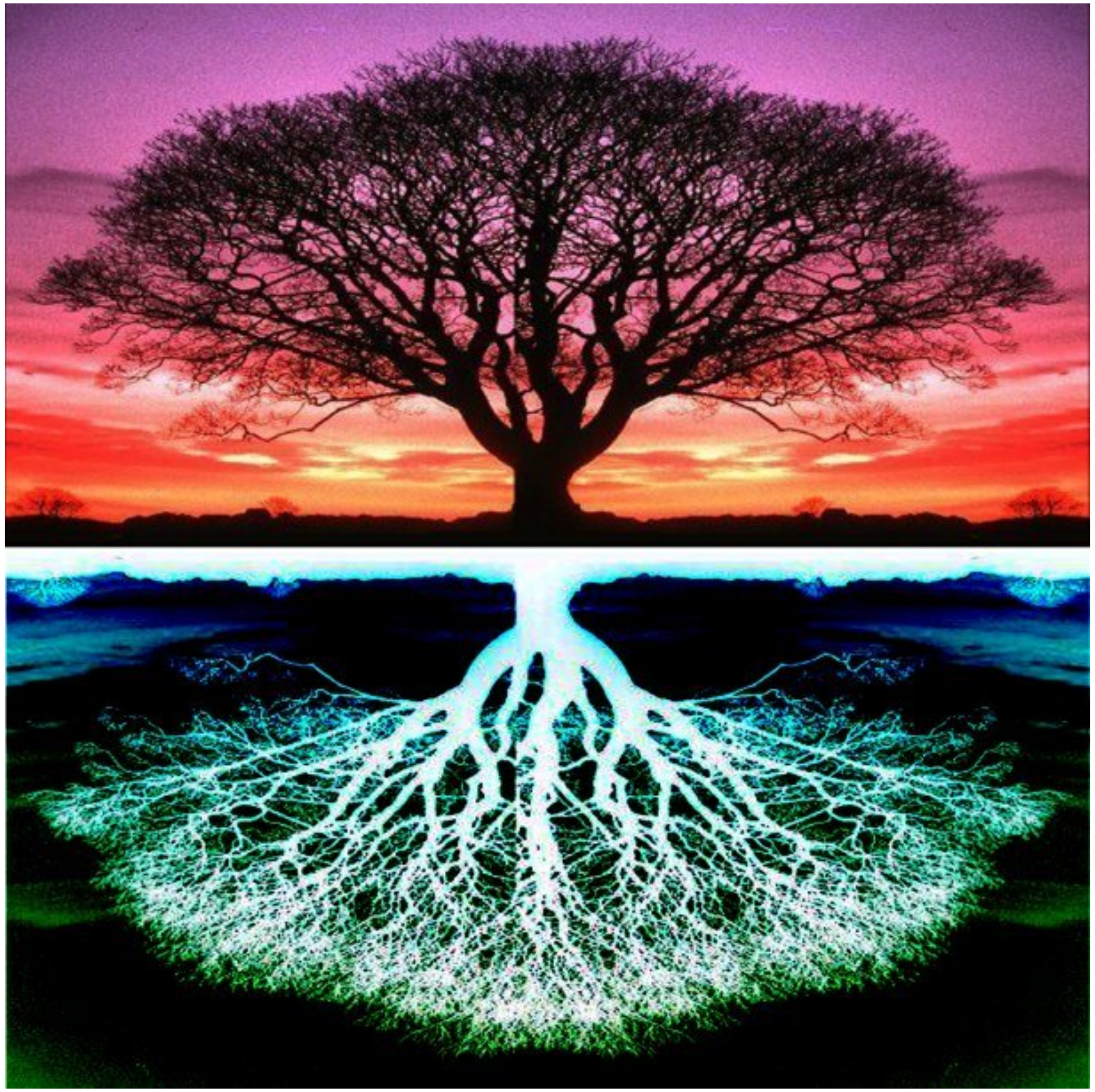


CSCI 241

Lecture 8
Introduction to Trees

Analyze ALL the Sorts!

	InsertionSort	SelectionSort	MergeSort	QuickSort	RadixSort
Best-case					
Average-case	same as worst	same as worst	same as worst		same as worst
Worst-case					
Stable?					
In-place?					

Announcements

- Nick’s office hours: Tuesdays 1-3pm, CF 167
- Events next week:
  - Monday, Oct. 15 – Tech Talk: Alaska Airlines – 5 pm in CF 110
  - Tuesday, Oct. 16 – Peer Lecture Series: CS Success Workshop – 4 pm in CF 420
  - Wednesday, Oct. 17 – Tech Talk: Integra Group – 5 pm in CF 125
  - Saturday & Sunday, Oct. 20 & 21 – Fall Game Jam! – 10 am in CF 105
- Regular club meetings:
  - AI Club - Tuesdays 6pm in PH 228 (talk to Sakari!)
  - Game Design Club - Mondays 6pm in CF 105 (talk to Kale!)
  - Others - see https://cse.wwu.edu/cs/cs-clubs

Goals:

- Understand the definition of a tree.
- Know the basic terminology associated with trees:
  - Root, child, parent, leaf, height, depth, subtree, descendent, ancestor
- Be able to write a tree class and some simple recursive processing methods.

Linked List

```
public class ListNode {
  int value;
  ListNode next;
}
```

Linked List

```
public class List {
  int value;
  List next;
}
```

The node ***is*** *the list*.

Next points to the **tail** of the list (also a list!)

Binary Tree

```
public class Tree {
  int value;
  Tree left;
  Tree right;
}
```

The node ***is*** *the tree*.

