CSCI 241

Lecture 3:
Loop Invariants
Insertion and Selection Sort

Roadmap

- Tools for reasoning about algorithms
- Sorting algorithms (A1)
    - Insertion
    - Merge
    - Quick
    - Radix
- Onward to more data structures

Today

- Quiz recap
- Tools for reasoning about algorithms
- Sorting algorithms (A1)
  - Insertion
  - Selection
  - Merge
  - Quick
  - Radix
- Onward to more data structures

```java
public class LinkedList {
  public Node head;

  /** if n is in the list, remove all elements
    * *following* it and return true.
    * otherwise, return false leaving the list unchanged
    * precondition: n is not null */
  public boolean truncateAfter(Node n) {
    Node current = head;
    while (current != null) {
      if (current == n) {
        // INSERT CODE HERE


        return true;
      }
      current = current.next;
    }
    return false;
  }
```

```java
public class Node {
  Node next;
  int value;
}
```

```java
public class Factorial {
  /** return n factorial. precondition: n > 0 */

  public static int fact2(int n) {
    return n * fact2(n-1);
  }

  public static int fact3(int n) {
    if (n > 0) {
      return n * fact3(n-1);
    }
    return 1;
  }

  public static int fact4(int n) {
    if (n > 1) {
      return n * fact4(n);
    }
  }
}
```

```java
public class Factorial {
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    return 1;
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    if (n > 1) {
      return n * fact4(n);
    }
  }
}
```

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      return n * fact3(n-1);
    }
    return 1;
  }

  public static int fact4(int n) {
    if (n > 1) {
      return n * fact4(n);
    }
  }
}
```

Goals:

- Be able to analyze an algorithm's correctness using a given loop invariant.
- Be able to describe in words, implement using a loop invariant, and analyze the runtime of:
  - Selection sort
  - Insertion sort

Tools for Reasoning about Algorithms

Precondition, Postcondition

```
/** return the max value in A
  * precondition: A is nonempty
  * postcondition: max value of A is returned */
public int findMax(int[] A) {
  int max = A[0];
  // invariant: max is the largest value in A[0..i]
  for (int i = 1; i < A.length; i++) {
    if (A[i] > max) {
      max = A[i];
    }
  }
  return max;
}
```

Interface, not *implementation*

The precondition is true **before** method execution.
The postcondition is true **after** method execution.

Loop Invariant

