5)
```

Searching a BST - the nonexistent case




```
search(t, 5)
```

$5 < 10$

```
search(left, 5)
```

$5 < 8$

```
search(left, 5)
```

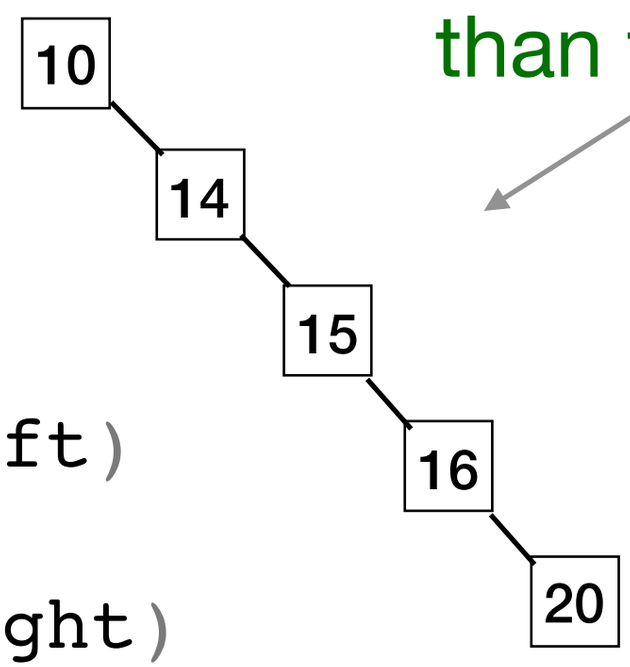
$5 > 4$

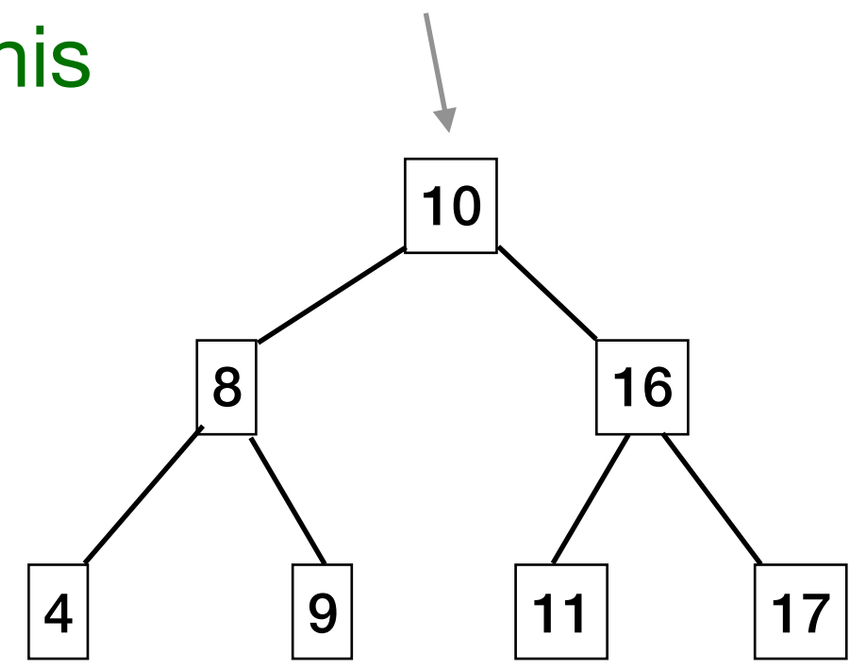
```
search(right, 5)
null - not found!
```

Searching a BST: What's the runtime?

```
boolean search(BST t, int v):
  if t == null:
    return false
  if t.value == v:
    return true
  if t.value < v:
    return search(t.left)
  else:
    return search(t.right)
```

We want our trees to look more like this than this

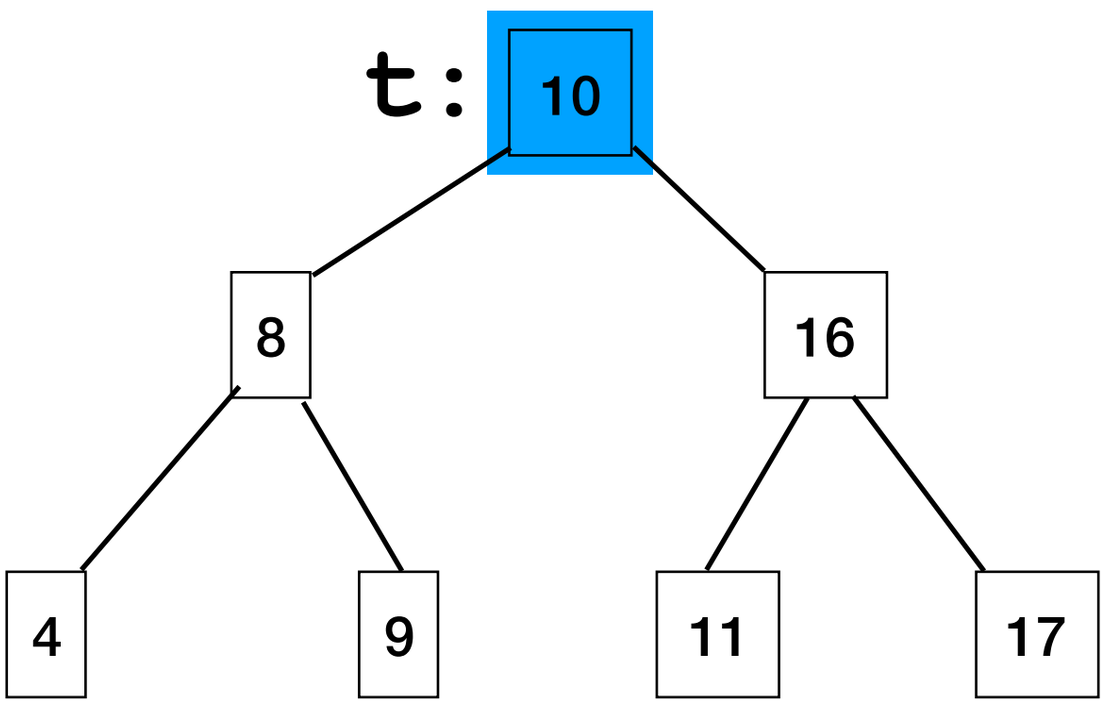




Runtime of search is O(h). Worst: **O(n)** Best: **O(log n)**

Inserting into a BST

Inserting into a BST

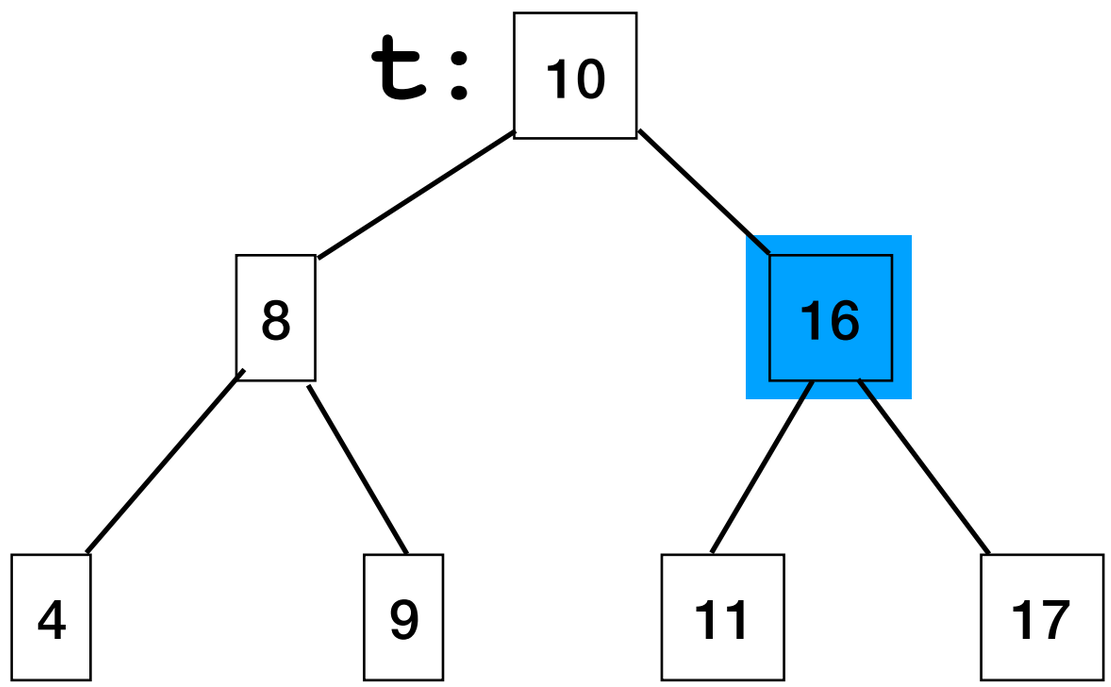



```
insert(t, 11)
```

11 > 10

```
insert(right, 11)
```

Inserting into a BST



`insert(t, 11)`

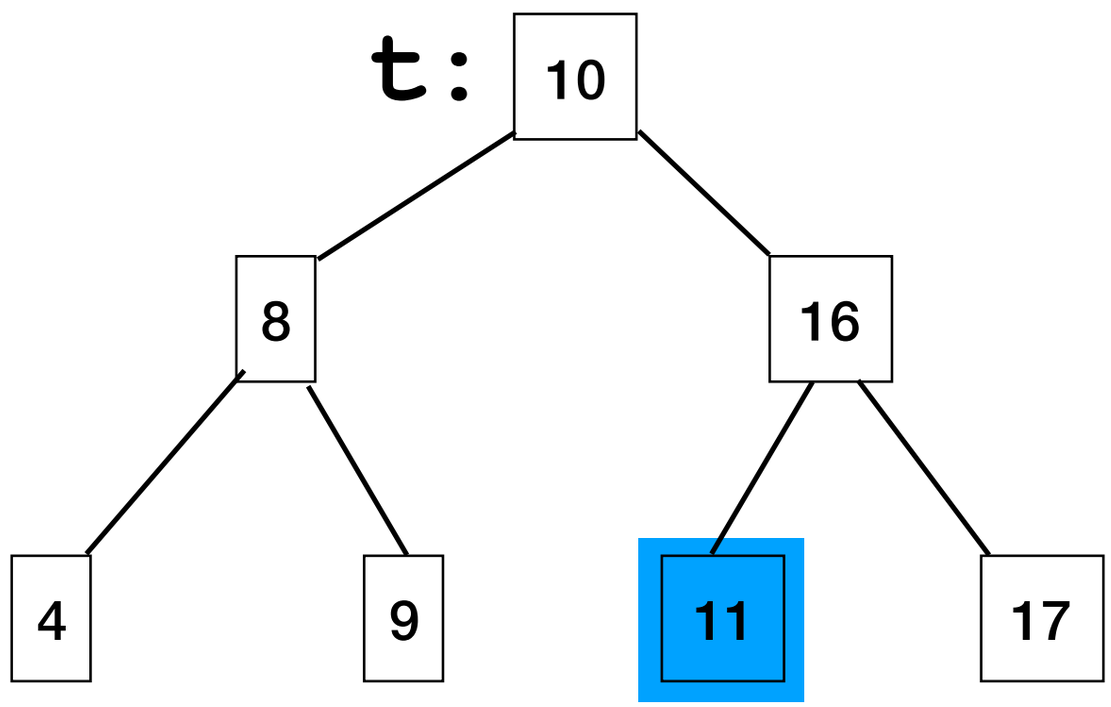
11 > 10

`insert(right, 11)`

11 < 16

`insert(left, 11)`

Inserting into a BST




```
insert(t, 11)
```

11 > 10

```
insert(right, 11)
```

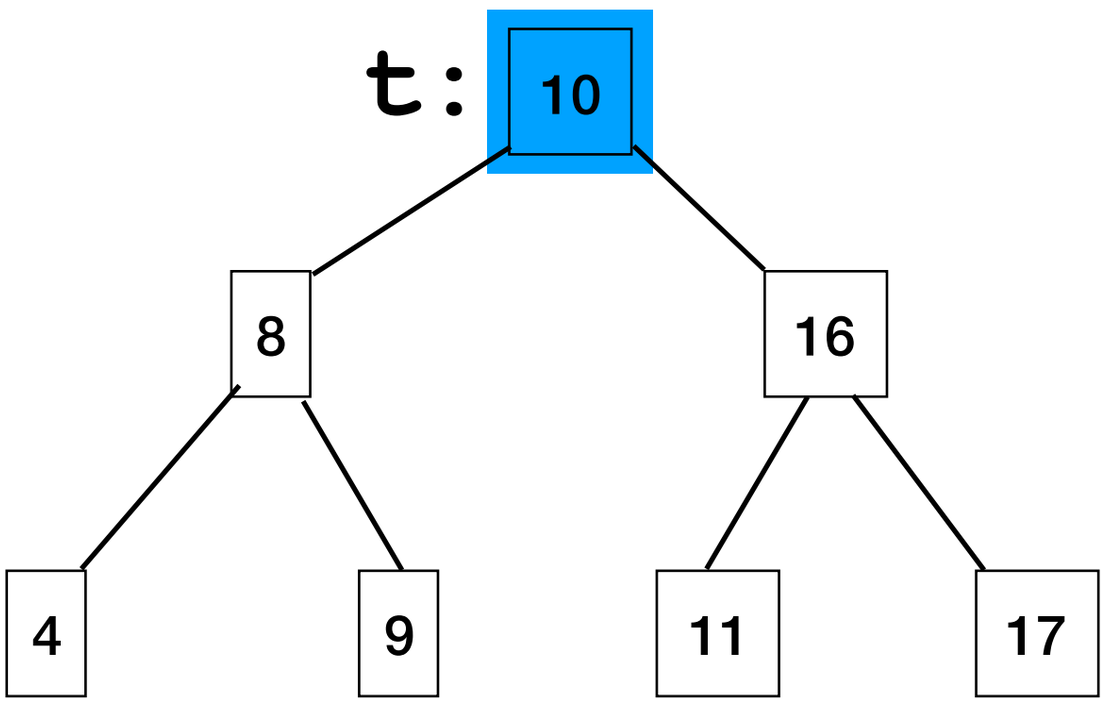
11 < 16

```
insert(left, 11)
```

11 == 11

```
found it! no duplicates,
allowed; nothing to do.
return.
```

Inserting into a BST - the nonexistent case

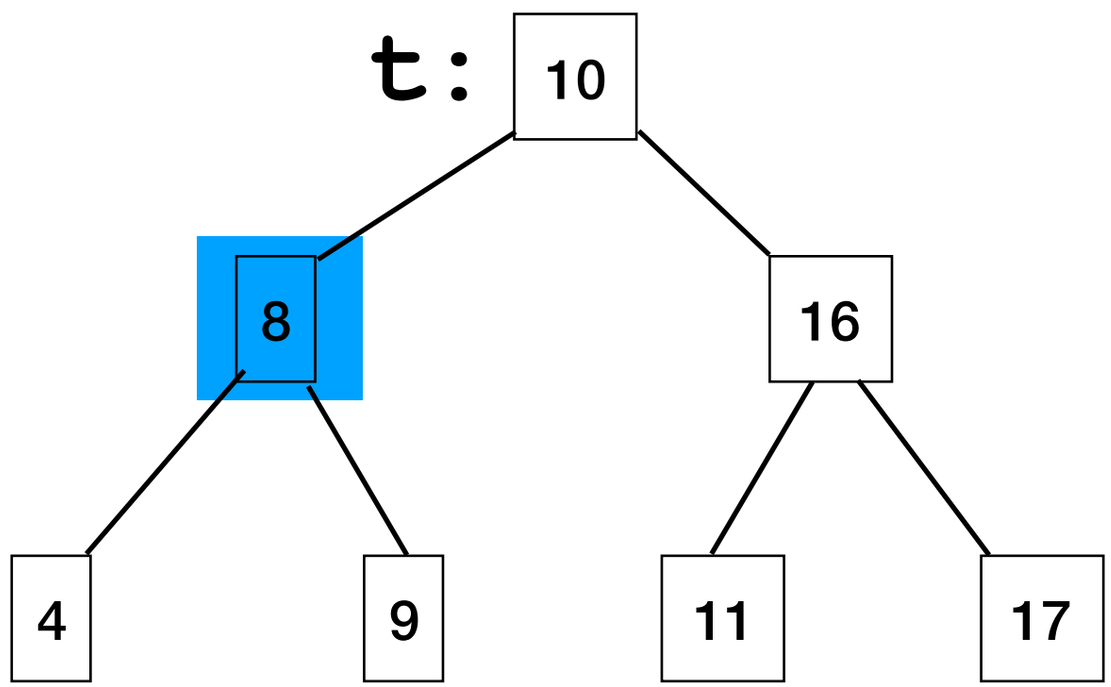



```
insert(t, 5)
```

$5 < 10$

```
insert(left, 5)
```

Inserting into a BST - the nonexistent case



`insert(t, 5)`

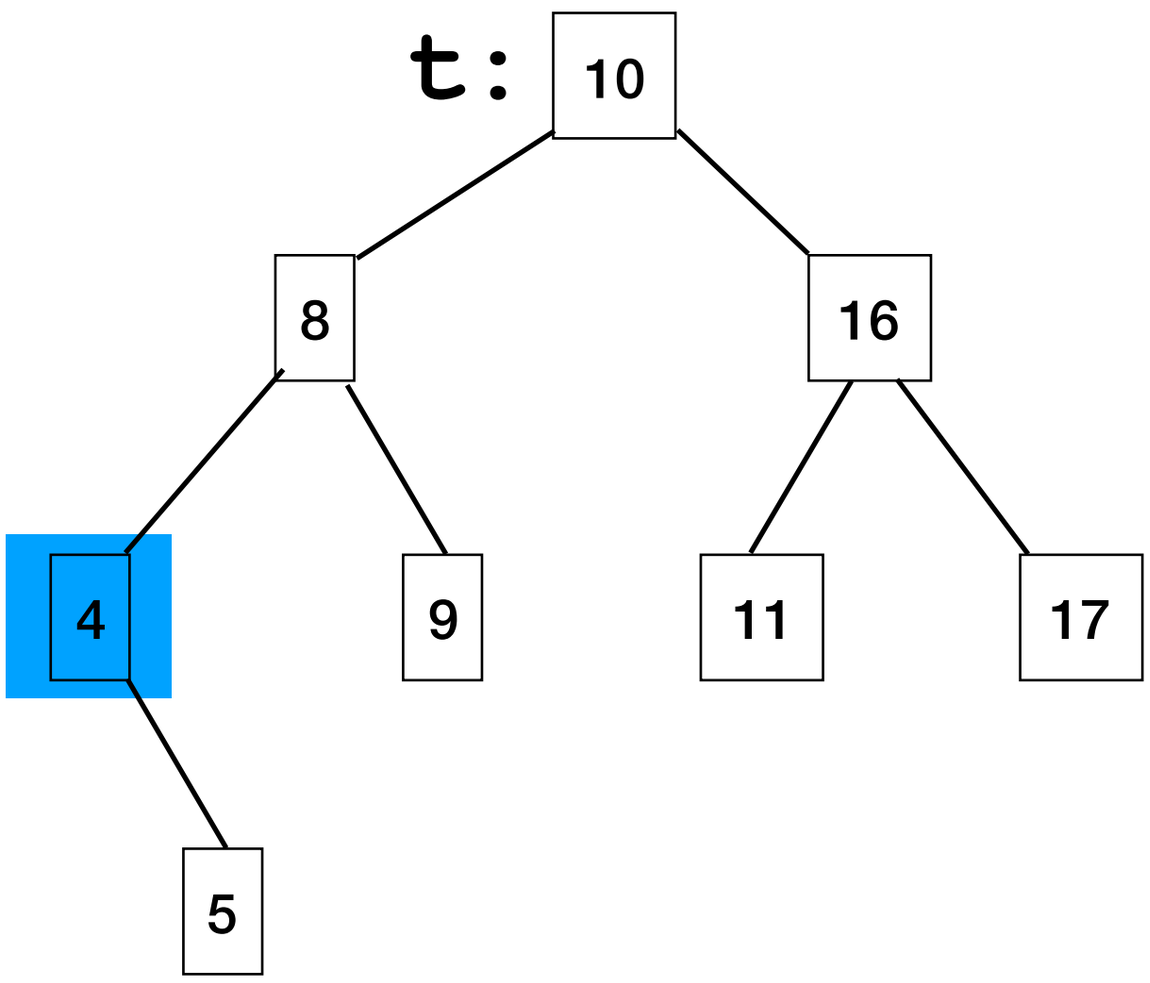
$5 < 10$

`insert(left, 5)`

$5 < 8$

`insert(left, 5)`

Inserting into a BST - the nonexistent case



`insert(t, 5)`

$5 < 10$

`insert(left, 5)`

$5 < 8$

`insert(left, 5)`

$5 > 4$

`insert(right, 5)`

`null - not found. insert it here!`

Warm-up

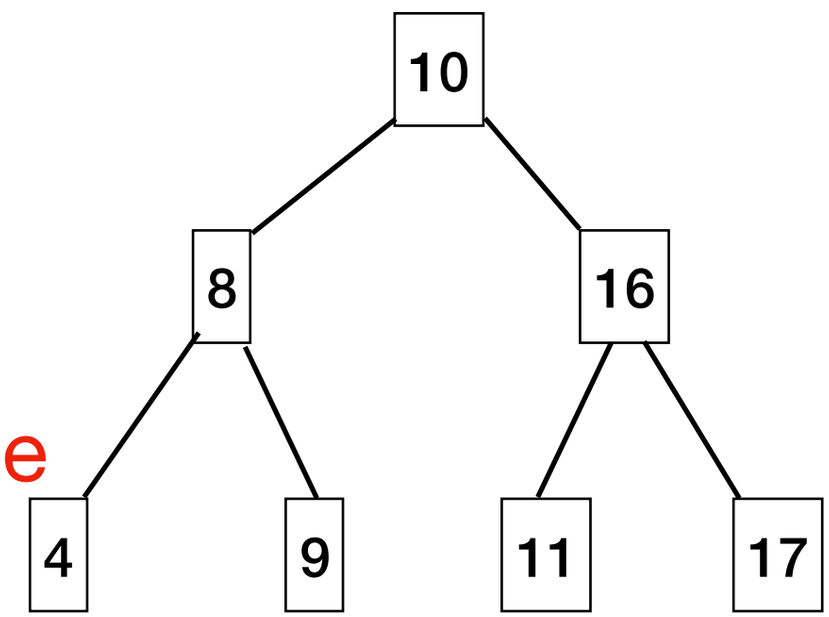
Write a method to find the smallest value in a BST:

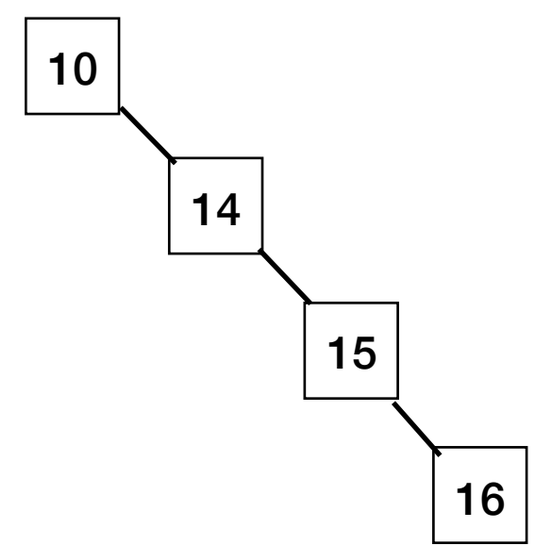
1. Spec