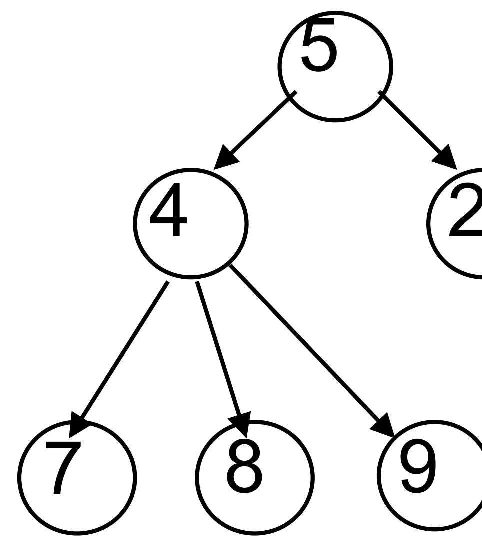
Tree - Definition

Tree: like a linked list, but:

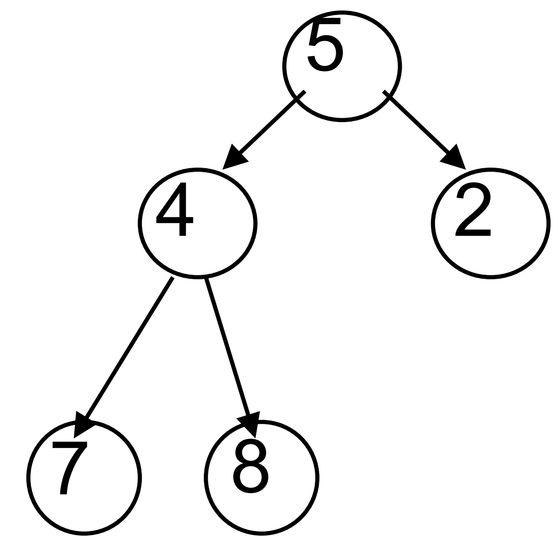
- Each node may have zero or more *successors* (**children**)
- Each node has exactly one *predecessor* (parent) except the *root*, which has none
- All nodes are reachable from *root*

Binary tree: A tree, but:

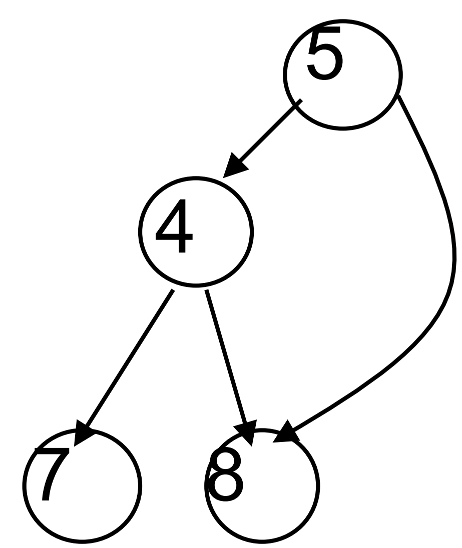
- Each can have at most **two** children (left child, right child)