```
/** return the max value in A
  * precondition: A is nonempty
  * postcondition: max value of A is returned */
public int findMax(int[] A) {
  int currentMax = A[0];
  // invariant: currentMax is the max of A[0..i]
  for (int i = 1; i < A.length; i++) {
    if (A[i] > max) {
      max = A[i];
    }
  }
  return max;
}
```

currentMax is the largest value in:

A[0..1]

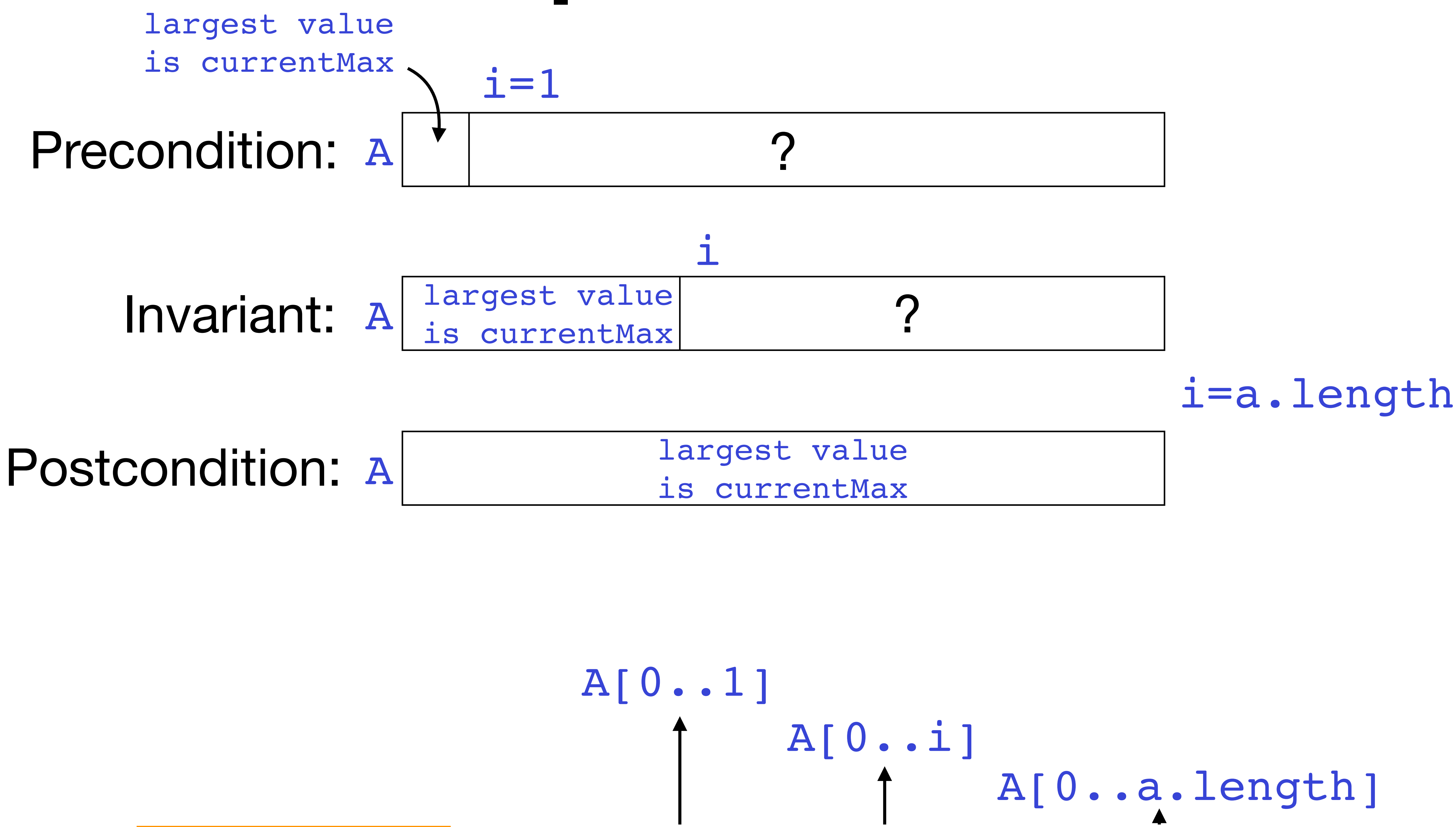
A[0..i]

A[0..a.length]

A loop invariant is true **before**, **during**, and **after** the loop.

(at the *end* of each iteration)

Loop Invariant



The loop invariant is true **before**, **during**, and **after** the loop.

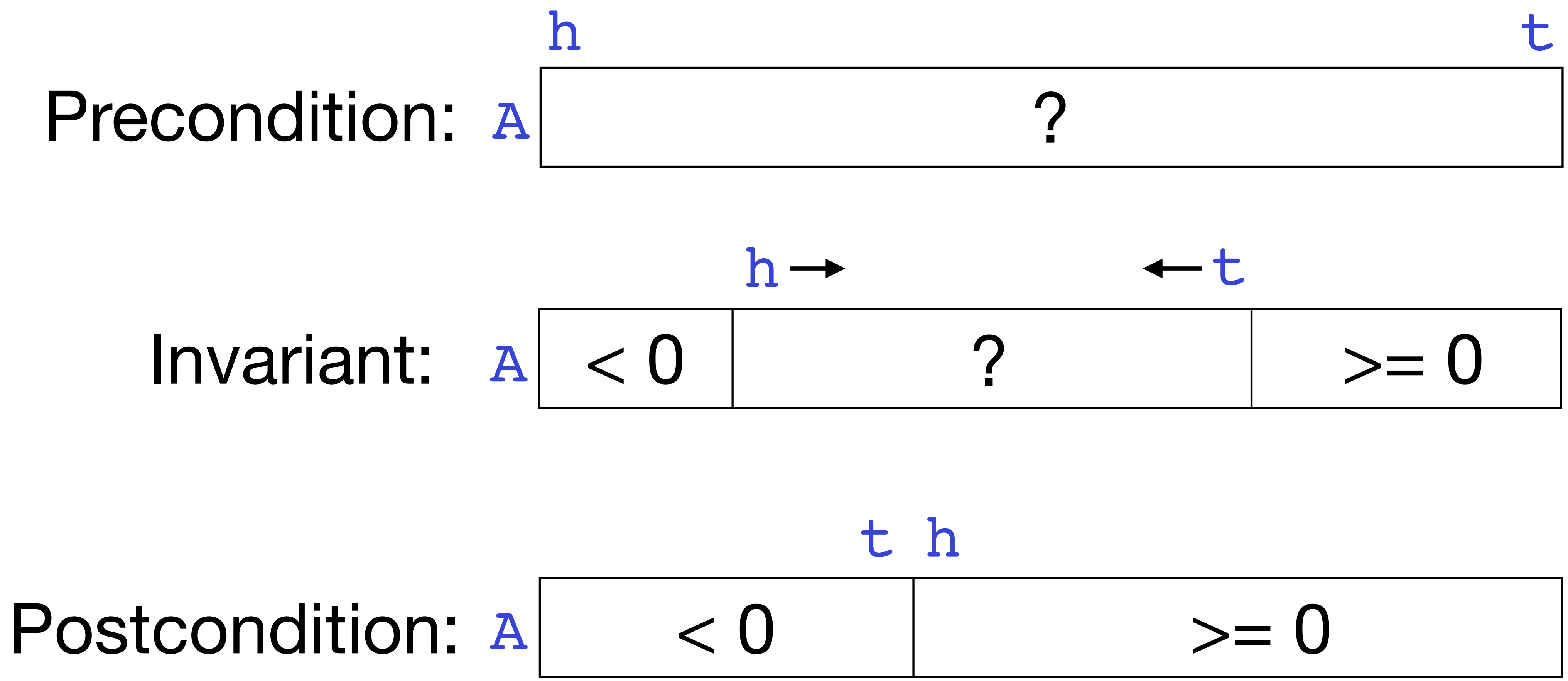
Another Example