```
/** min value in n's subtree.
  * pre: n is not null */
public int minimum(Node n) {

}
```

2. Base case





Warm-up

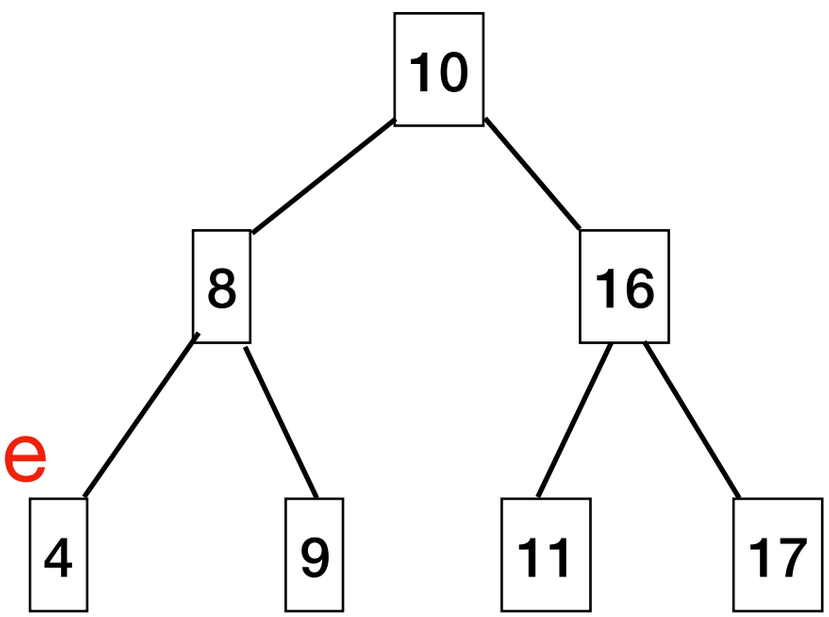
Write a method to find the smallest value in a BST:

1. Spec

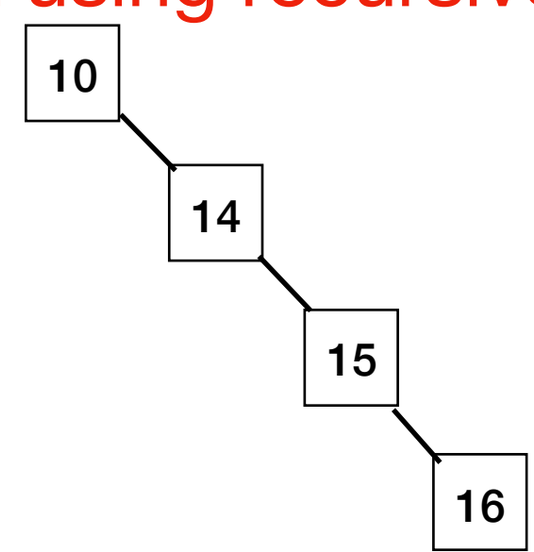
```
/** min value in n's subtree.
  * pre: n is not null */
public int minimum(Node n) {
  if (n.left == null)
    return n.value;

}
```

2. Base case



4. Implement using recursive call



3. Recursive definition:
Smallest(n) is the smallest value in the left subtree, or n.value if no left subtree exists.

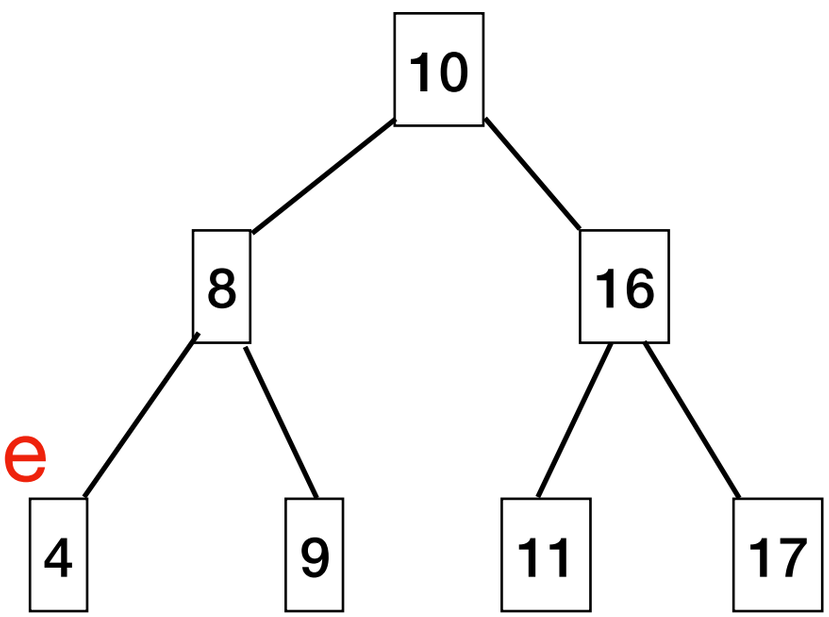
Warm-up

Write a method to find the smallest value in a BST:

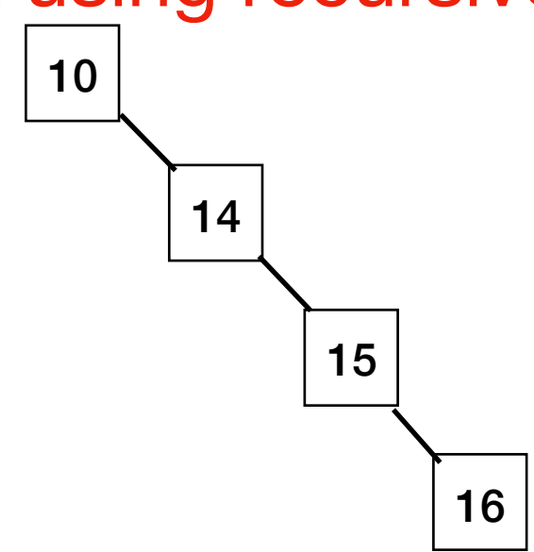
1. Spec

```
/** min value in n's subtree.
  * pre: n is not null */
public int minimum(Node n) {
  if (n.left == null)
    return n.value;
  return minimum(n.left);
}
```

2. Base case



4. Implement using recursive call

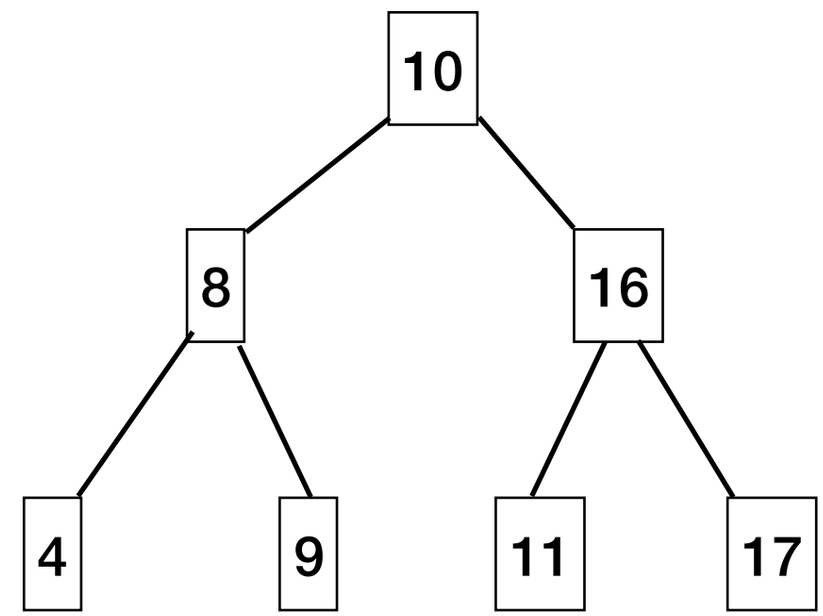


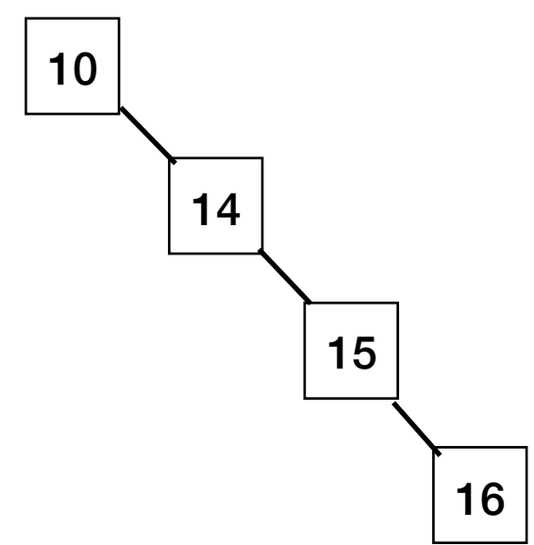
3. Recursive definition:
Smallest(n) is the smallest value in the left subtree, or n.value if no left subtree exists.

Warm-up

Write a method to find the smallest value in a BST:

```
/** min value in n's subtree.
  * pre: n is not null */
public int minimum(Node n) {
  if (n.left == null)
    return n.value;
  return minimum(n.left);
}
```

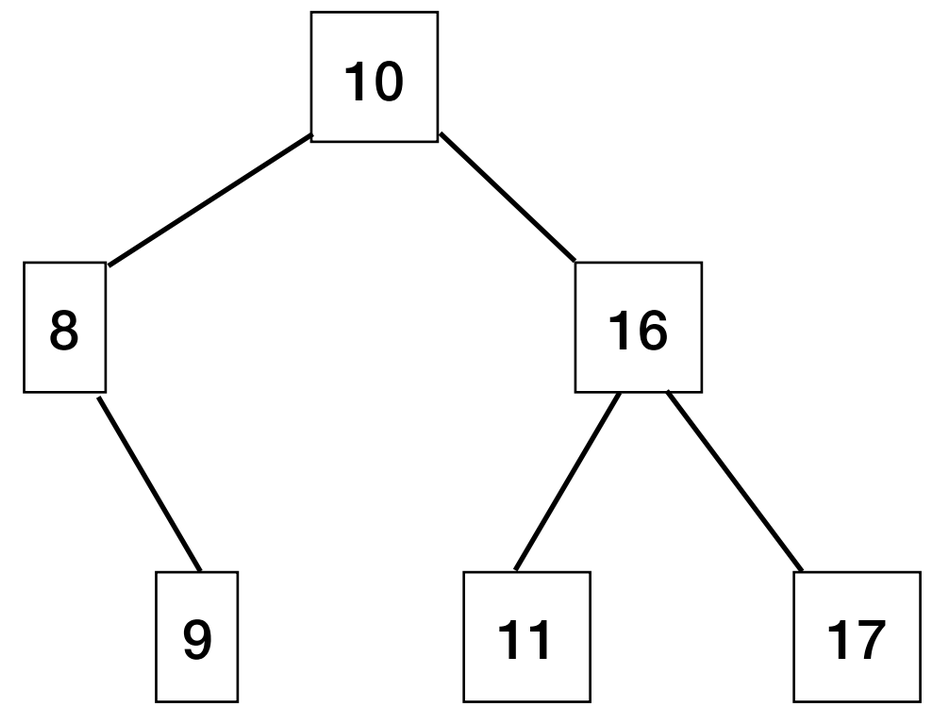




Deleting a node from a BST

Three possible cases:

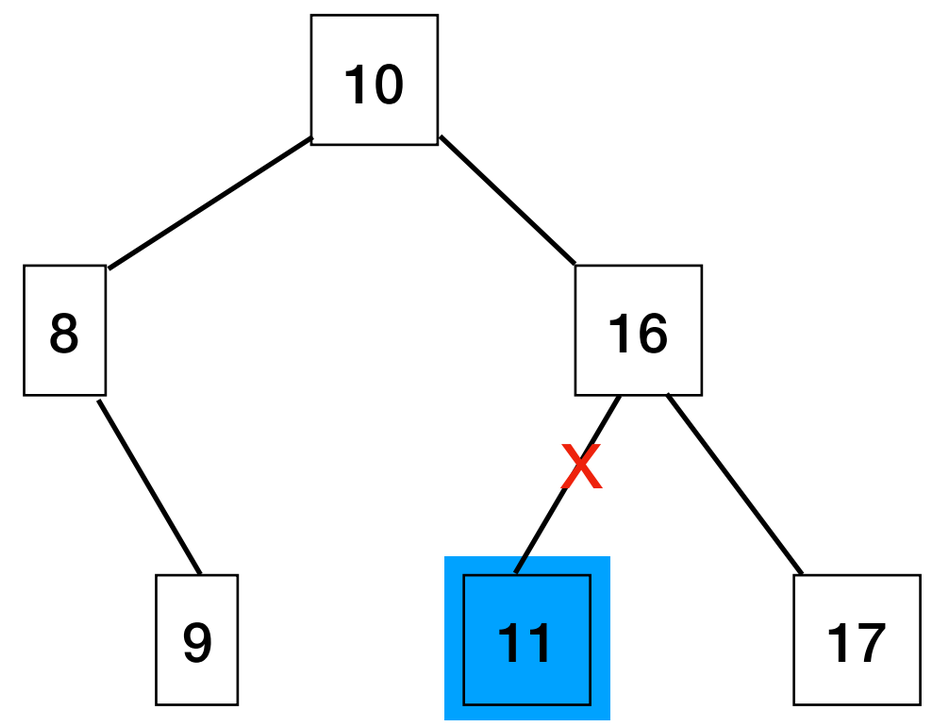
1. n has no children (is a leaf)
2. n has one child
3. n has two children



Deleting a node from a BST: Case 1

Three possible cases:

1. **n has no children (is a leaf)**
2. n has one child
3. n has two children

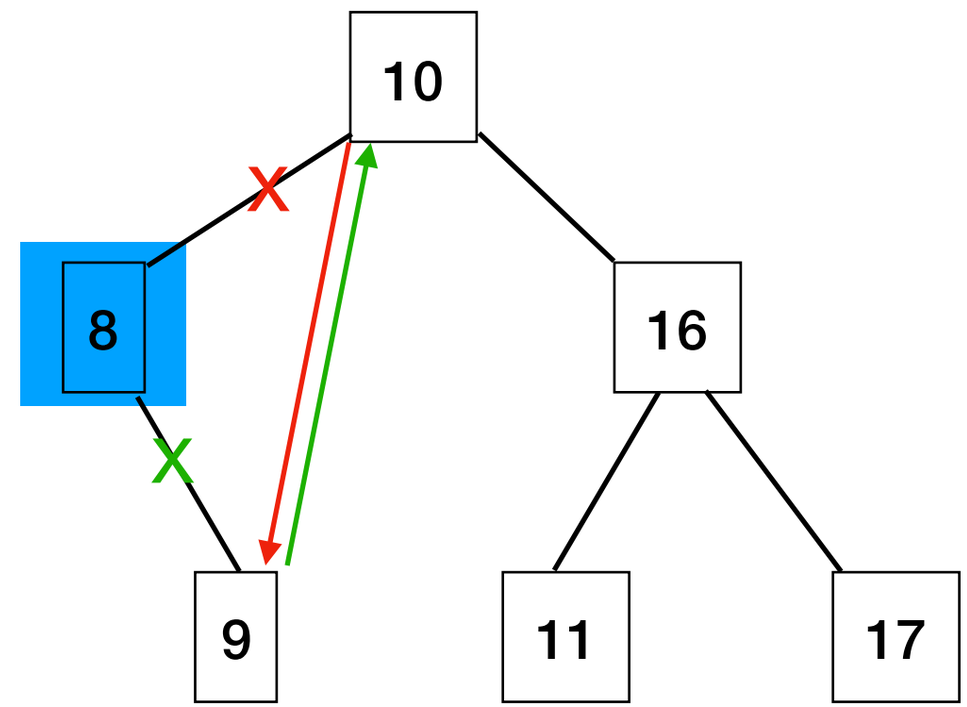



```
if (n is a leaf)
   replace parent's child with null
```

Deleting a node from a BST: Case 2

Three possible cases:

1. n has no children (is a leaf)
2. **n has one child**
3. n has two children

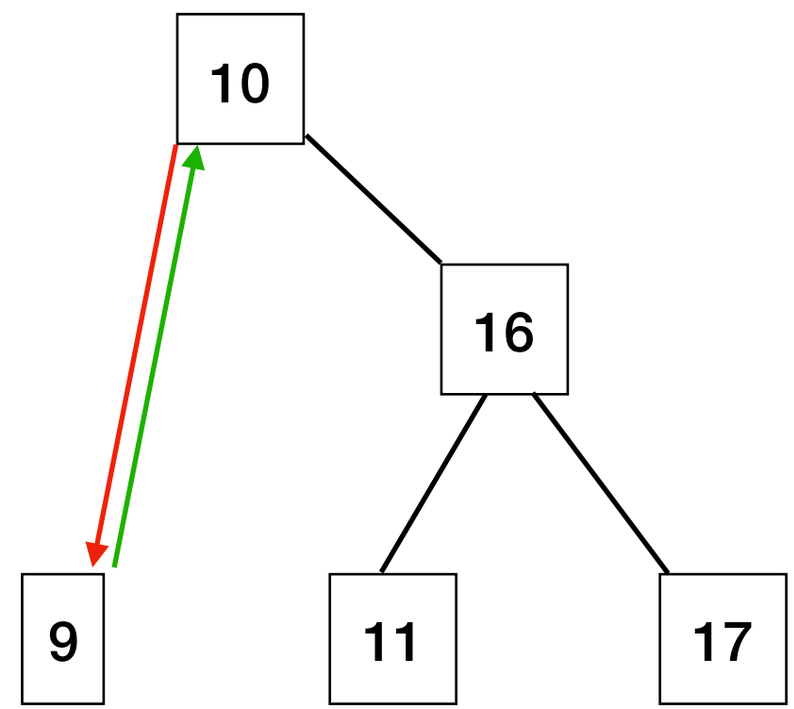