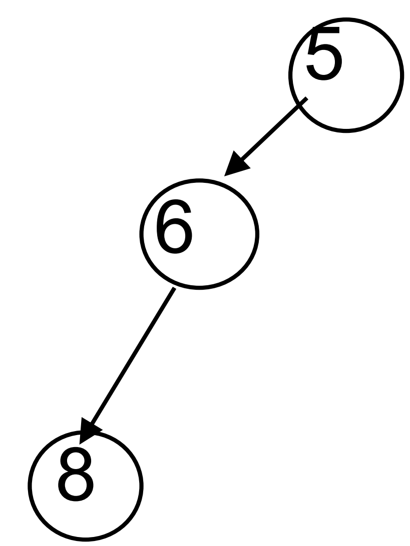
General tree



Binary tree



Not a tree



List-like tree

Tree Terminology

M is the ***root*** of this tree

G is the ***root*** of the ***left subtree*** of M

B, H, J, N, S are ***leaves*** *(have no children)*

N is the ***left child*** of P

S is the **right *child*** of P

P is the **parent** of N

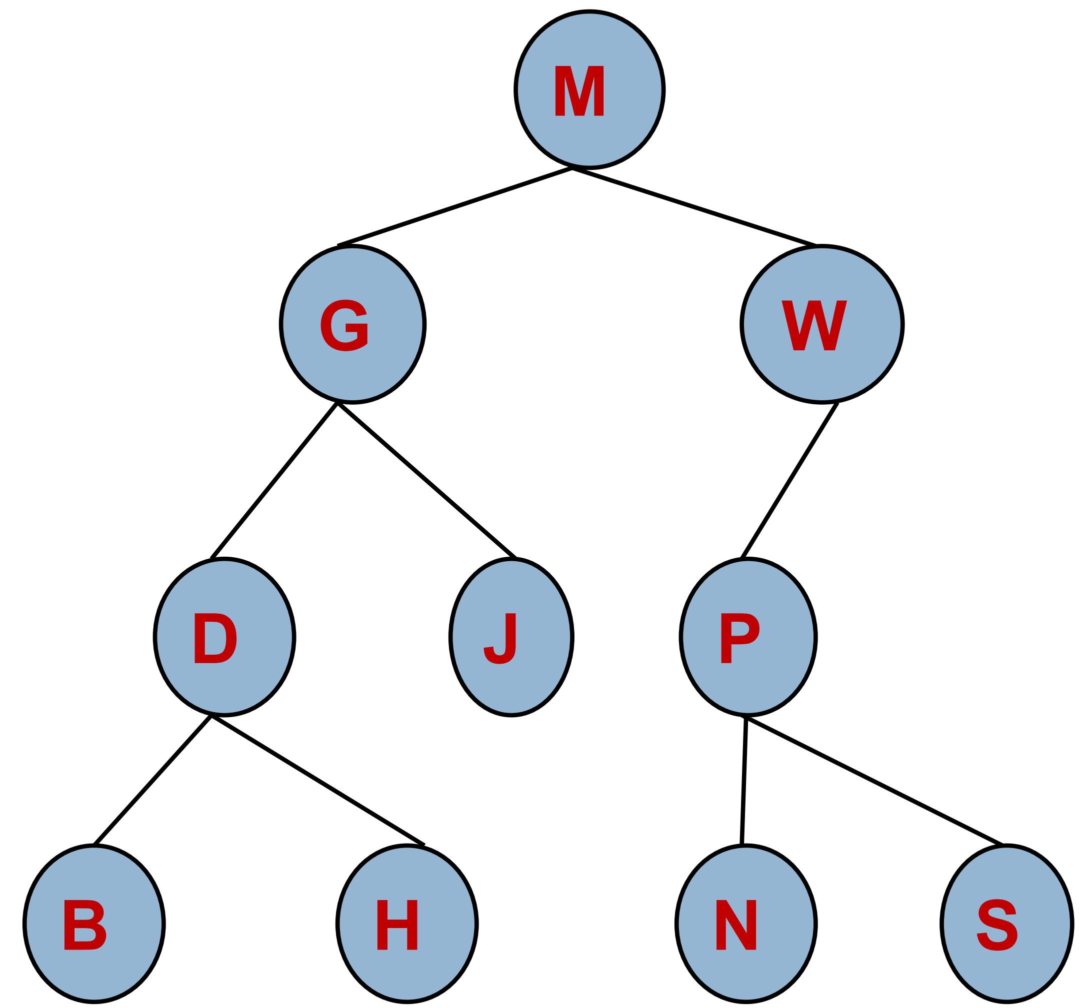
M and G are ***ancestors*** of D

P, N, S are ***descendants*** of W

J is at ***depth*** 2 (length of path from root)

The subtree rooted at W has ***height*** (length of longest path to a leaf) of 2

A collection of several trees is called a ____?



```
public class BinaryTreeNode {
  private int value;
  private BinaryTreeNode parent;
  private BinaryTreeNode left; // left subtree
  private BinaryTreeNode right; // right subtree
}
```

(null if no left child)

(null if no right child)

```
public class GeneralTreeNode {
  private int value;
  private GeneralTreeNode parent;
  private List<GeneralTreeNode> children;
}
```

Why do we need these?

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to represent **hierarchical structure**.

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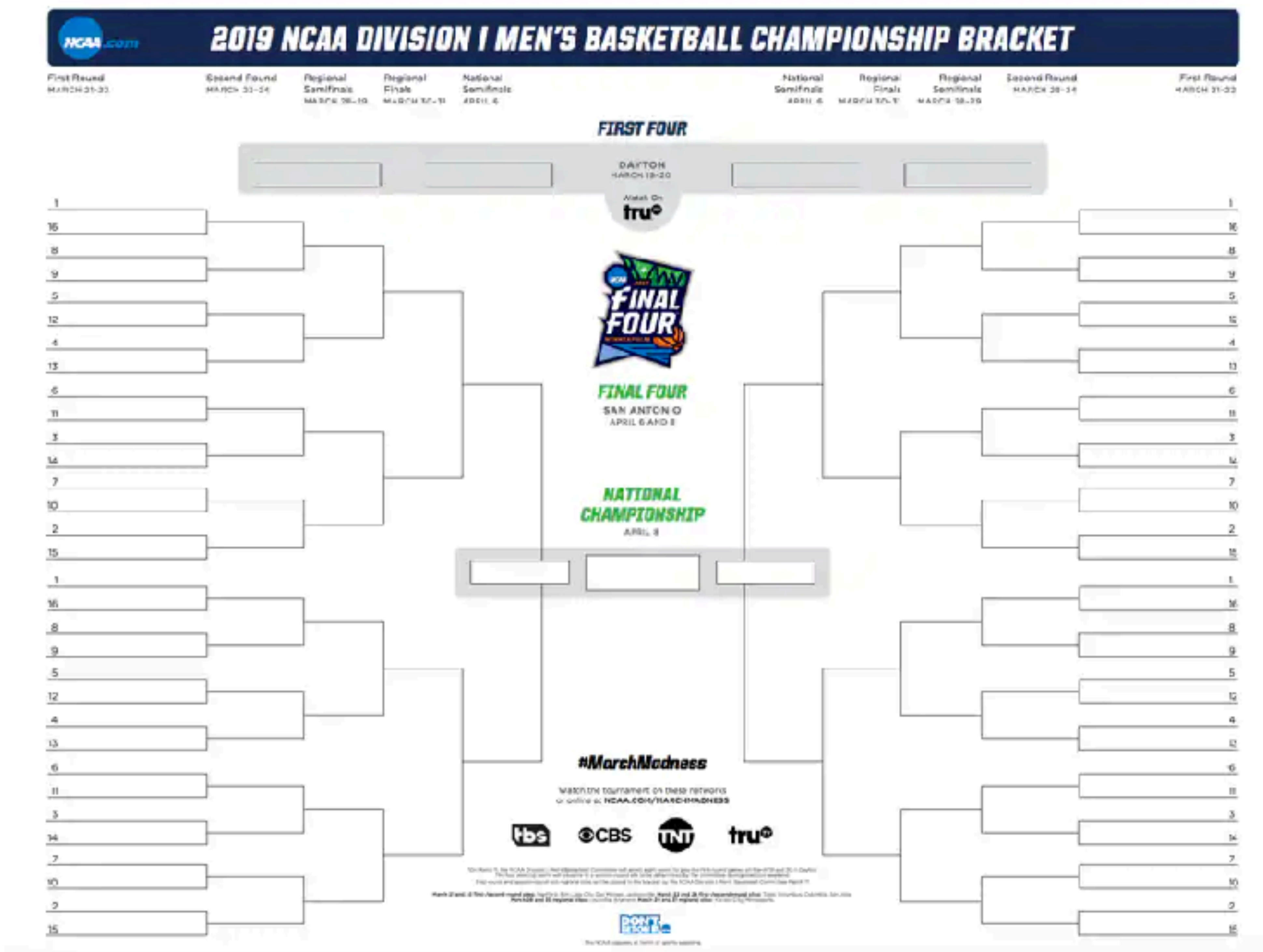