```
if (n has exactly one child)
  replace parent's child with n's child
  replace n's child's parent with n's parent
```

Deleting a node from a BST: Case 2

Three possible cases:

1. n has no children (is a leaf)
2. **n has one child**
3. n has two children

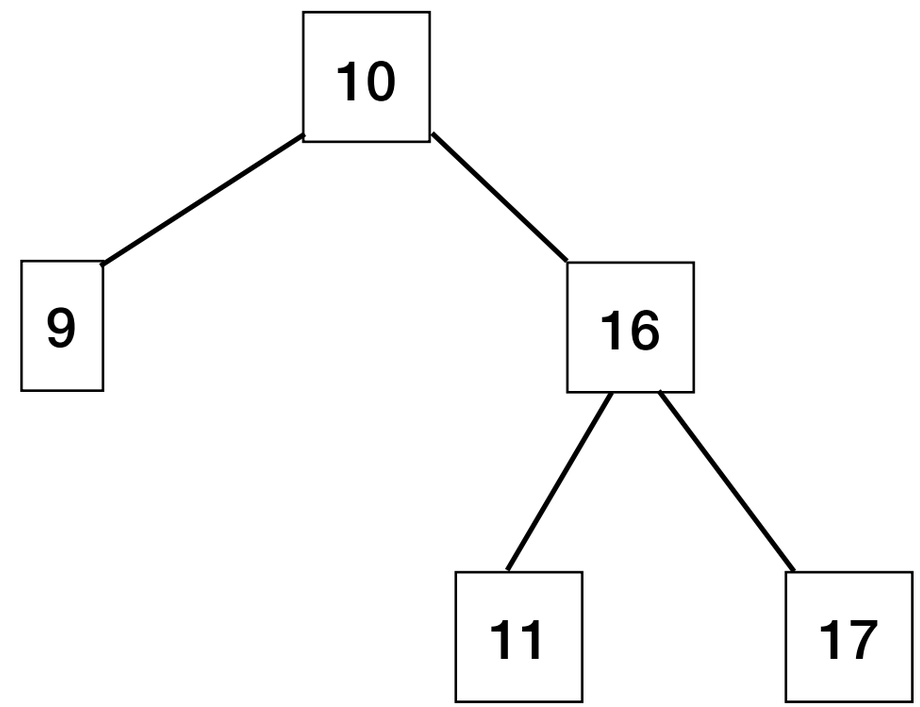



```
if (n has exactly one child)
  replace parent's child with n's child
  replace n's child's parent to n's parent
```

Deleting a node from a BST: Case 2

Three possible cases:

1. n has no children (is a leaf)
2. **n has one child**
3. n has two children



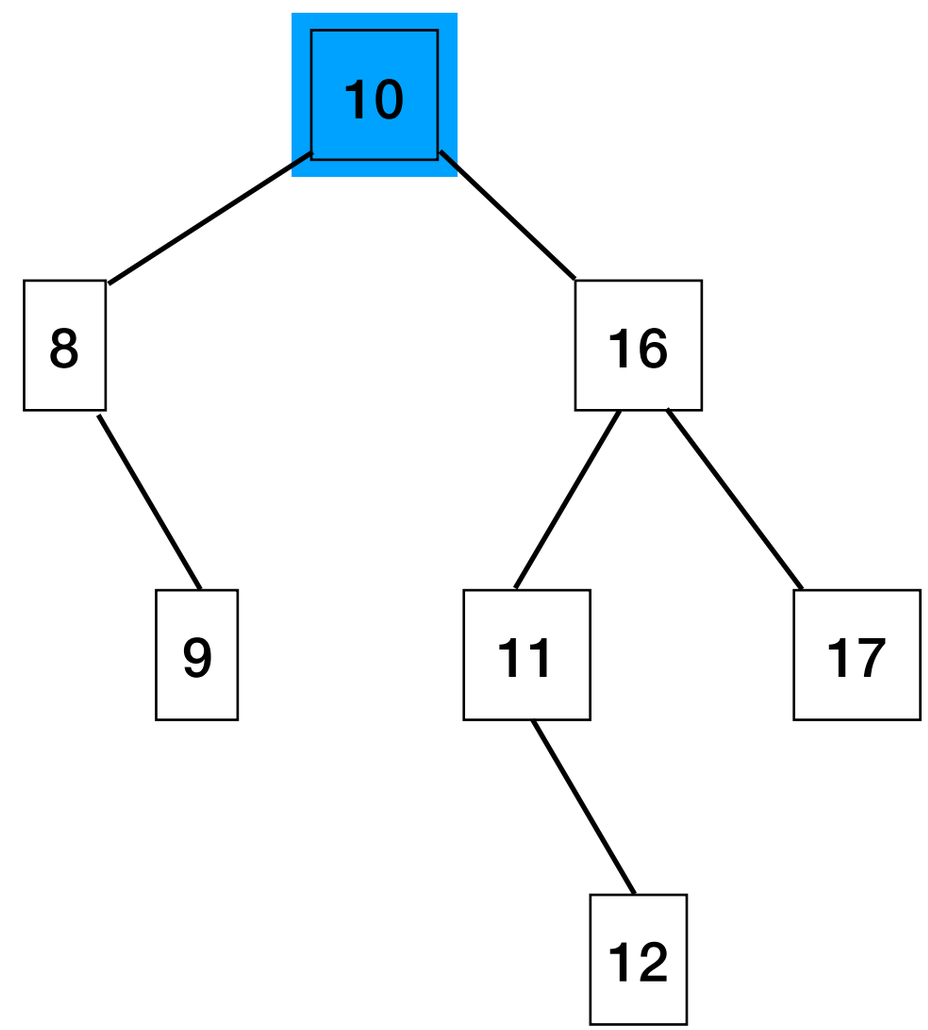

```
if (n has exactly one child)
  replace parent's child with n's child
  replace n's child's parent to n's parent
```

Deleting a node from a BST: Case 3

Three possible cases:

1. n has no children (is a leaf)
2. n has one child
3. **n has two children**

```
if (n has two children)
```

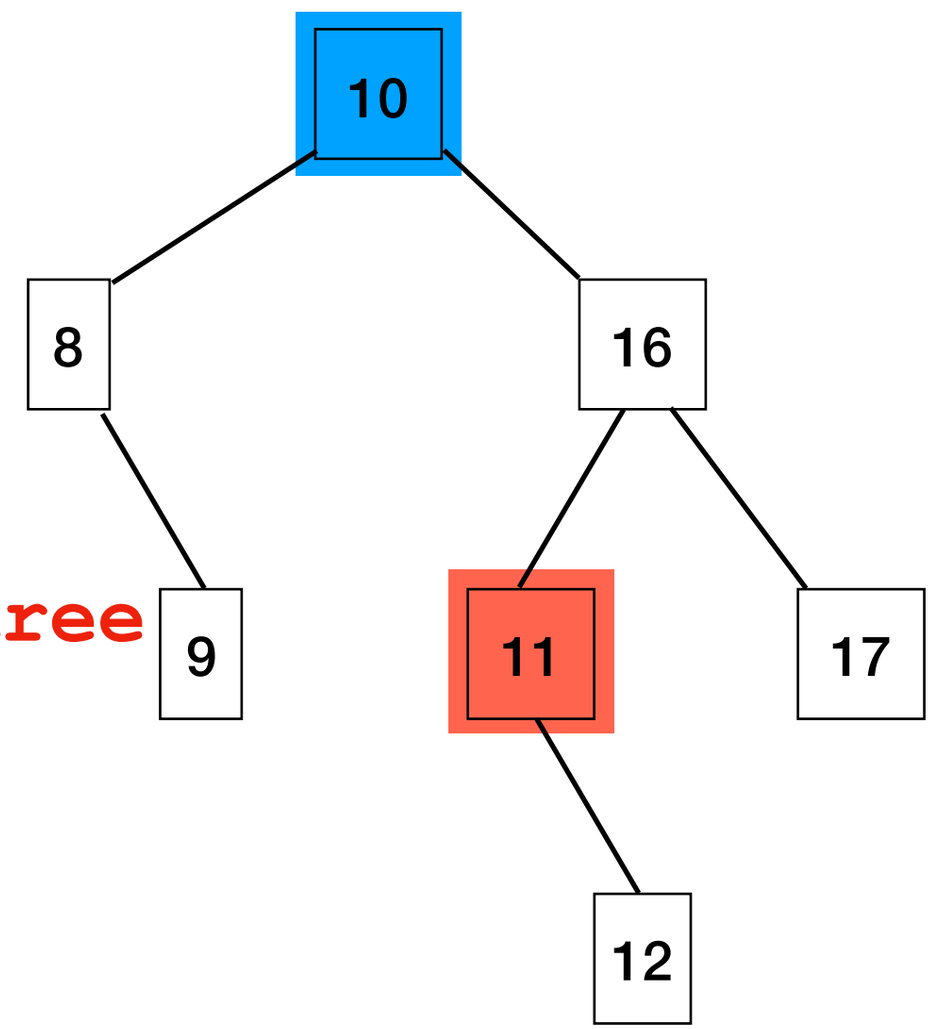


Deleting a node from a BST: Case 3

Three possible cases:

1. n has no children (is a leaf)
2. n has one child
3. **n has two children**

```
if (n has two children)
   let k = min node in right subtree
```

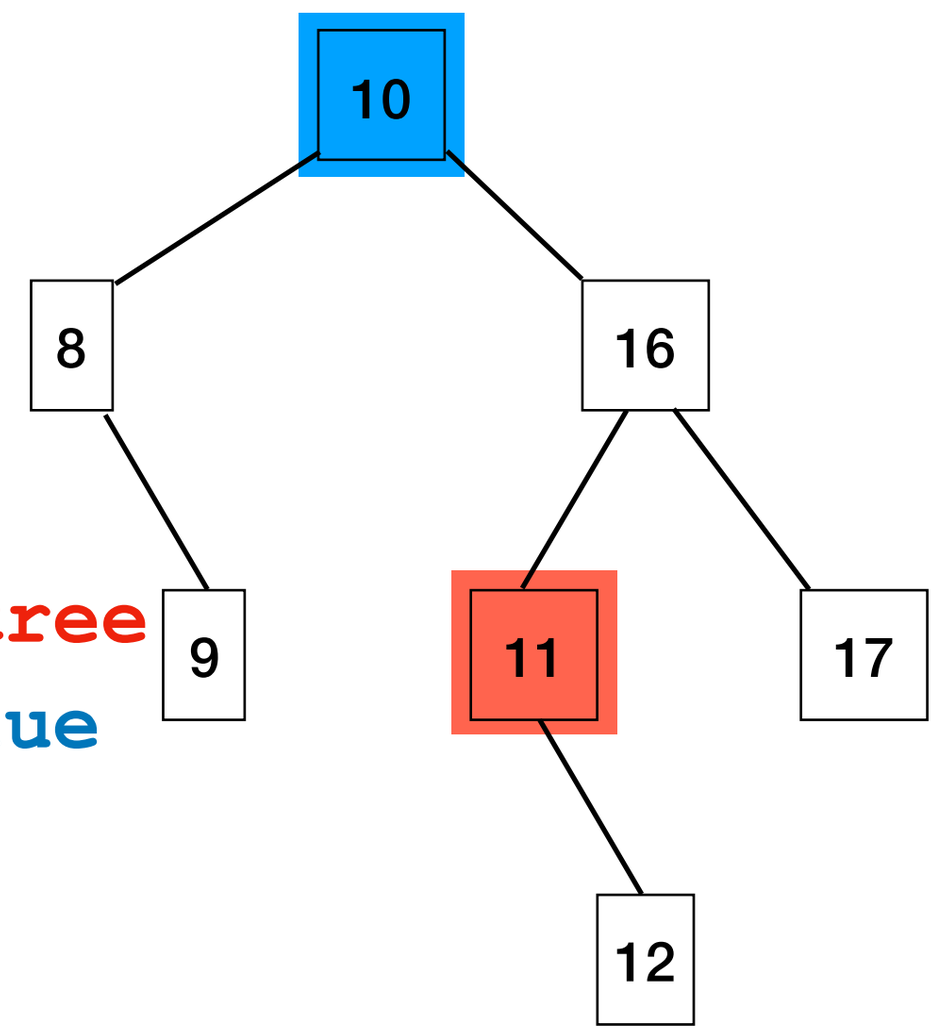


Deleting a node from a BST: Case 3

Three possible cases:

1. n has no children (is a leaf)
2. n has one child
3. **n has two children**

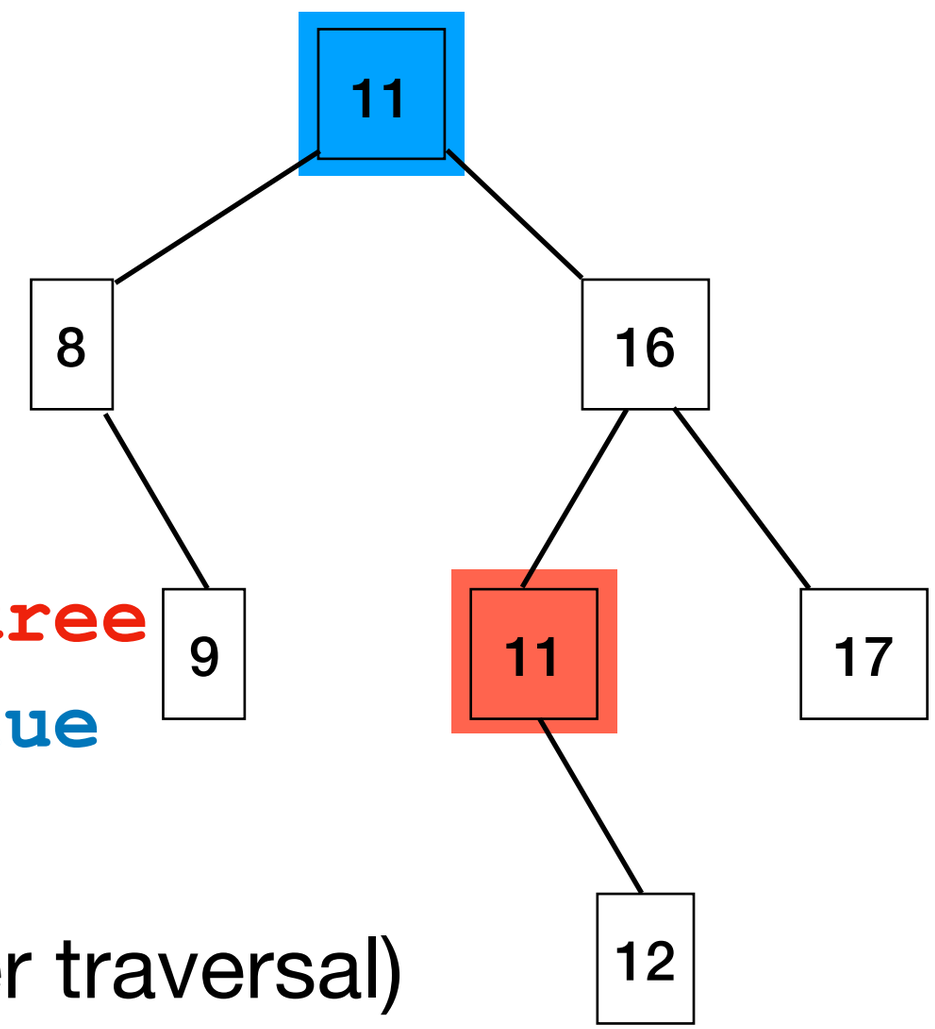
```
if (n has two children)
   let k = min node in right subtree
   replace n's value with k's value
```



Deleting a node from a BST: Case 3

Three possible cases:

1. n has no children (is a leaf)
2. n has one child
3. **n has two children**




```
if (n has two children)
  let k = min node in right subtree
  replace n's value with k's value
```

Can we do that?

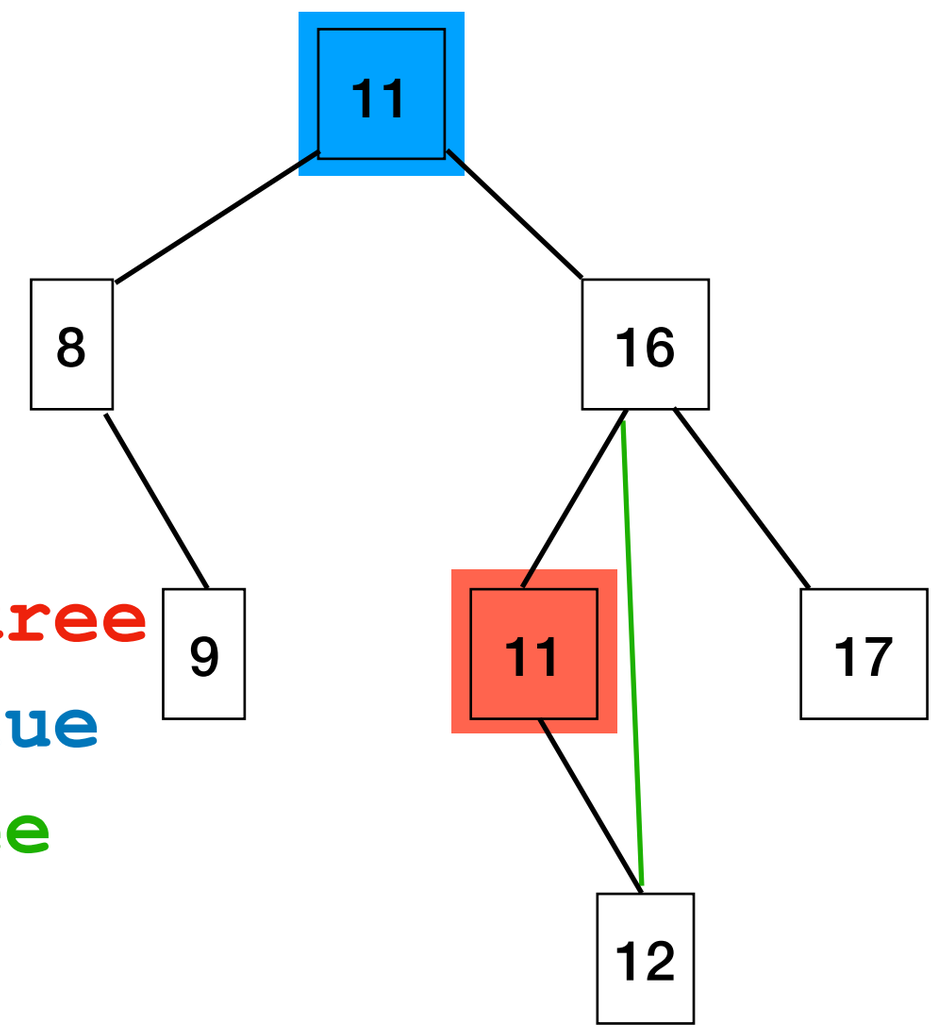
- **k** is **n**'s **successor** (next in an in-order traversal)
- Everything *else* in **n**'s right subtree is bigger than it
- Everything in **n**'s left subtree is smaller than it
- **k**'s value can safely replace **n**'s…but now we have a duplicate.

Deleting a node from a BST: Case 3

Three possible cases:

1. n has no children (is a leaf)
2. n has one child
3. **n has two children**