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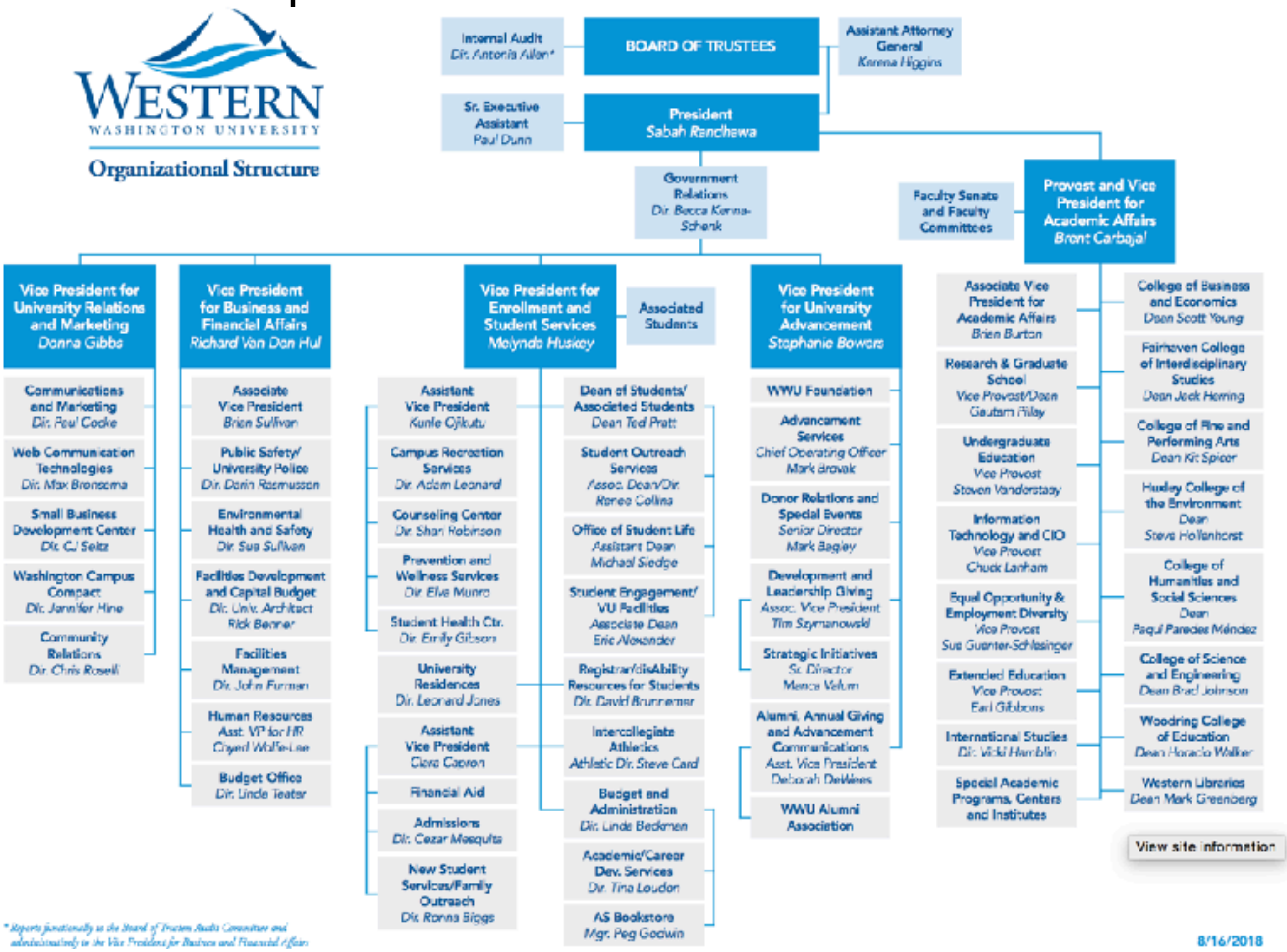