```
if (n has two children)
   let k = min node in right subtree
   replace n's value with k's value
   remove k from n's right subtree
```

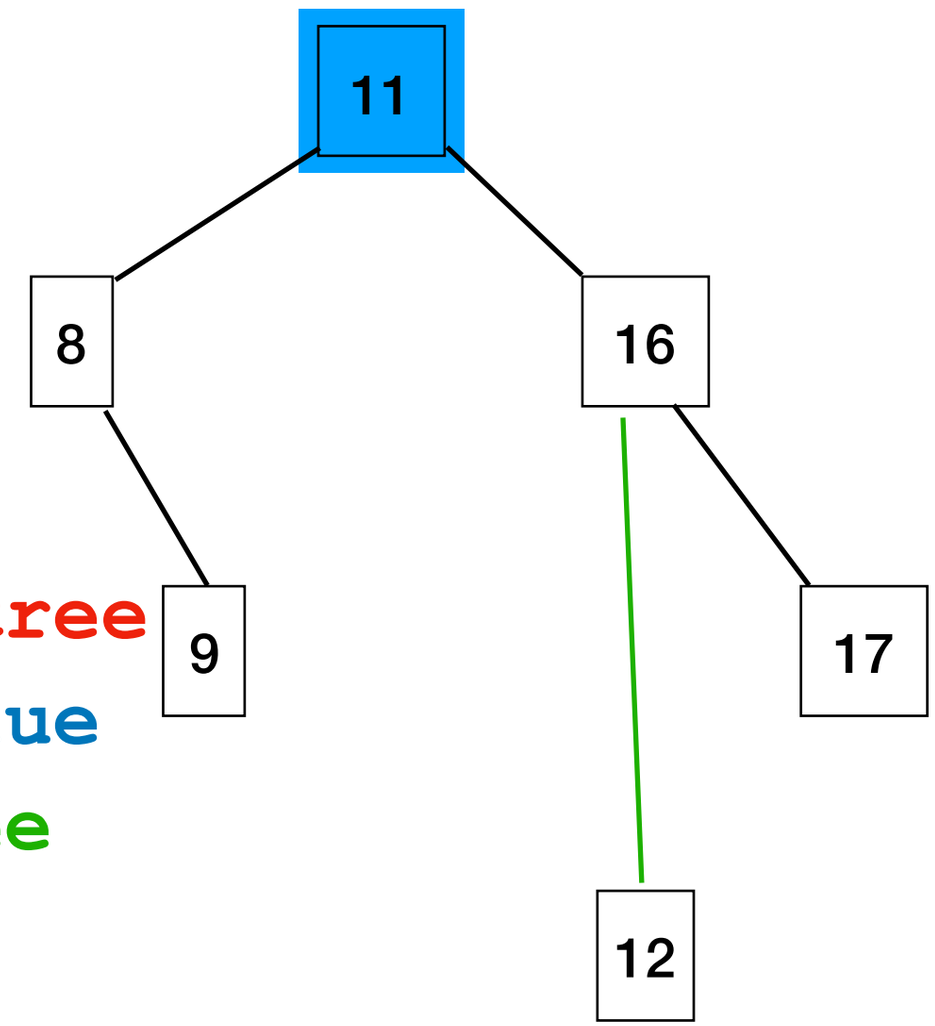


Deleting a node from a BST: Case 3

Three possible cases:

1. n has no children (is a leaf)
2. n has one child
3. **n has two children**

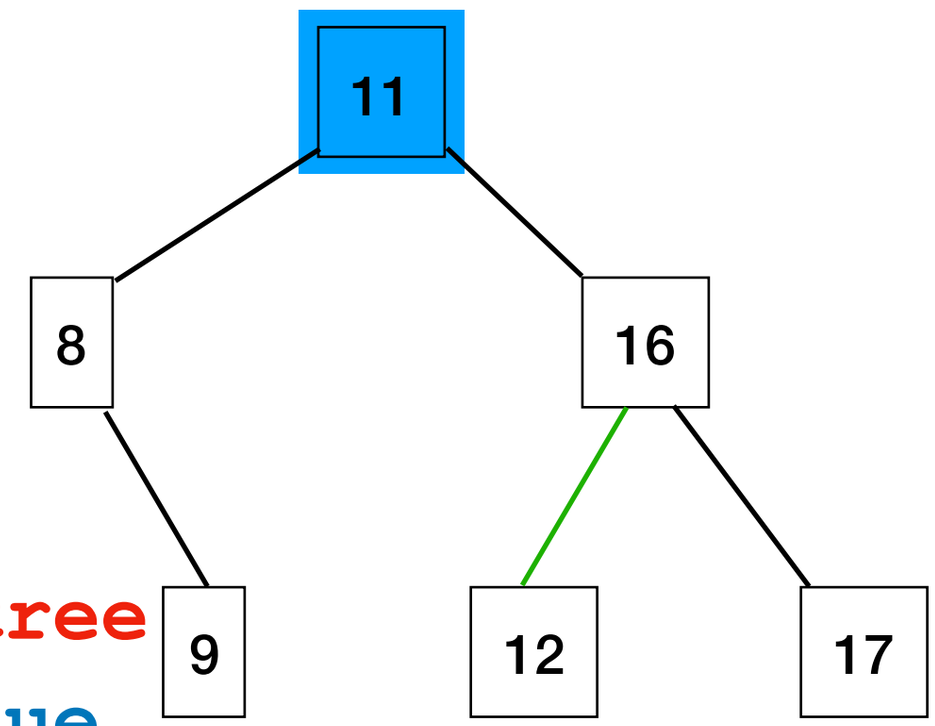
```
if (n has two children)
   let k = min node in right subtree
   replace n's value with k's value
   remove k from n's right subtree
```



Deleting a node from a BST: Case 3

Three possible cases:

1. n has no children (is a leaf)
2. n has one child
3. **n has two children**

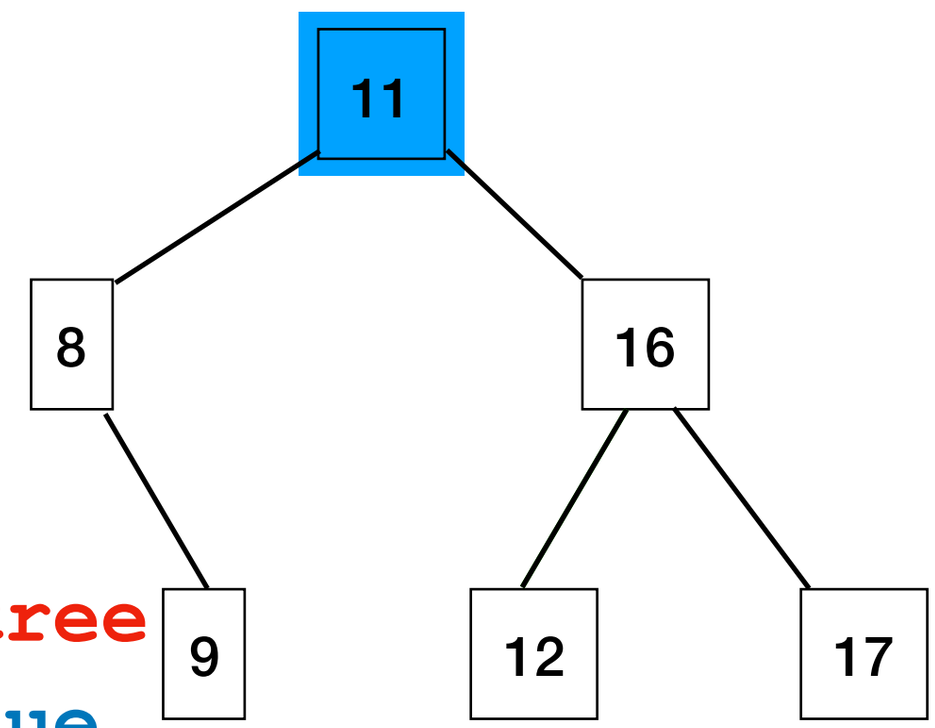



```
if (n has two children)
   let k = min node in right subtree
   replace n's value with k's value
   remove k from n's right subtree (recursively!)
```

Deleting a node from a BST: Case 3

Three possible cases:

1. n has no children (is a leaf)
2. n has one child
3. **n has two children**



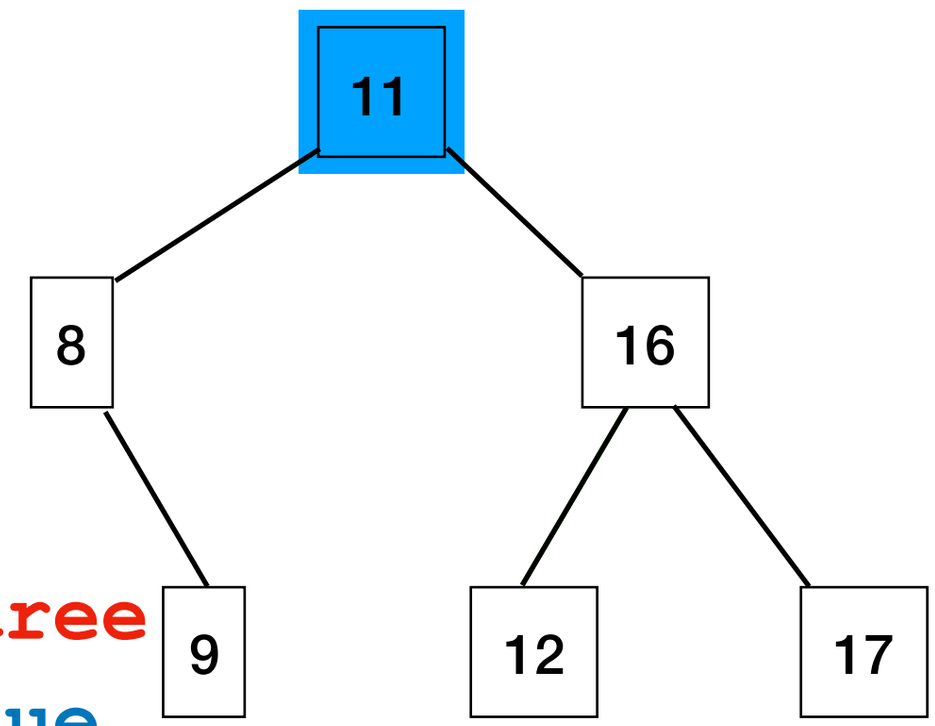

```
if (n has two children)
   let k = min node in right subtree
   replace n's value with k's value
   remove k from n's right subtree
```

this **has to be** either Case 1 or Case 2!

Deleting a node from a BST: Case 3

Three possible cases:

1. n has no children (is a leaf)
2. n has one child
3. **n has two children**




```
if (n has two children)
  let k = min node in right subtree
  replace n's value with k's value
  remove k from n's right subtree
```

this **has to be** either Case 1 or Case 2!

Why?

Deleting a node from a BST: Case 3

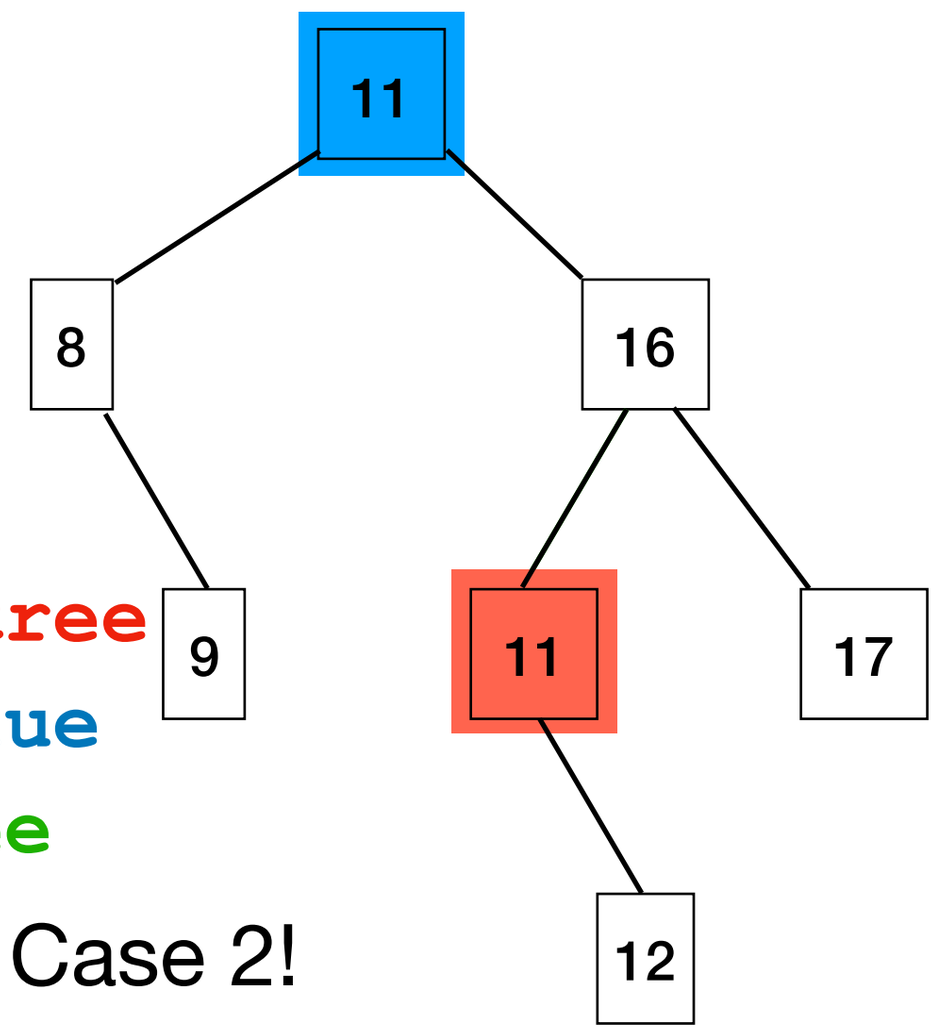
Three possible cases:

1. n has no children (is a leaf)
2. n has one child
3. **n has two children**

```
if (n has two children)
  let k = min node in right subtree
  replace n's value with k's value
  remove k from n's right subtree
```

this **has to be** either Case 1 or Case 2!

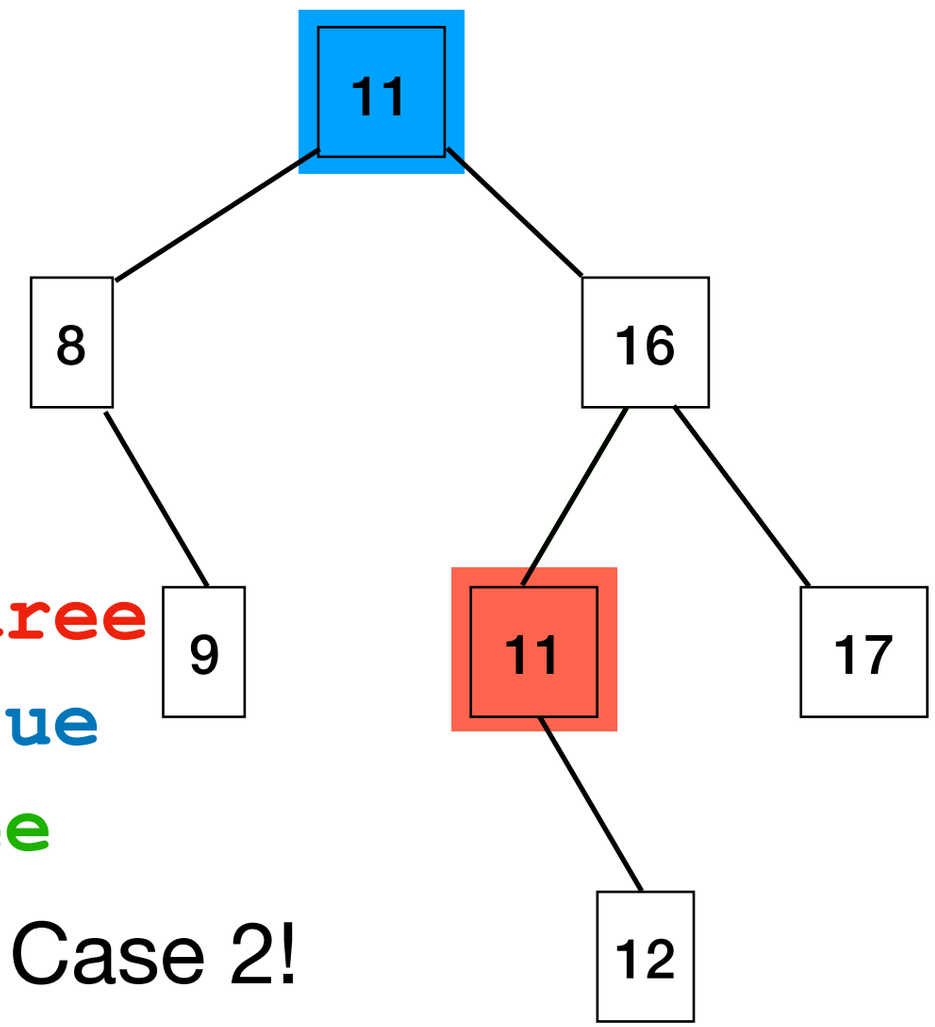
Why? Rewind to before we removed it:



Deleting a node from a BST: Case 3

Three possible cases:

1. n has no children (is a leaf)
2. n has one child
3. **n has two children**




```
if (n has two children)
  let k = min node in right subtree
  replace n's value with k's value
  remove k from n's right subtree
```

this **has to be** either Case 1 or Case 2!

Why? Rewind to before we removed it:

- **k** is the smallest node in **n**'s right subtree.
- if it had a left child, that child would have to be smaller!

Details

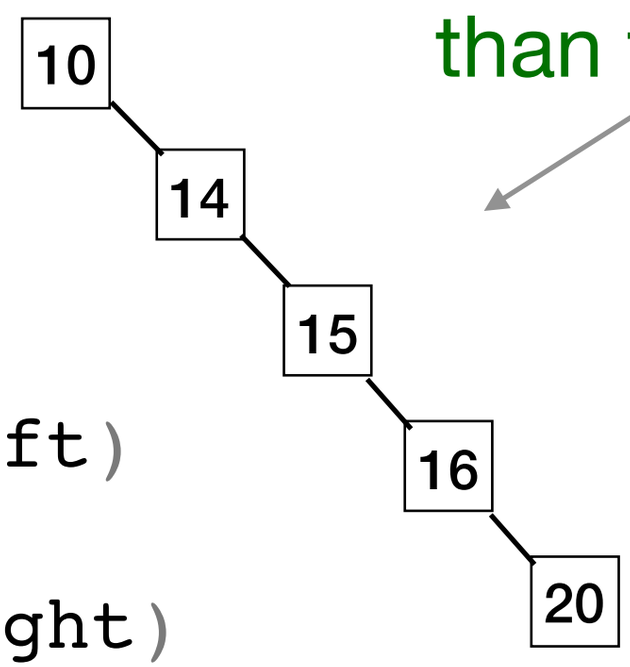
- Need to update root pointer if root is removed.
- Often can’t assume n.parent isn’t null - n may be root
- To update parent’s child pointer, you need to know which (L or R) child pointer to update.
- The approach presented differs from that in CLRS and some other resources.

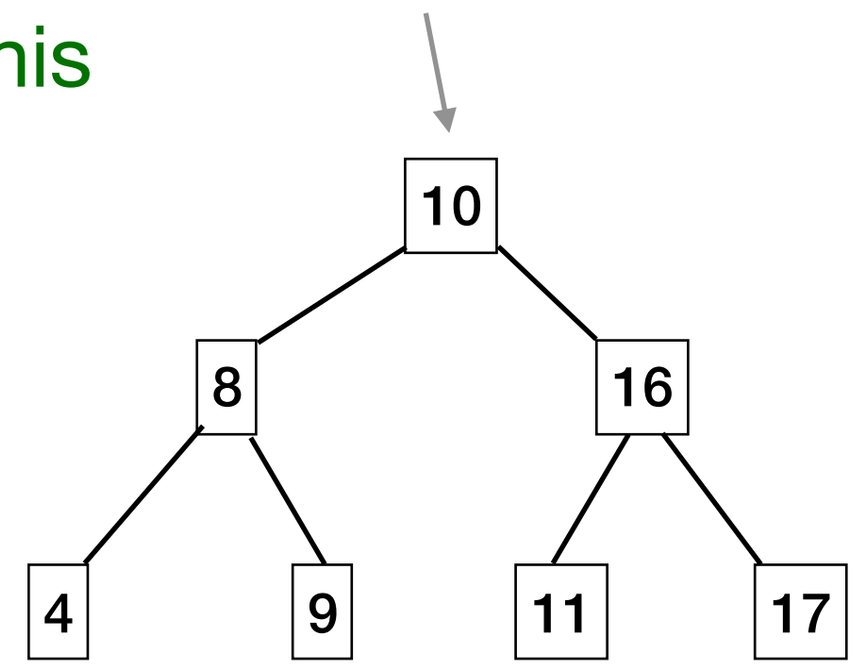
30 second kitten break

Searching a BST: What's the runtime?