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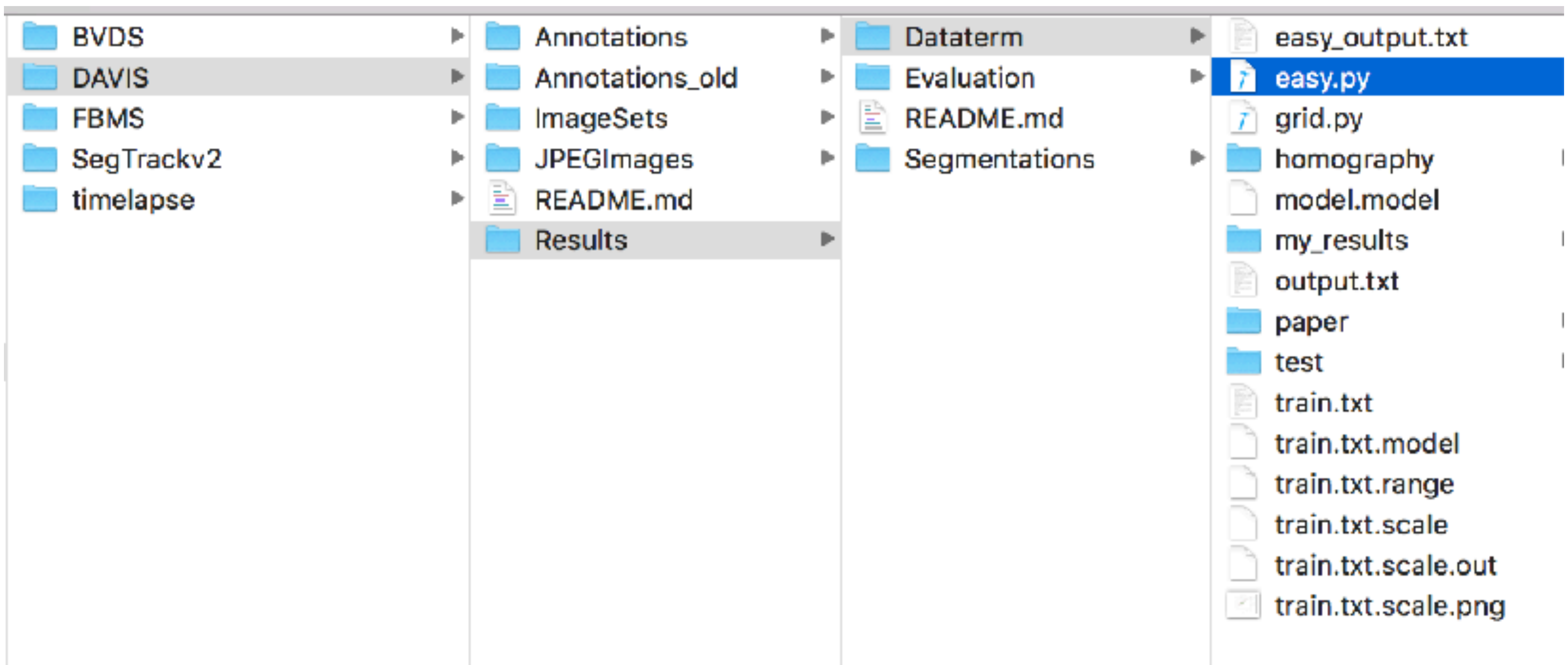


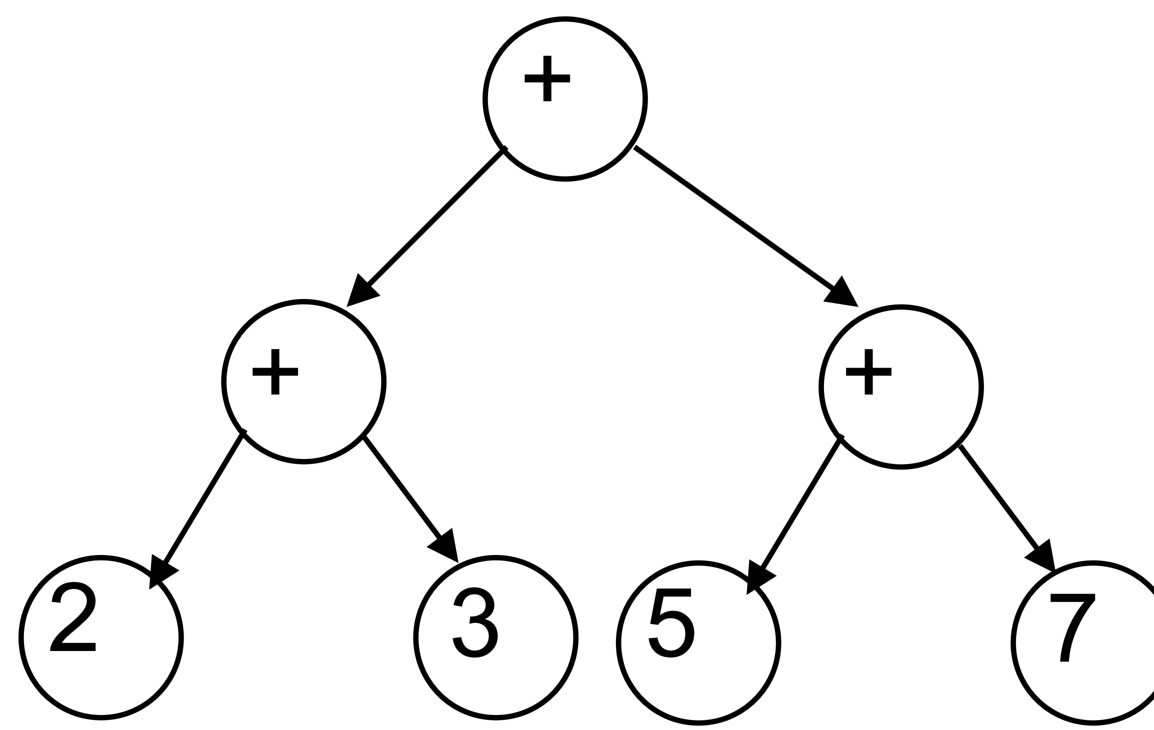
Why do we need these?

to represent **hierarchical structure**.

Syntax Trees:

- In textual representation, **parentheses** show hierarchical structure
- In tree representation, hierarchy is explicit in the tree's **structure**

((2+3) + (5+7))



Also used for **natural languages** and **programming languages**!