```
boolean search(BST t, int v):
  if t == null:
    return false
  if t.value == v:
    return true
  if t.value < v:
    return search(t.left)
  else:
    return search(t.right)
```

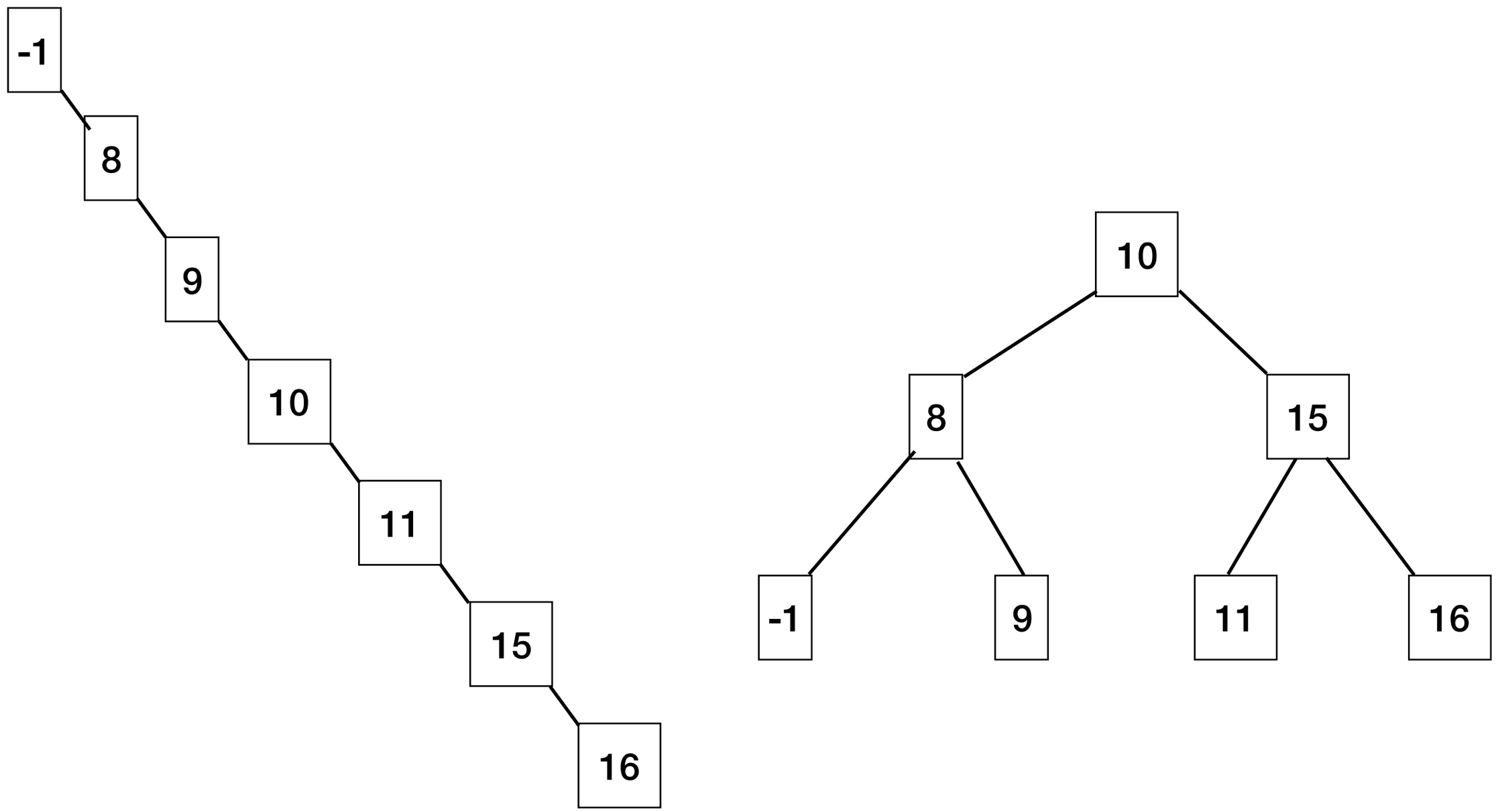
We want our trees to look more like this than this





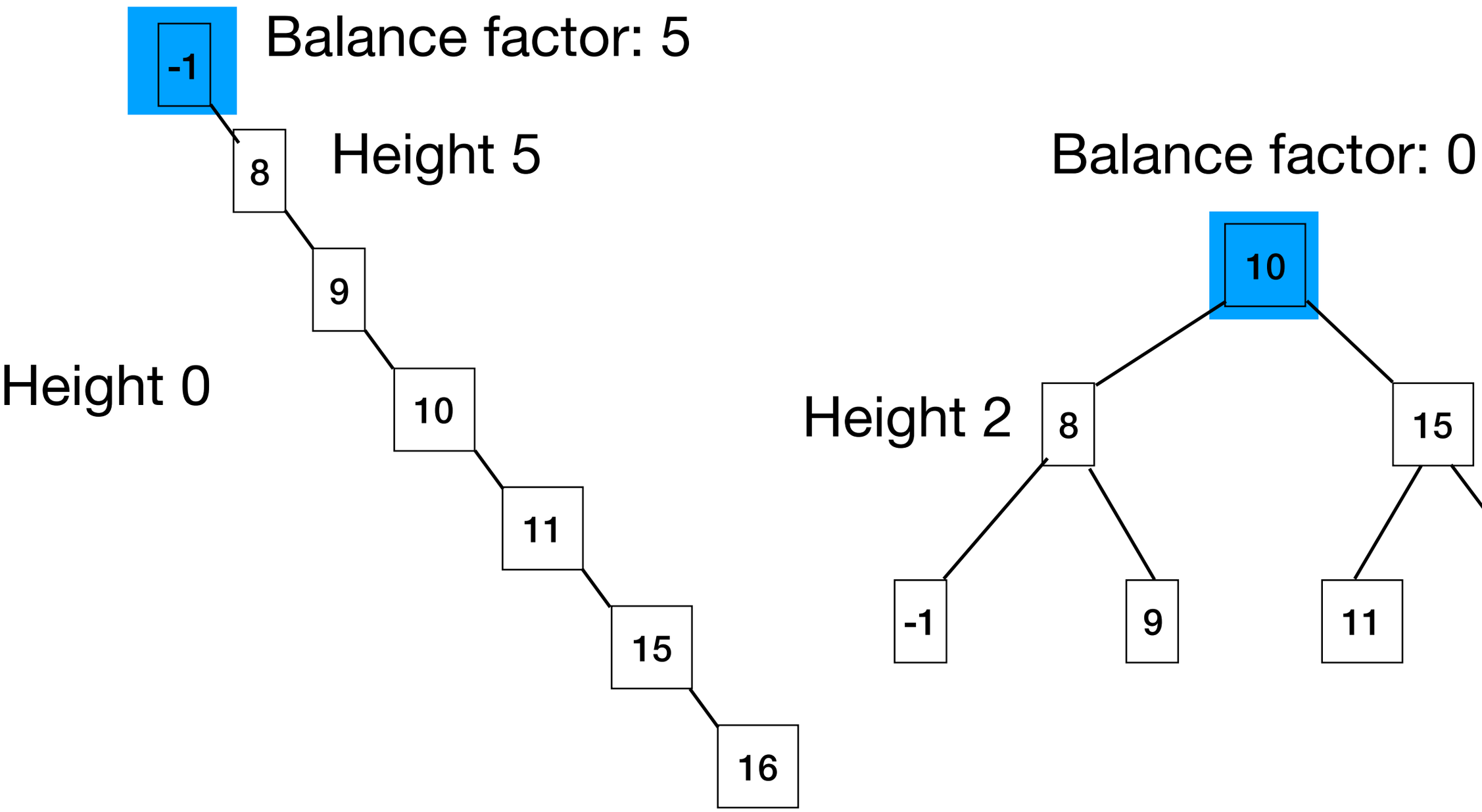
Runtime of search is O(h). **Worst: O(n)** **Best: O(log n)**

Same values, different trees



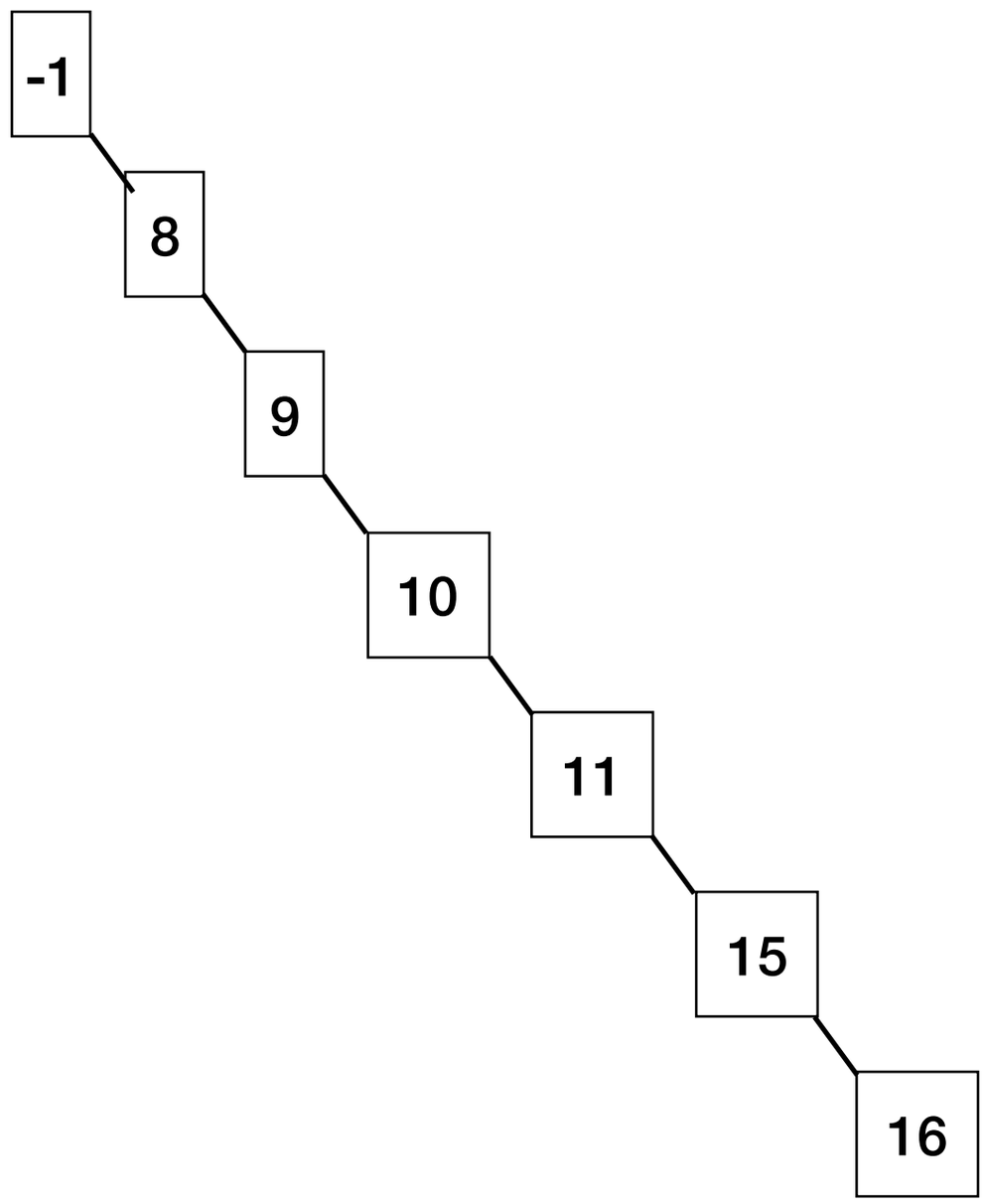
Same values, different trees

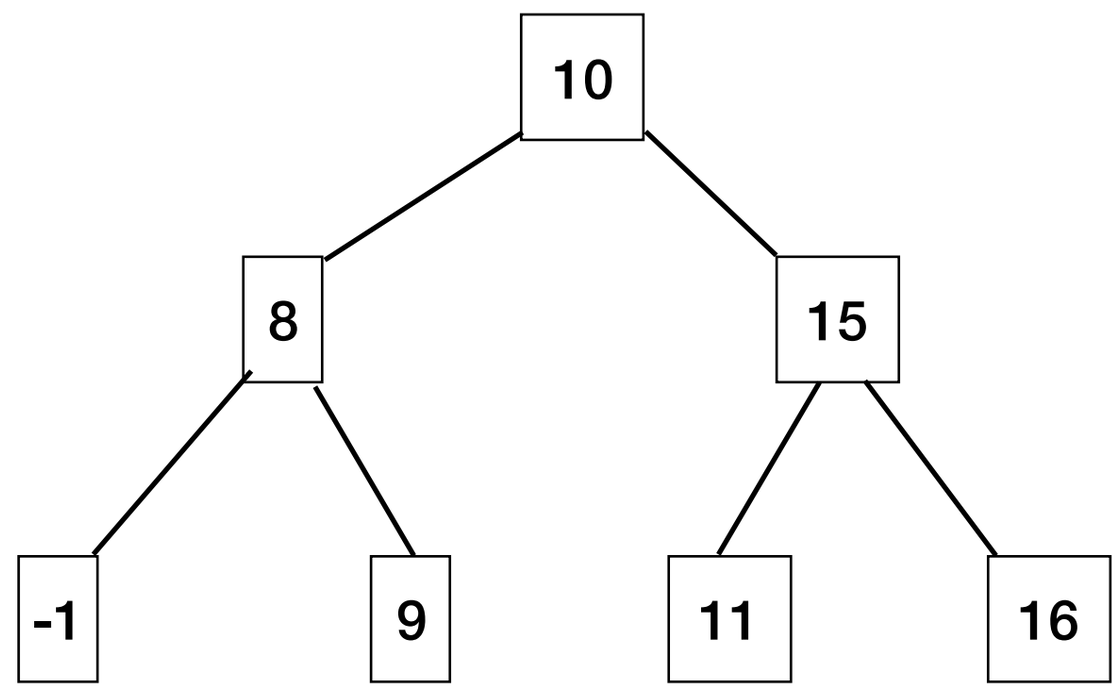
Balace Factor(n) = height(n.right) - height(left)



Can we improve balance?

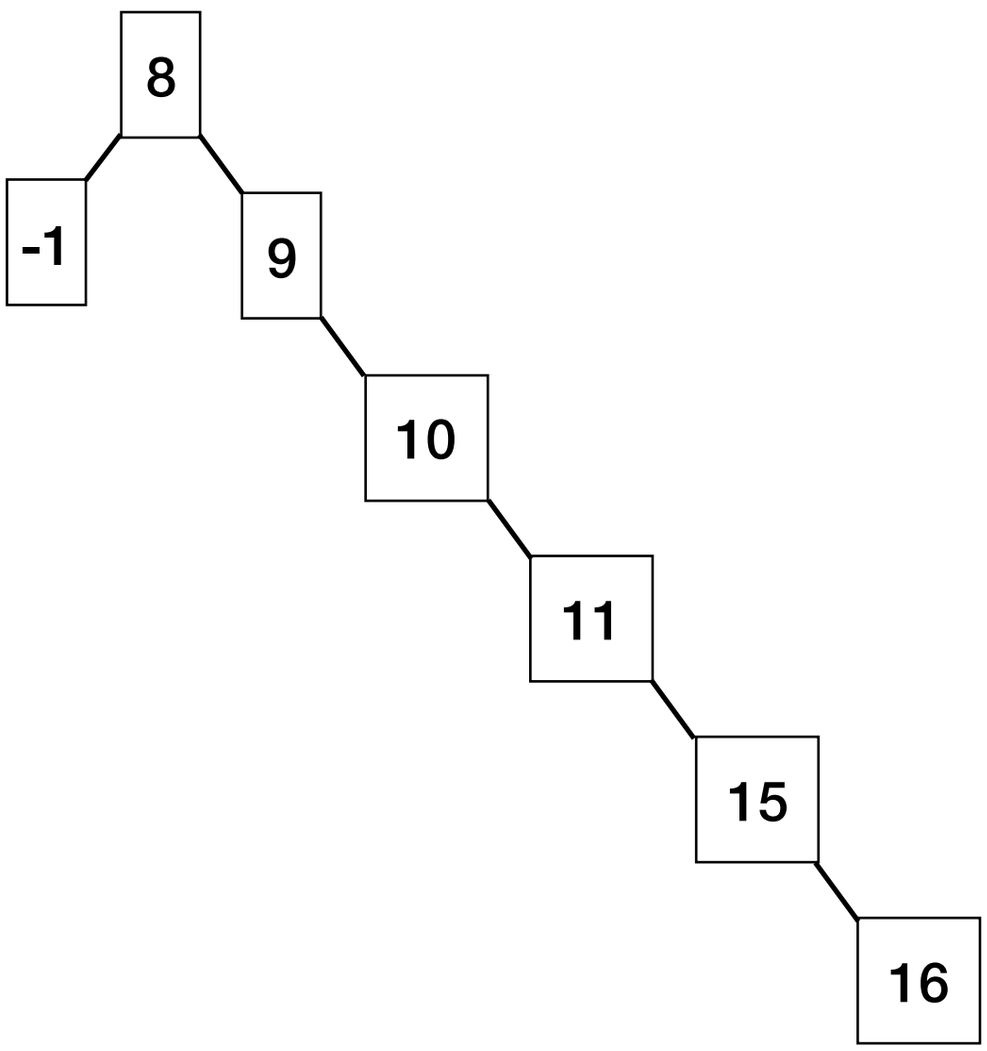
Balace Factor(n) = height(n.right) - height(left)

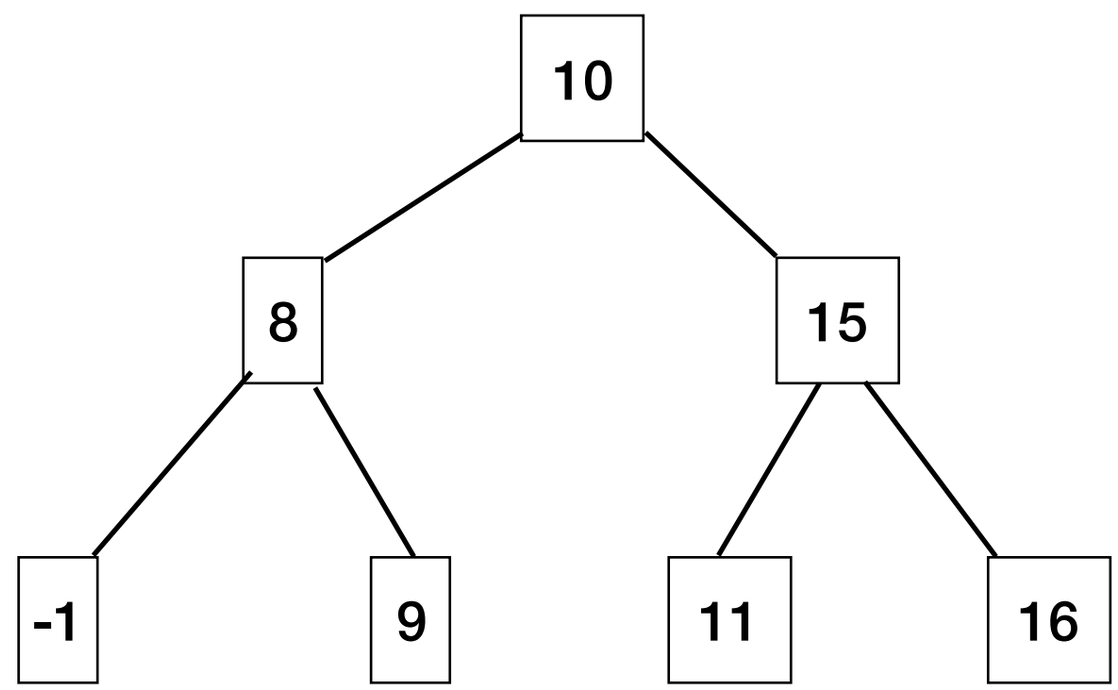




Can we improve balance?

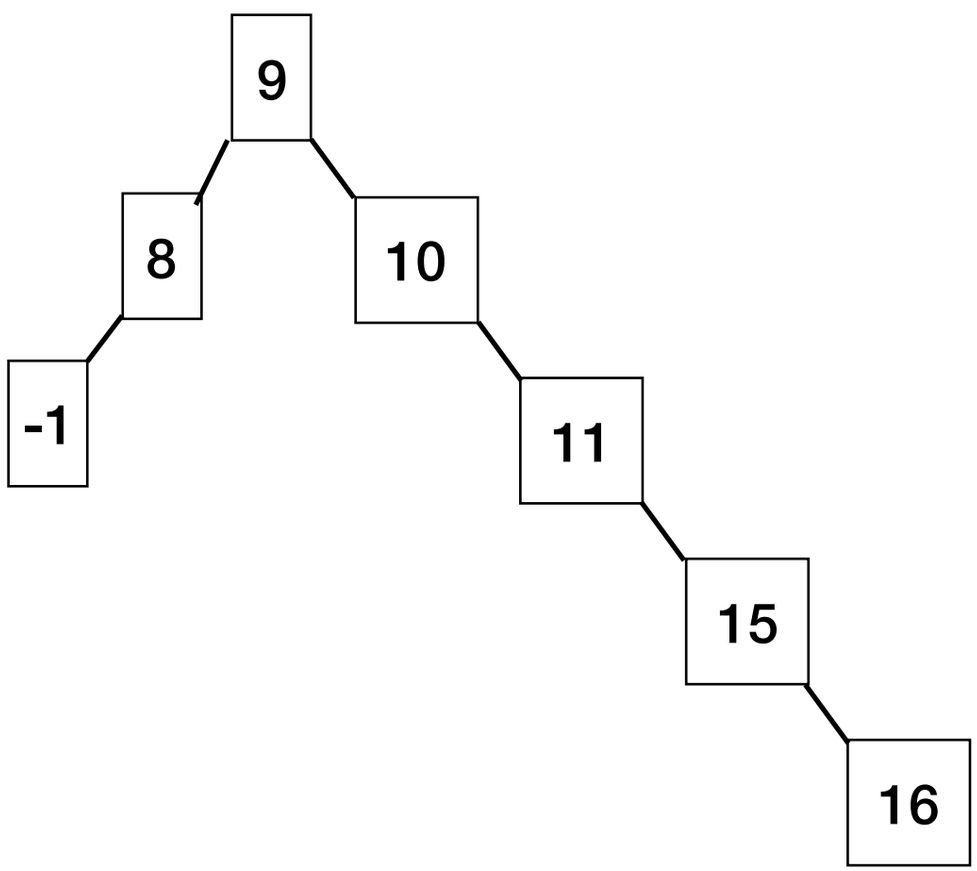
Balace Factor(n) = height(n.right) - height(left)

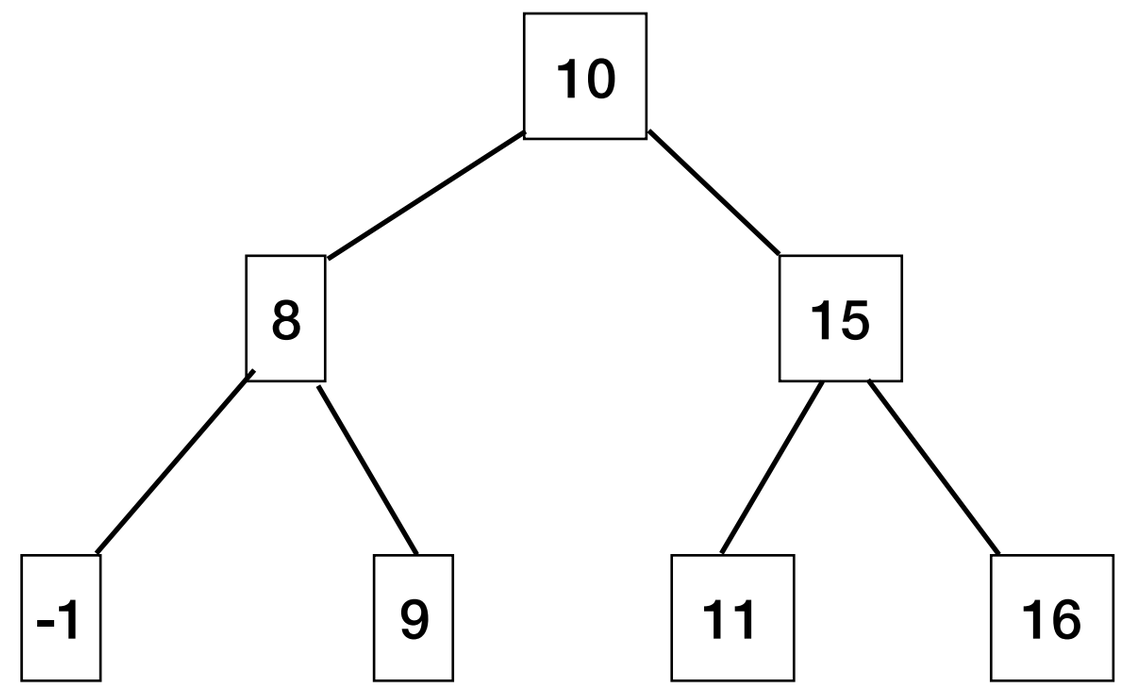




Can we improve balance?

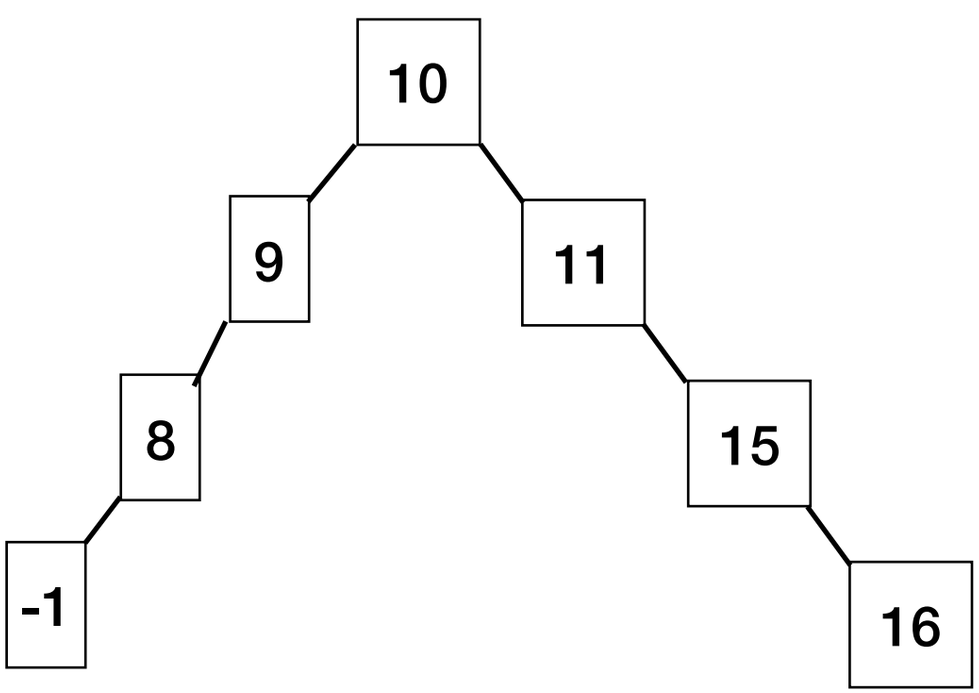
Balace Factor(n) = height(n.right) - height(left)

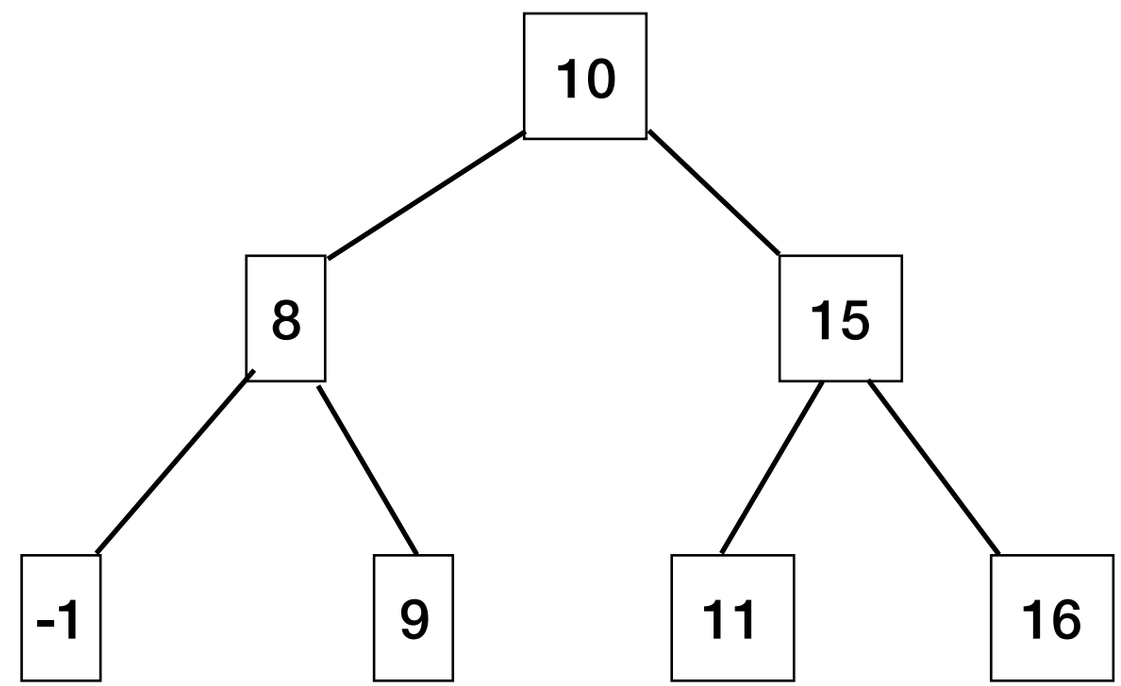




Can we improve balance?

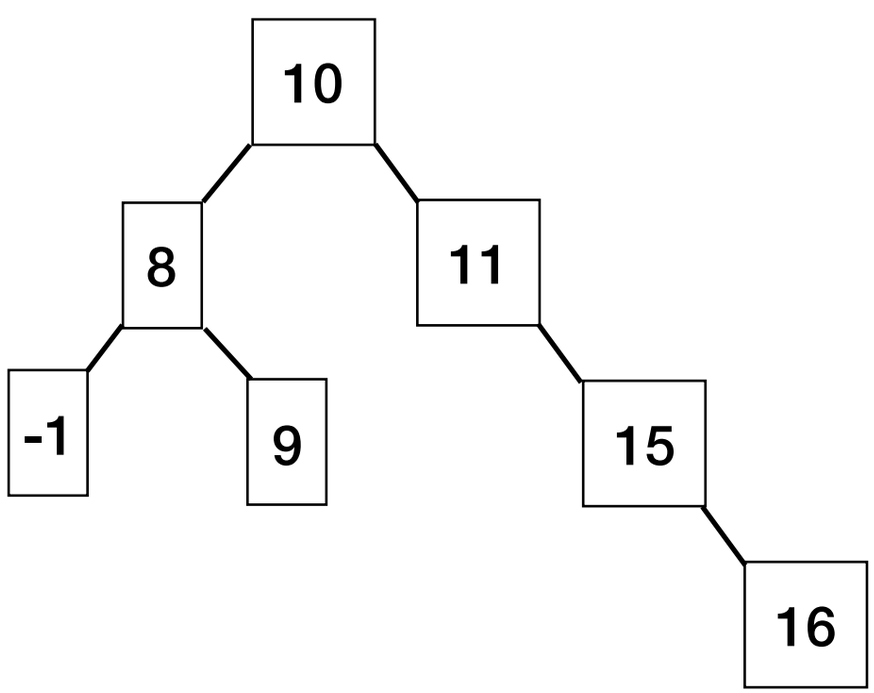
Balace Factor(n) = height(n.right) - height(left)

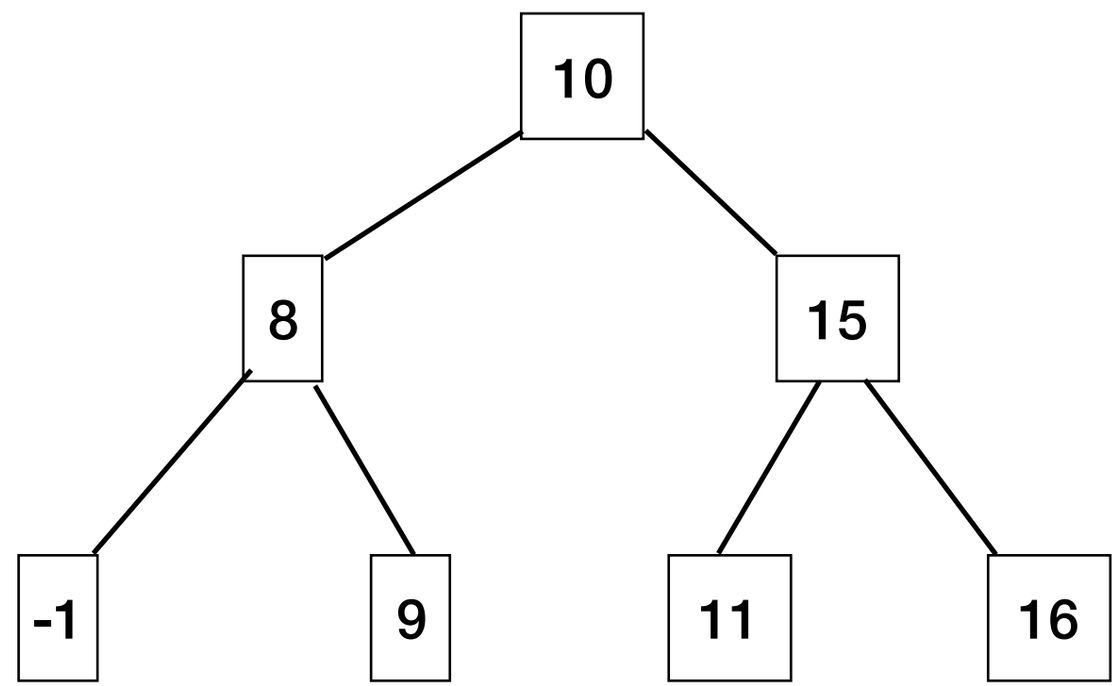




Can we improve balance?

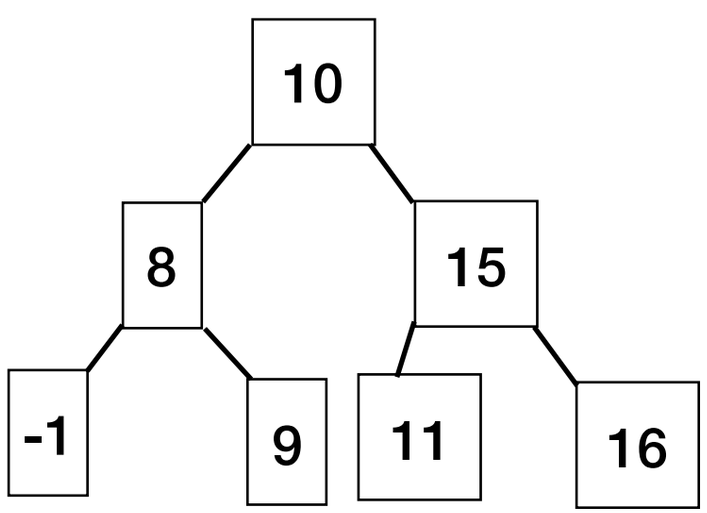
Balace Factor(n) = height(n.right) - height(left)

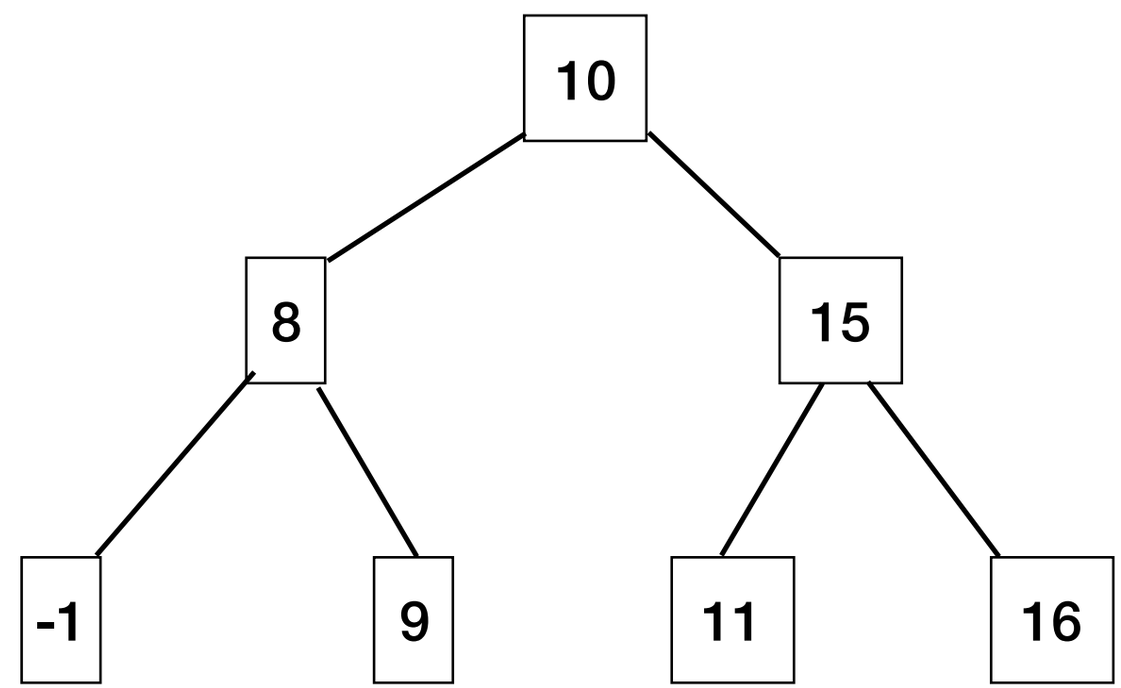




Can we improve balance?

Balace Factor(n) = height(n.right) - height(left)



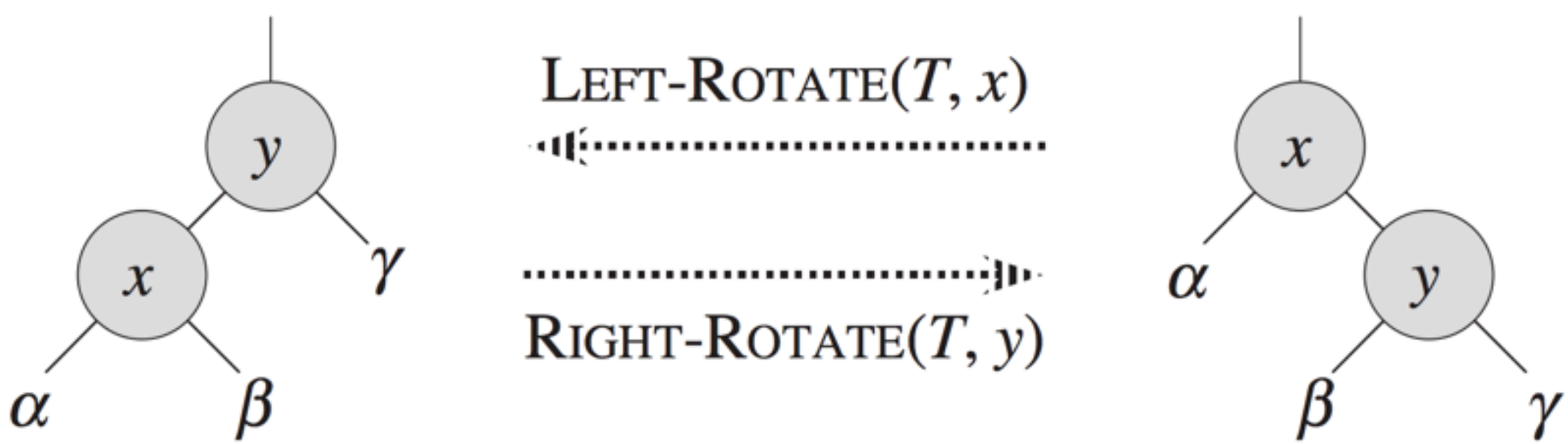


Tree Rotations

modify the structure without violating the BST property.

Steps in left rotation (move y up to its parent's position):

1. Transfer β: x's right subtree becomes y's old left subtree (β)