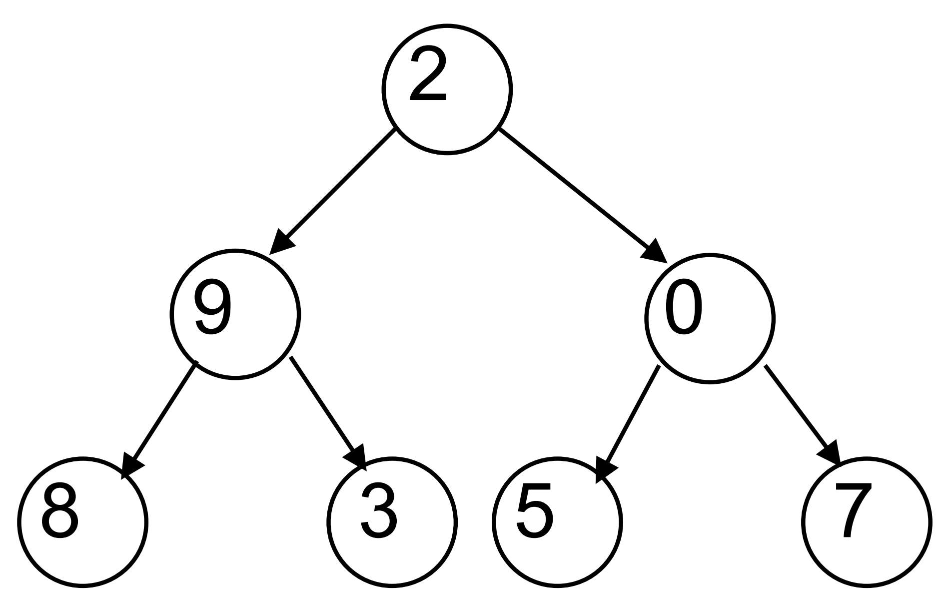
Why do we need these?

to implement various ADTs **efficiently**.

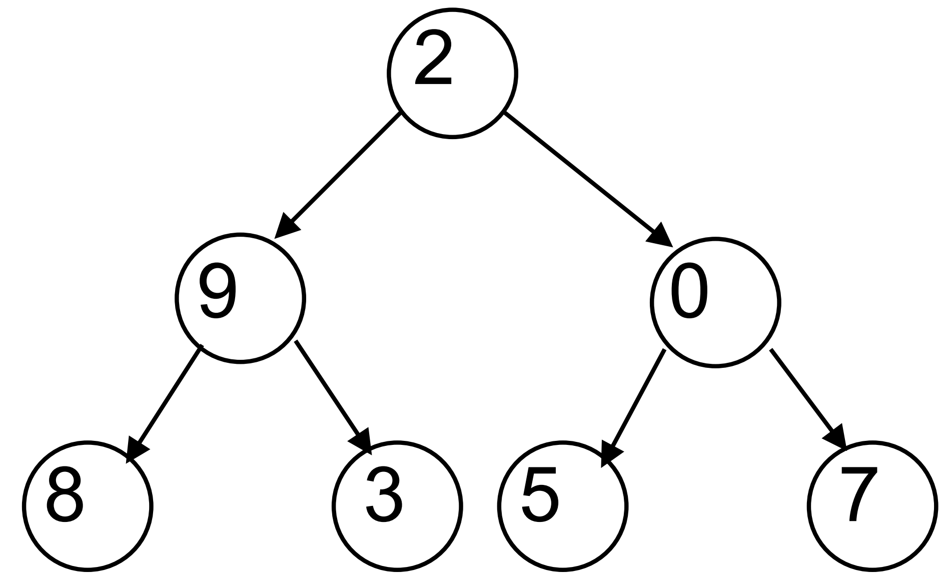
Tree**Set**, Tree**Map**

Height of a balanced binary tree is O(log n)

Consequence: Many operations (find, insert, …) can be done in **O(log n)** in carefully-designed trees.

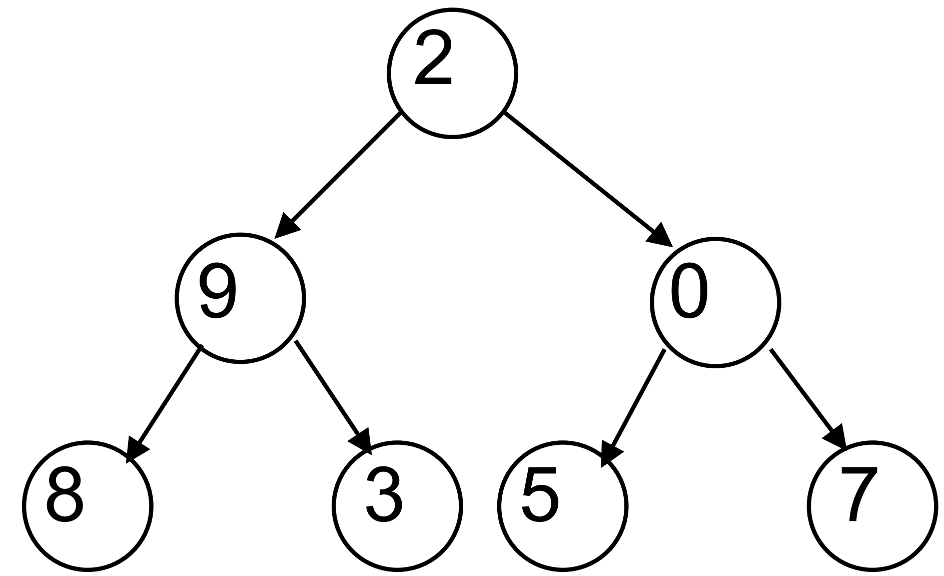


Thinking about trees recursively



- A **binary tree** is
  - Empty, or
  - Three things:
    - value
    - a left **binary tree**
    - a right **binary tree**

Thinking about trees recursively



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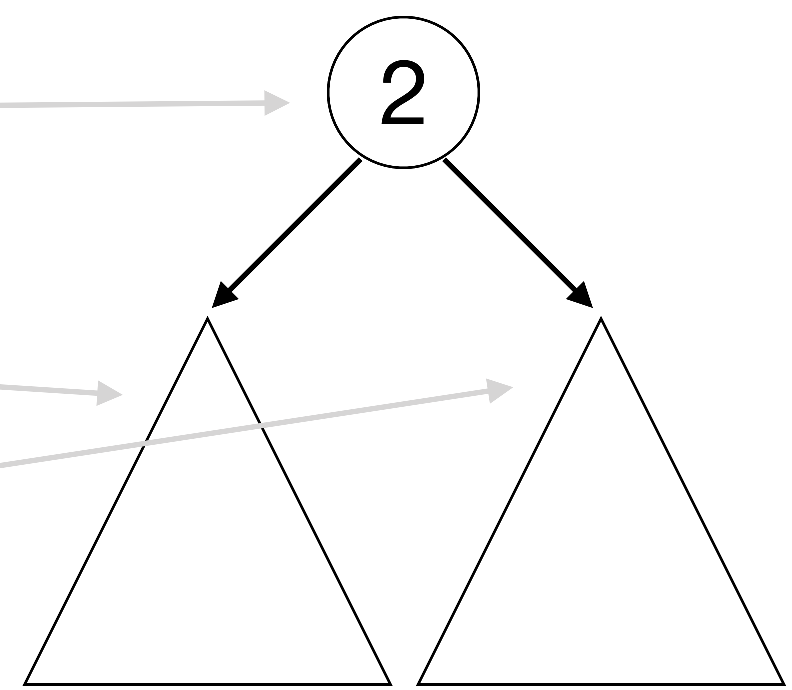

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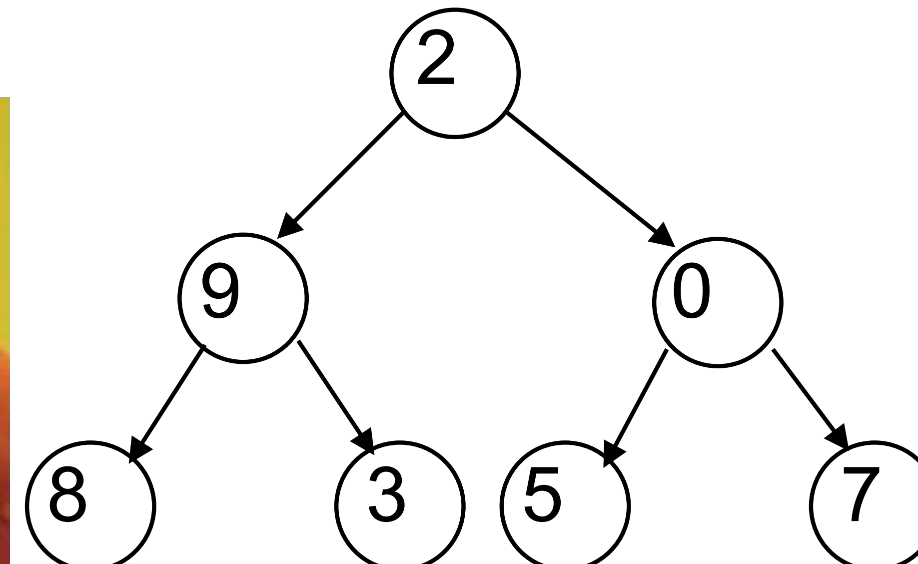

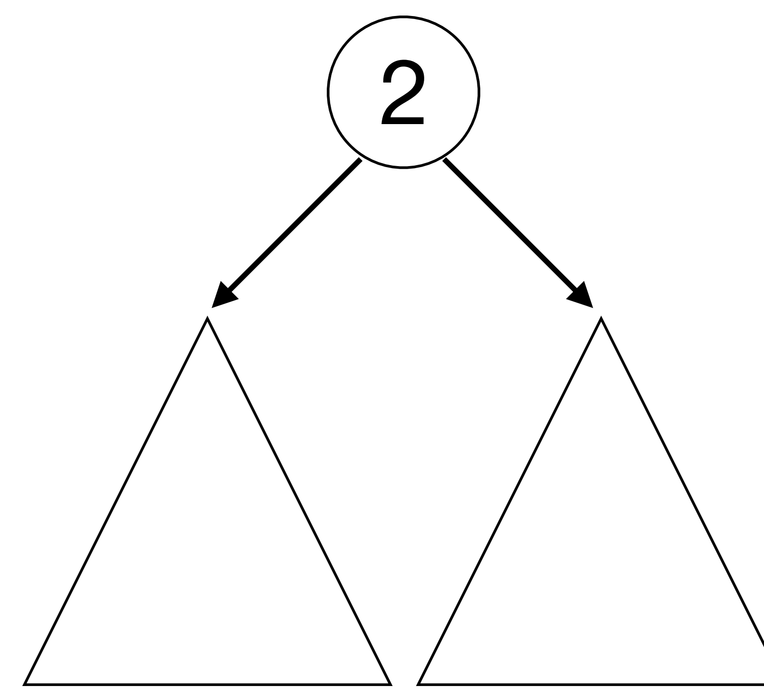

Operations on trees

often follow naturally from the definition of a tree:

- A **binary tree** is
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    - a right **binary tree**

Find v in a binary tree:

(base case - not found!)