2. Transfer the parent: y's parent becomes x's old parent
3. Transfer x itself: x becomes y's left subtree



CLRS Fig 13.2, pg 313

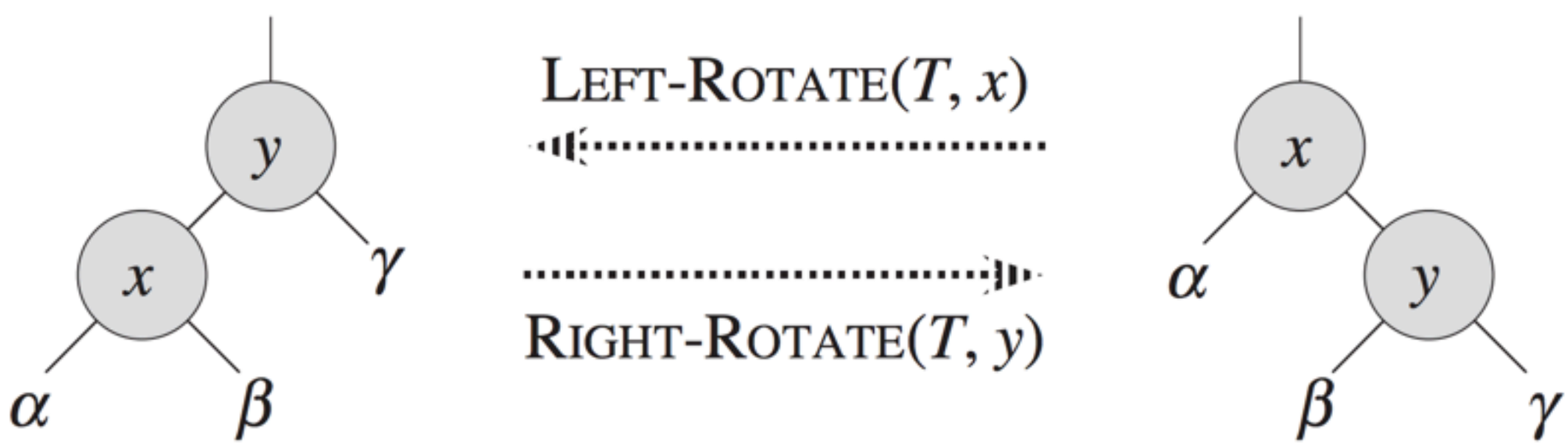
Tree Rotations

modify the structure without violating the BST property.

Steps in left rotation (move y up to its parent's position):

1. Transfer β: x's right subtree becomes y's old left subtree (β)
2. Transfer the parent: y's parent becomes x's old parent
3. Transfer x itself: x becomes y's left subtree

Details: need to update child, parent, and (possibly) root pointers.



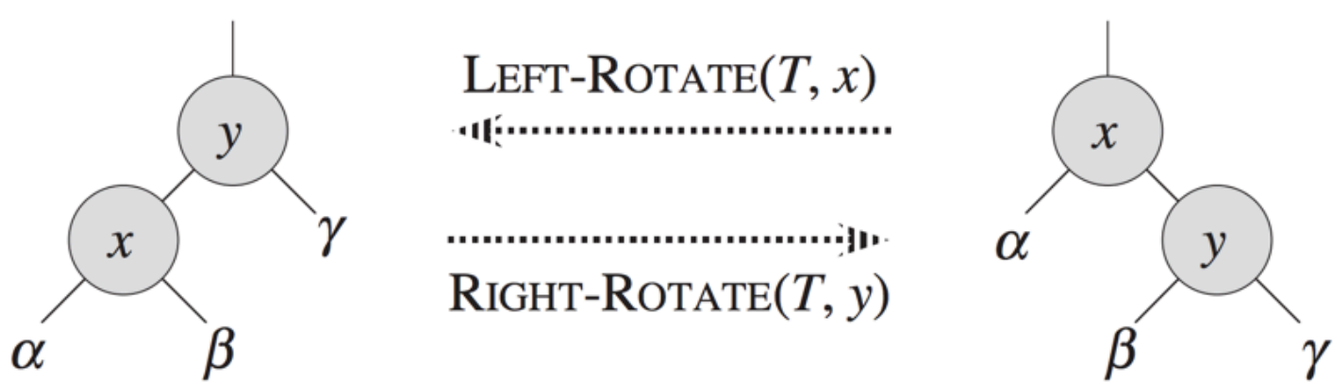
CLRS Fig 13.2, pg 313

Tree Rotations

Steps in left rotation (move y up to its parent's position):

1. Transfer β: x's right subtree becomes y's old left subtree (β)
2. Transfer the parent: y's parent becomes x's old parent
3. Transfer x itself: x becomes y's left subtree

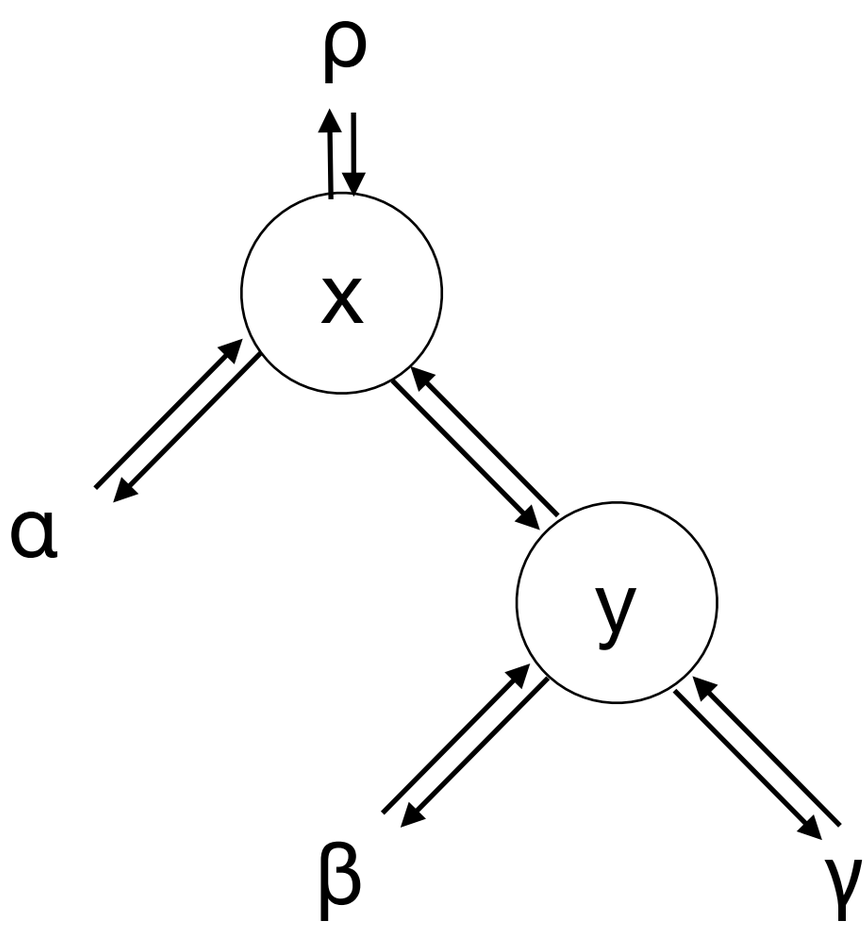
Details: need to update child, parent, and (possibly) root pointers.



Tree Rotations

Steps in left rotation (move y up to its parent's position):

1. Transfer β: x's right subtree becomes y's old left subtree (β)
2. Transfer the parent: y's parent becomes x's old parent
3. Transfer x itself: x becomes y's left subtree

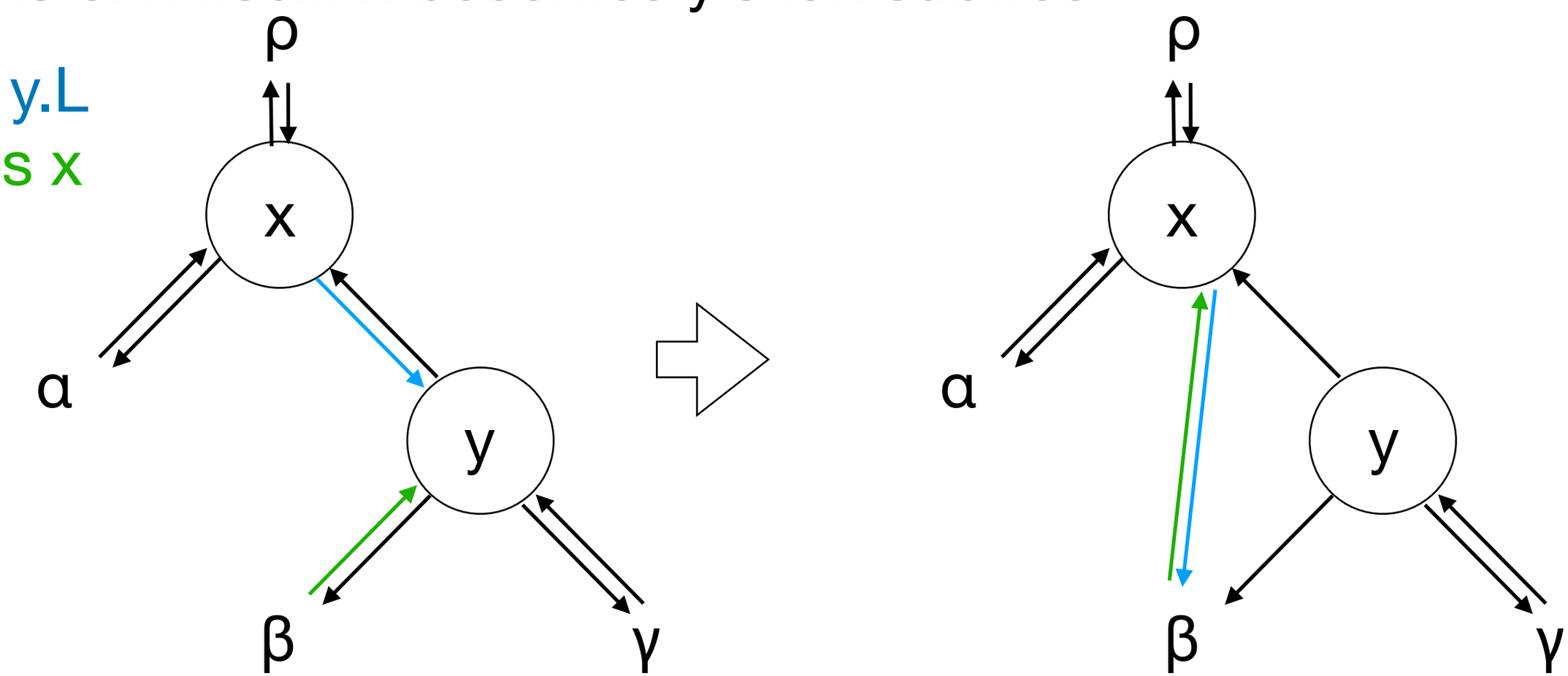


Tree Rotations

Steps in left rotation (move y up to its parent's position):

1. **Transfer β:** x's right subtree becomes y's old left subtree (β)
2. Transfer the parent: y's parent becomes x's old parent
3. Transfer x itself: x becomes y's left subtree

x.R gets y.L
y.L.p gets x

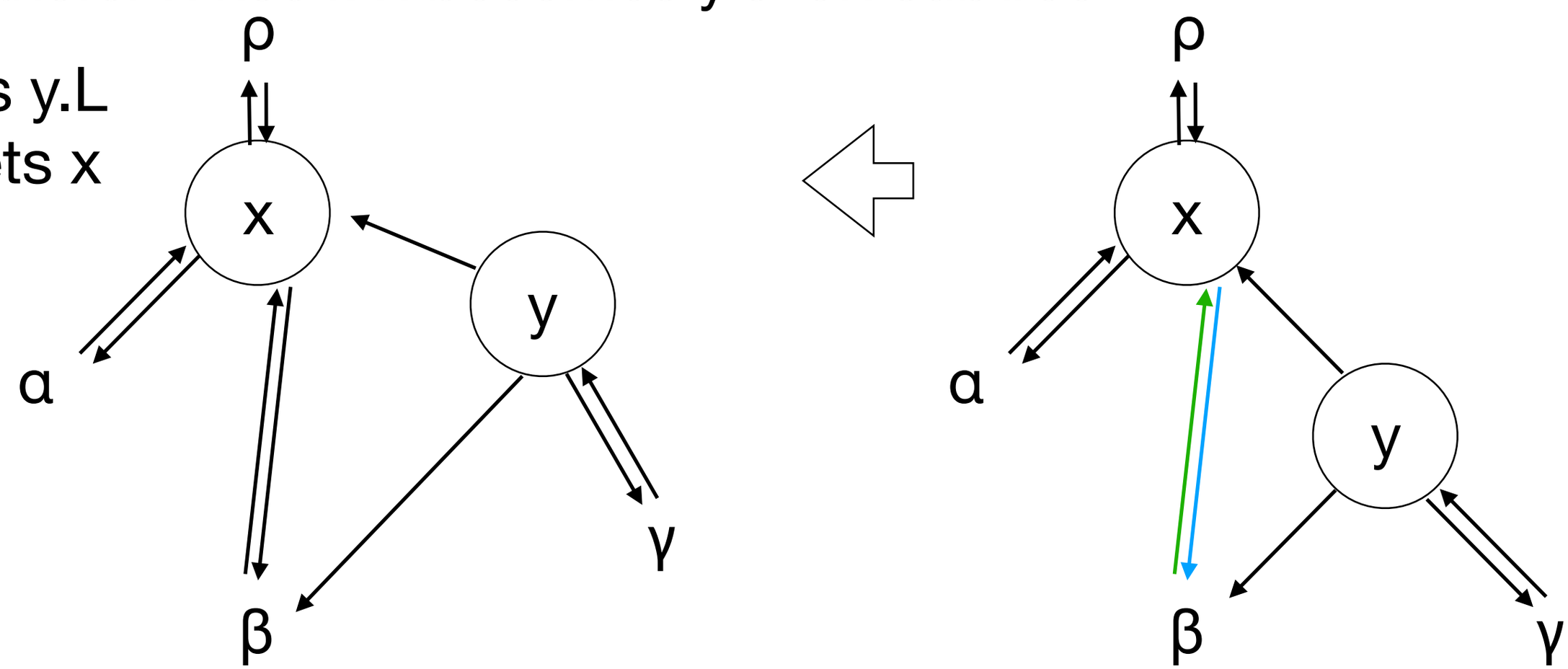


Tree Rotations

Steps in left rotation (move y up to its parent's position):

1. Transfer β: x's right subtree becomes y's old left subtree (β)
2. Transfer the parent: y's parent becomes x's old parent
3. Transfer x itself: x becomes y's left subtree

x.R gets y.L
y.L.p gets x



(**only** rearranged the picture)

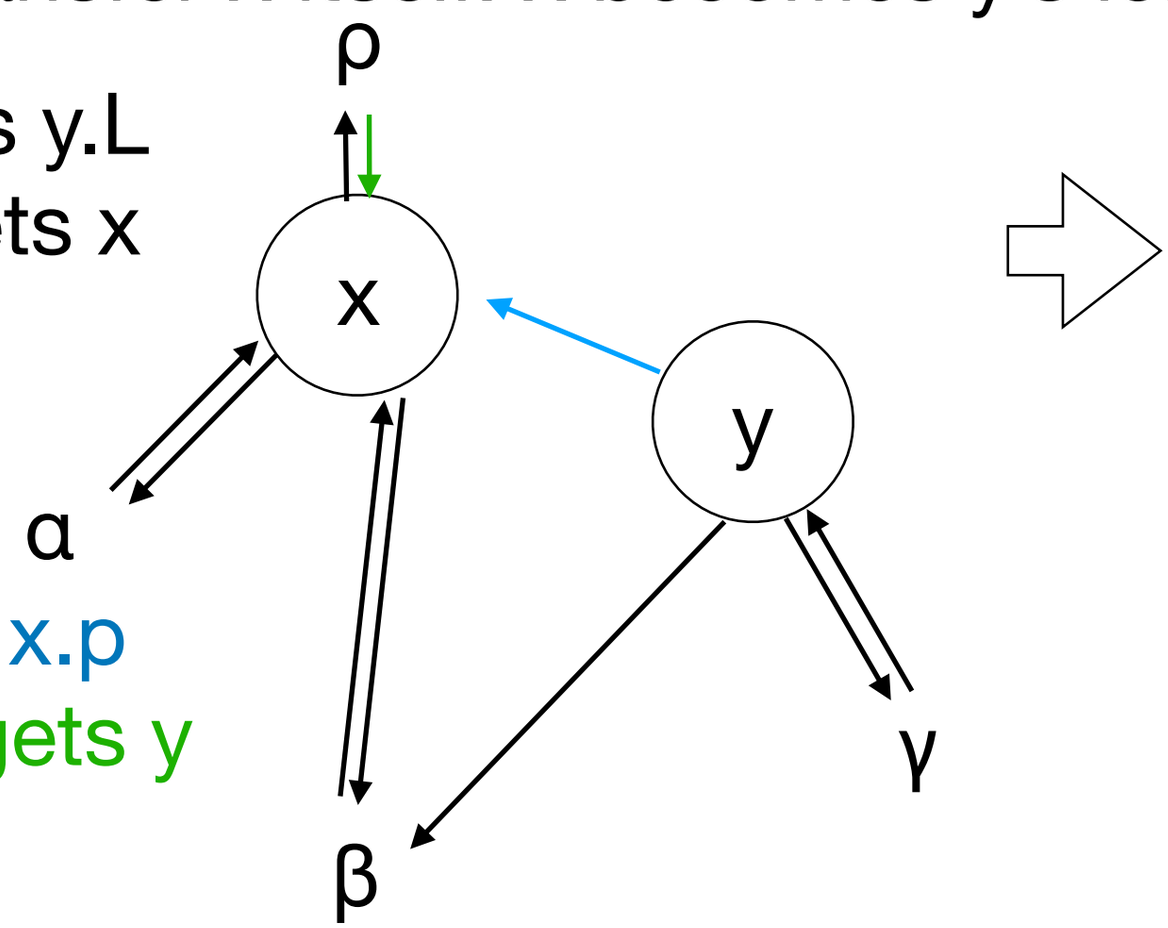
Tree Rotations

Steps in left rotation (move y up to its parent's position):

1. Transfer β: x's right subtree becomes y's old left subtree (β)
2. **Transfer the parent:** y's parent becomes x's old parent
3. Transfer x itself: x becomes y's left subtree

x.R gets y.L
y.L.p gets x

y.p gets x.p
p.[L/R] gets y



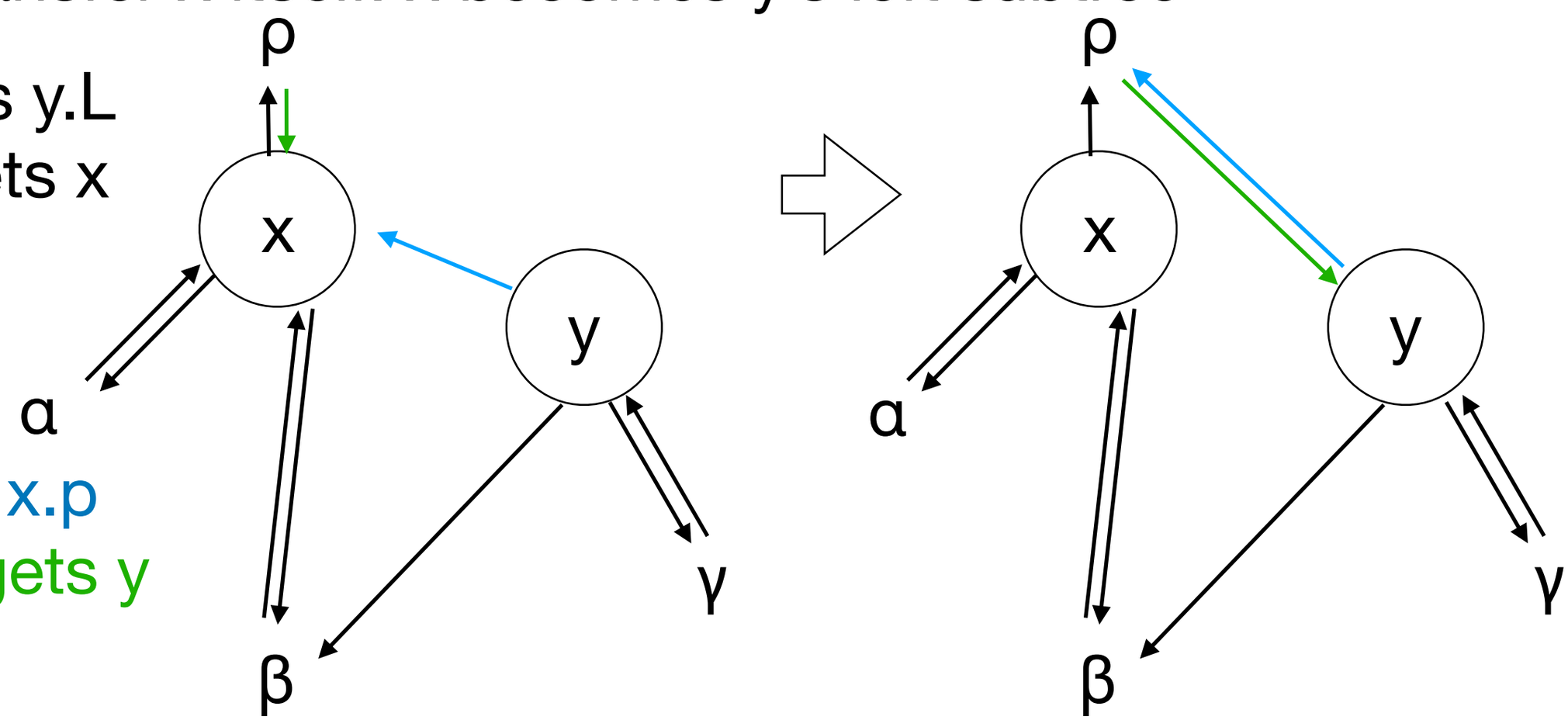
Tree Rotations

Steps in left rotation (move y up to its parent's position):

1. Transfer β: x's right subtree becomes y's old left subtree (β)
2. **Transfer the parent:** y's parent becomes x's old parent
3. Transfer x itself: x becomes y's left subtree

x.R gets y.L
y.L.p gets x

y.p gets x.p
p.[L/R] gets y



(what if ρ is null / x was root?)

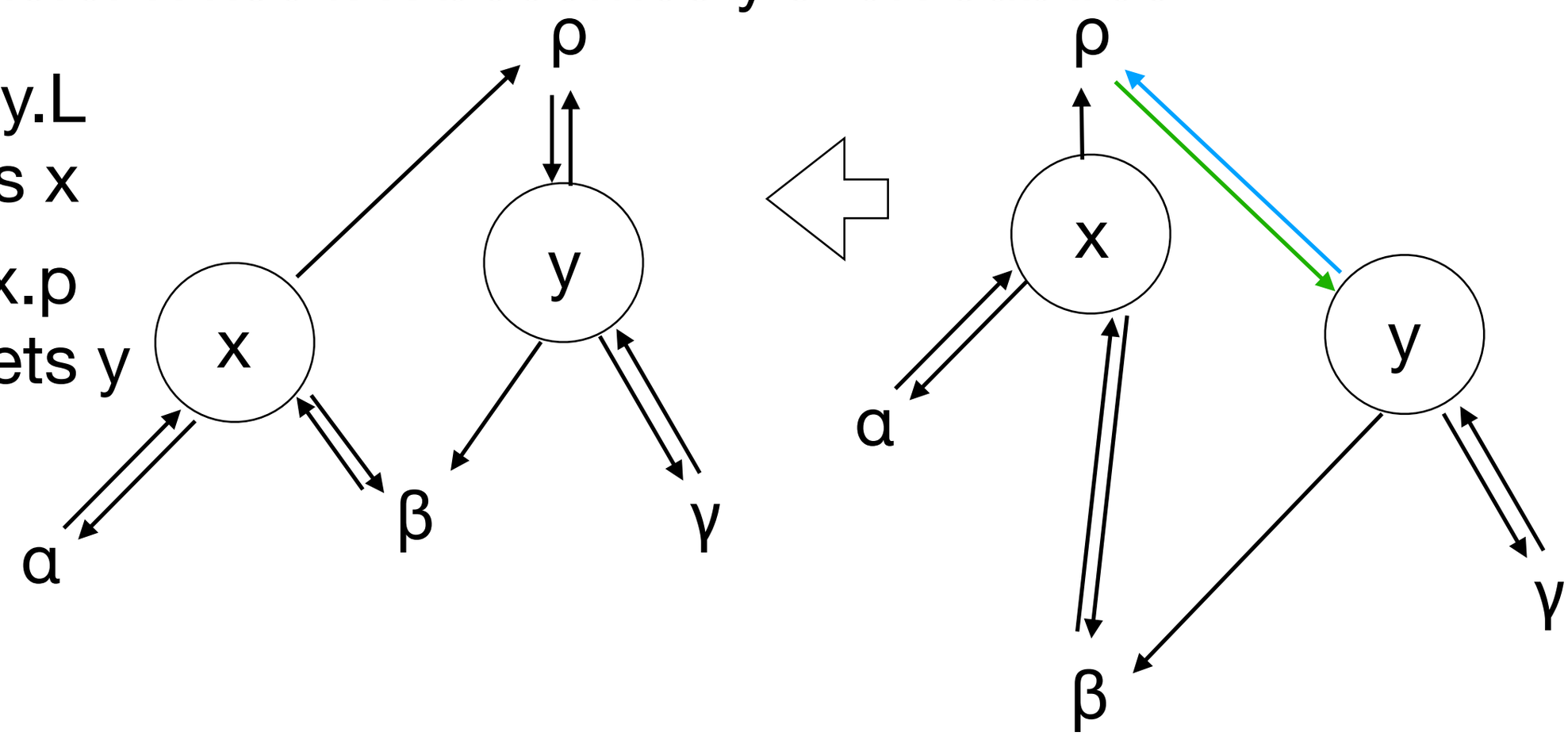
Tree Rotations

Steps in left rotation (move y up to its parent's position):

1. Transfer β: x's right subtree becomes y's old left subtree (β)
2. Transfer the parent: y's parent becomes x's old parent
3. Transfer x itself: x becomes y's left subtree

x.R gets y.L
y.L.p gets x

y.p gets x.p
p.[L/R] gets y



(**only** rearranged the picture)

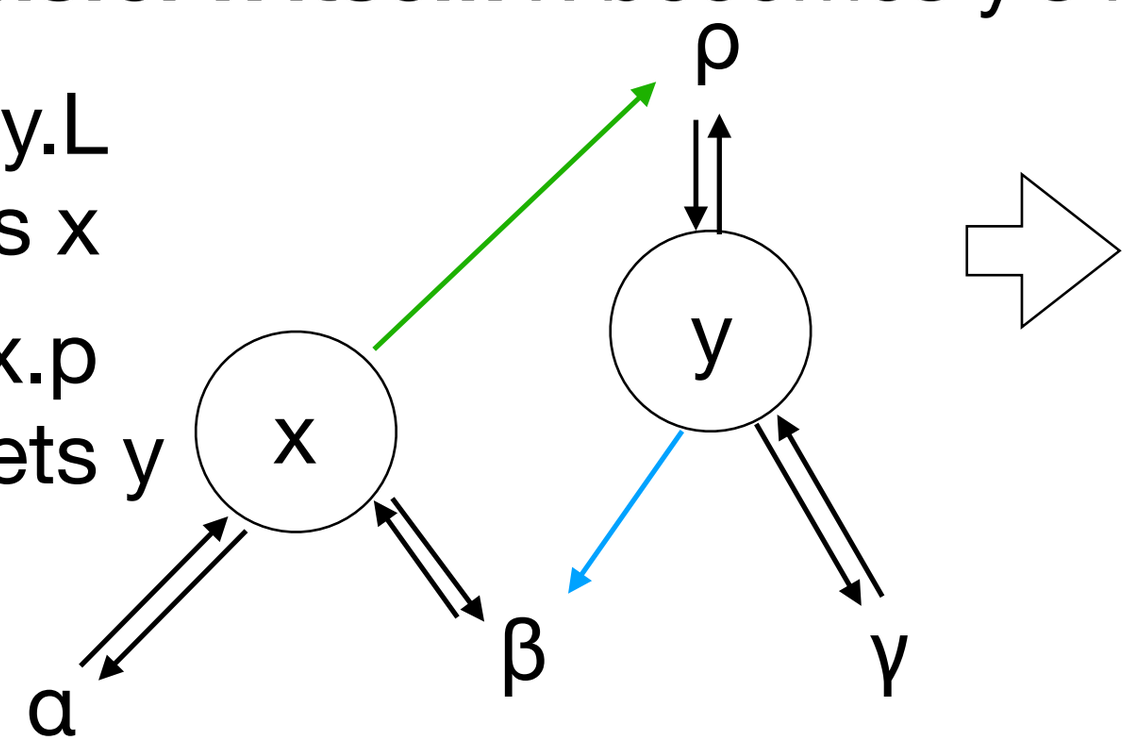
Tree Rotations

Steps in left rotation (move y up to its parent's position):

1. Transfer β: x's right subtree becomes y's old left subtree (β)
2. Transfer the parent: y's parent becomes x's old parent
3. **Transfer x itself:** x becomes y's left subtree

x.R gets y.L