(base case - is this v?)

(recursive call - is v in left?)

(recursive call - is v in right?)

Operations on trees

often follow naturally from the definition of a tree:

- A **binary tree** is
  - Empty, or
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    - value
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    - a right **binary tree**

Find v in a binary tree:

```
boolean findVal(Tree t, int v):
```

(base case - not found!)

```
if t == null:
   return false
```

(base case - is this v?)

```
if t.value == v: return true
```

(recursive call - is v in left?)

```
return findVal(t.left)
    || findVal(t.right)
```

(recursive call - is v in right?)

Tree Traversals

Print (or otherwise process) every node in a tree:

- A **binary tree** is
  - Empty, or
  - Three things:
    - value
    - a left **binary tree**
    - a right **binary tree**

Print all nodes in a binary tree:

```
boolean printTree(Tree t):
```

(base case - nothing to print)

```
if t == null:
   return
```

(print this node's value)

```
System.out.println(t.value)
```

(recursive call - print left subtree)

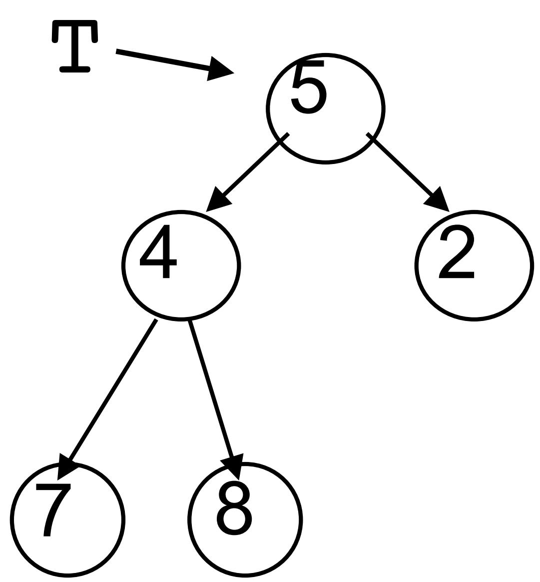
```
printTree(t.left)
```

(recursive call - print left subtree)

```
printTree(t.right)
```

Tree Traversals

Print (or otherwise process) every node in a tree:



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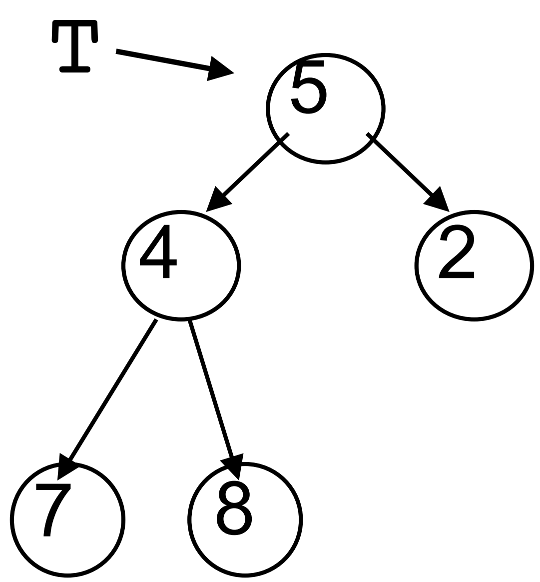
```

Tree Traversals

Print (or otherwise process) every node in a tree:



ABCD: `T` is a reference to the node with value 5. What is printed by the call `printTree(T)`?

A. 5 4 2 7 8
B. 7 4 8 5 2
C. 7 8 4 2 5
D. 5 4 7 8 2

Print all nodes in a binary tree:

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boolean printTree(Tree t):
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if t == null:
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(recursive call - print left subtree)

```
printTree(t.left)
```

(recursive call - print left subtree)

```
printTree(t.right)
```

Tree Traversals

"Walking" over the whole tree is called a tree traversal
This is done often enough that there are standard names.
Previous example was a **pre-order traversal**:

1. Process root
2. Process left subtree
3. Process right subtree

Other common traversals:

in-order traversal:

1. Process left subtree
2. Process root
3. Process right subtree

post-order traversal:

1. Process left subtree
2. Process right subtree
3. Process root

Why do we need these?

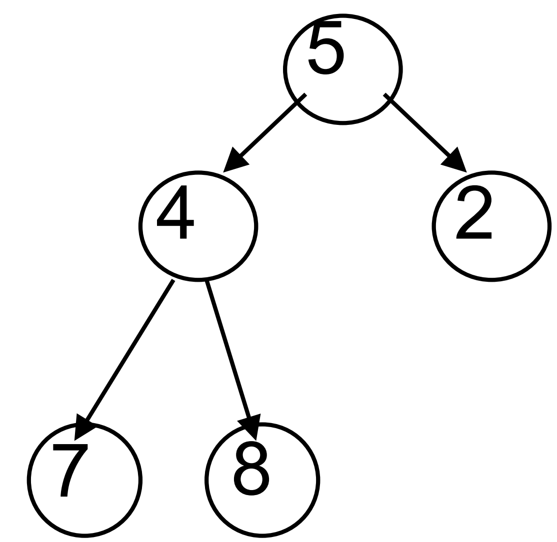
to represent **hierarchical structure**.

Quadtrees in graphics and simulation:
https://www.youtube.com/watch?v=fuexOsLOfl0

Practice Exercise

- Write the values printed by a:
  - pre-order
  - in-order
  - post-order

  traversal of this tree.



Terminology - Self-Quiz

root

subtree

leaf

child

parent

ancestor

descendant

depth

height