y.L.p gets x

y.p gets x.p
p.[L/R] gets y



y.L gets x
x.p gets y

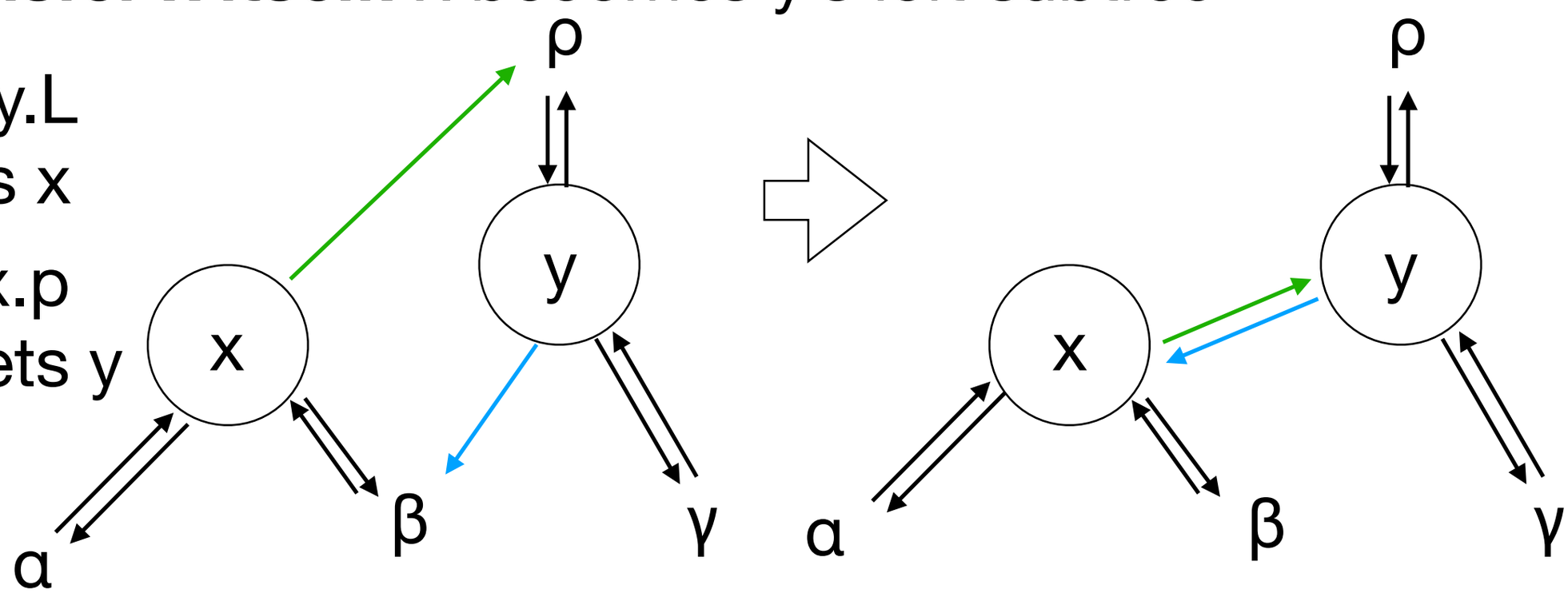
Tree Rotations

Steps in left rotation (move y up to its parent's position):

1. Transfer β: x's right subtree becomes y's old left subtree (β)
2. Transfer the parent: y's parent becomes x's old parent
3. **Transfer x itself:** x becomes y's left subtree

x.R gets y.L
y.L.p gets x

y.p gets x.p
p.[L/R] gets y



y.L gets x
x.p gets y

Tree Rotations

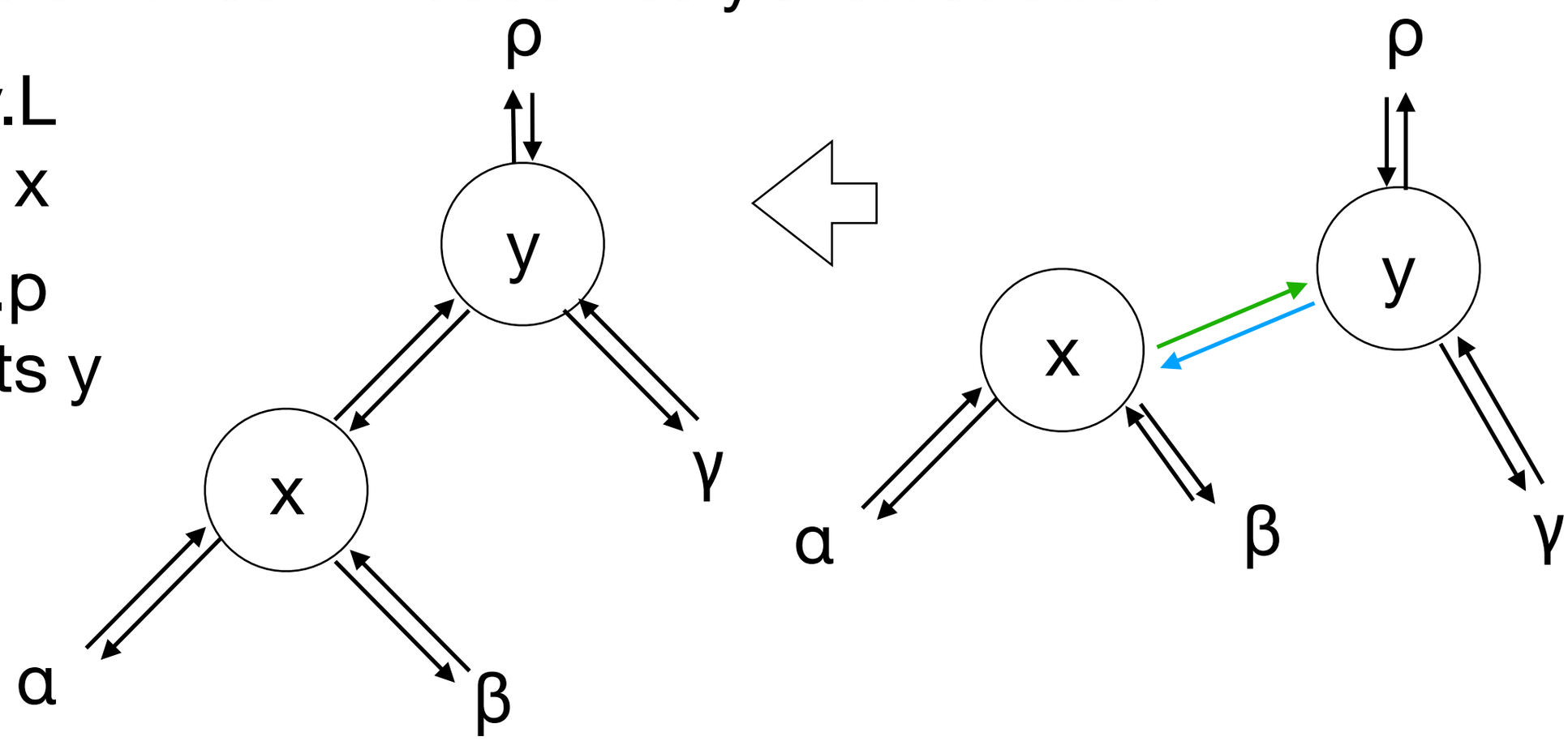
Steps in left rotation (move y up to its parent's position):

1. Transfer β: x's right subtree becomes y's old left subtree (β)
2. Transfer the parent: y's parent becomes x's old parent
3. Transfer x itself: x becomes y's left subtree

x.R gets y.L
y.L.p gets x

y.p gets x.p
p.[L/R] gets y

y.L gets x
x.p gets y



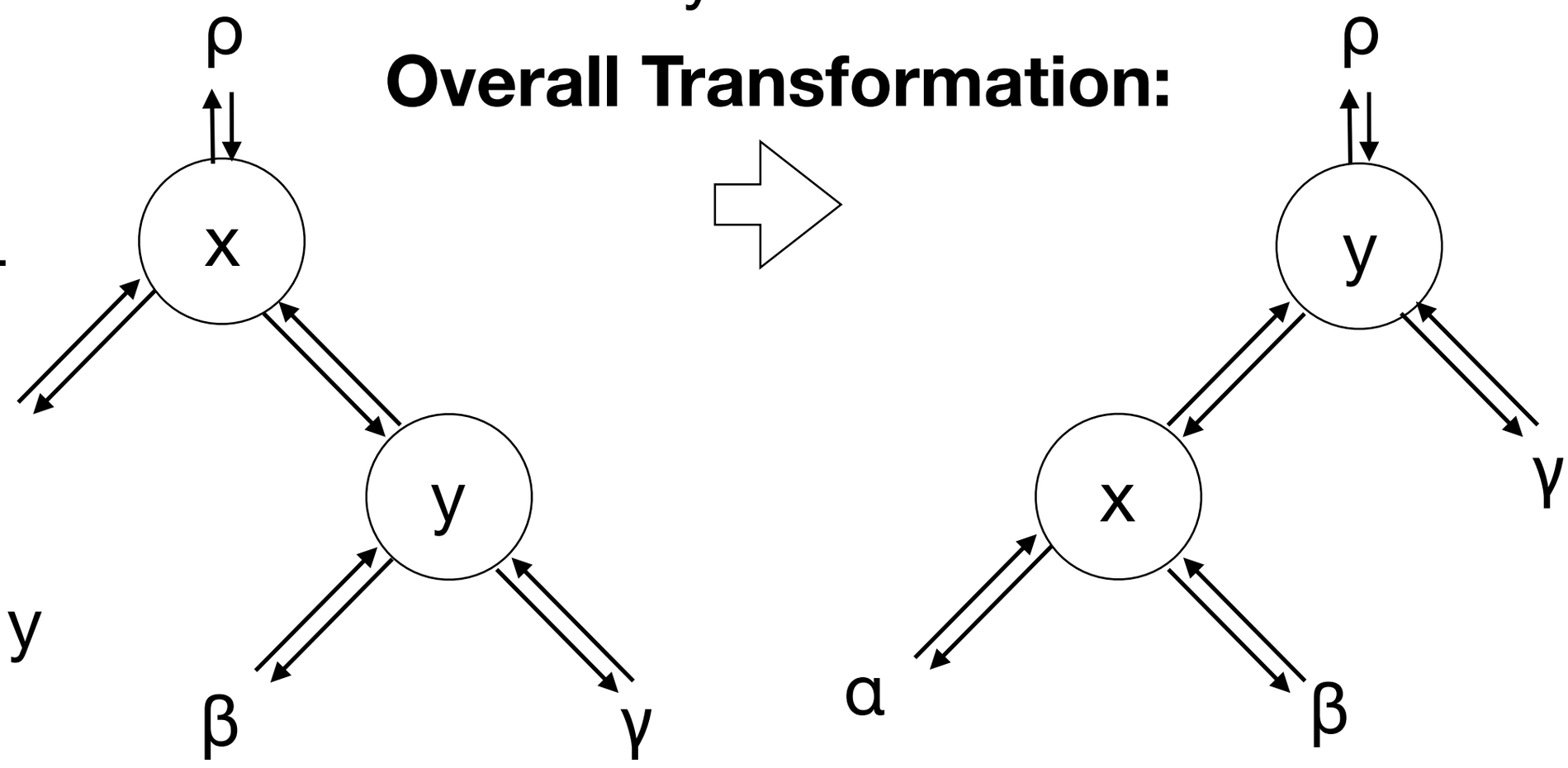
(**only** rearranged the picture)

Tree Rotations

Steps in left rotation (move y up to its parent's position):

1. Transfer β: x's right subtree becomes y's old left subtree (β)
2. Transfer the parent: y's parent becomes x's old parent
3. Transfer x itself: x becomes y's left subtree

Overall Transformation:



x.R gets y.L
y.L.p gets x

y.p gets x.p
p.[L/R] gets y

y.L gets x
x.p gets y

Pseudocode from CLRS

```
LEFT-ROTATE(T, x)
y = x.right                    // set y
x.right = y.left               // turn y's left subtree into x's right subtree
if y.left ≠ T.nil
    y.left.p = x
y.p = x.p                      // link x's parent to y
if x.p == T.nil
    T.root = y
elseif x == x.p.left
    x.p.left = y
else x.p.right = y
y.left = x                     // put x on y's left
x.p = y
```

1. xfer β

2. xfer parent

3. xfer x