```java
/** rearrange A so all negative values are to
  * the left of all non-negative values */
public void separateSign(int[] A);
```

Postcondition: `A`

< 0	>= 0

```java
/** rearrange A so all negative values are to
  * the left of all non-negative values */
public void separateSign(int[] A) {
```



Four concerns:

1. **Initialization**: Make the invariant true at the start.
2. **Termination**: Make the loop end when the postcondition is true.
3. **Progress**: Make progress towards the postcondition.
4. **Maintenance**: Make the invariant true after each iteration.

Insertion Sort

Insert A[i] into the sorted sublist A[0..i-1].

Selection Sort

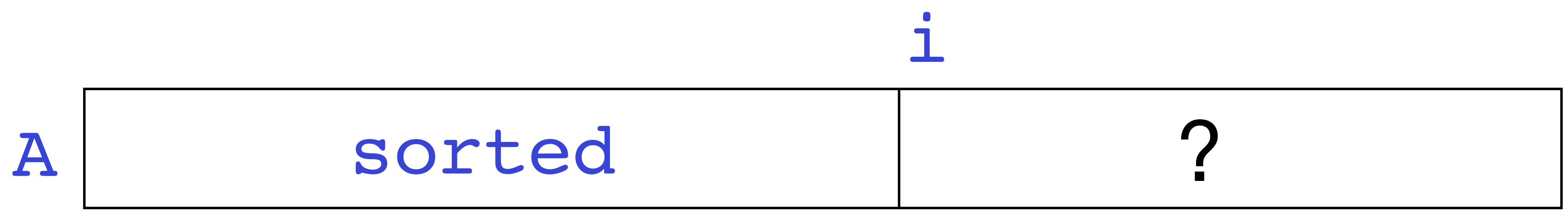
Find the smallest element in A[i..n] and place it at A[i].

https://visualgo.net/bn/sorting

Insertion Sort

Insert A[i] into the sorted sublist A[0..i-1].

Invariant:



Selection Sort

Find the smallest element in A[i..n] and place it at A[i].

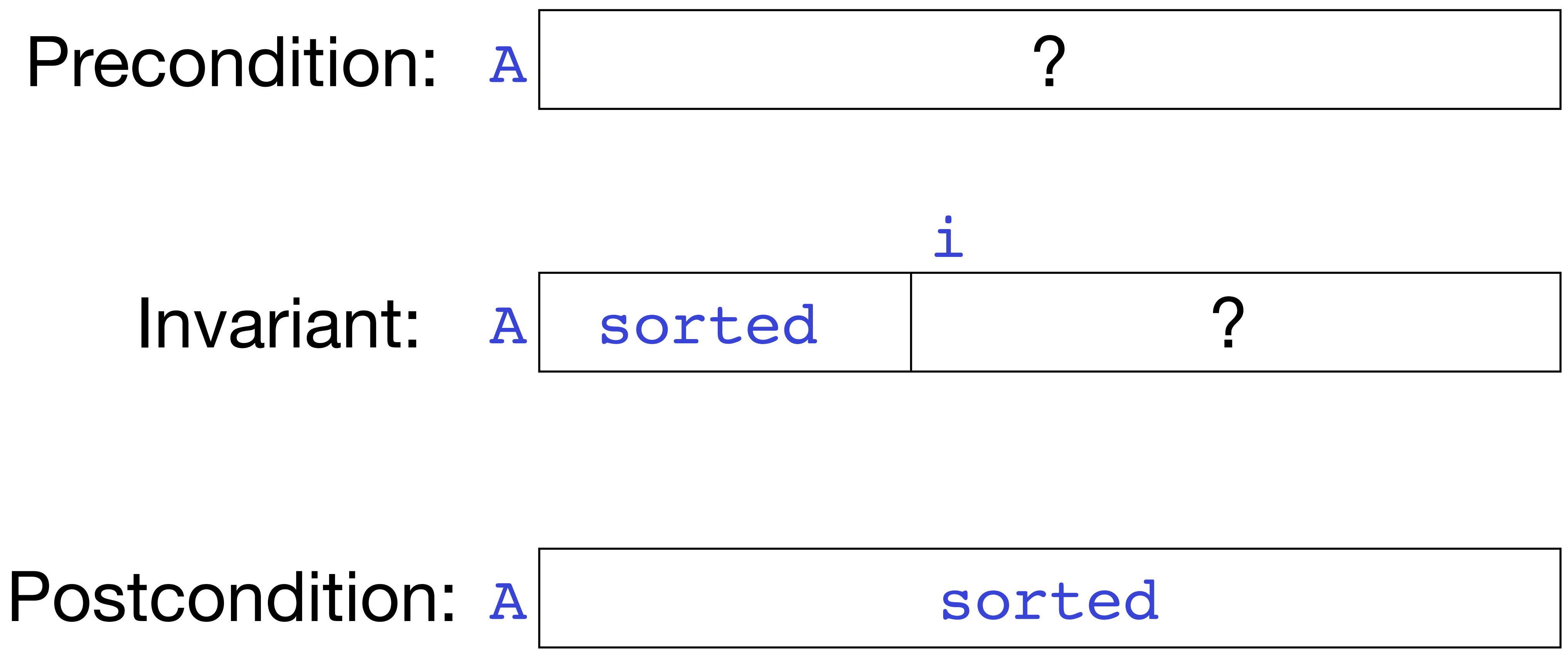
Invariant:

https://visualgo.net/bn/sorting

```
insertionSort(A):
  i = 0;
  while i < A.length:
    // push A[i] to its sorted position
    // increment i


selectionSort(A):
  i = 0;
  while i < A.length:
    // find min of A[i..A.length]
    // swap it with A[i]
    // increment i
```

Developing InsertionSort



Four tasks:

1. **Initialization**: Make the invariant true at the start.
2. **Termination**: Make the loop end when the postcondition is true.
3. **Progress**: Make progress towards the postcondition.
4. **Maintenance**: Make the invariant true after each iteration.

Developing SelectionSort

Precondition: `A` | ? |

Invariant: `A` | `sorted, <= A[i..n]` | ? | (with `i` marking the boundary)

Postcondition: `A` | `sorted` |

Four concerns:

1. **Initialization**: Make the invariant true at the start.
2. **Termination**: Make the loop end when the postcondition is true.
3. **Progress**: Make progress towards the postcondition.
4. **Maintenance**: Make the invariant true after each iteration.

```
insertionSort(A):
  i = 0;
  while i < A.length:
    j = i;
    while j > 0 and A[j] > A[j-1]:
      swap(A[j], A[j-1])
      j--
    i++
```

ABCD: What's the best and worst-case asymptotic runtime complexity of insertionSort?

	Best	Worst
A	O(n)	O(n)
B	$O(n^2)$	O(n)
C	O(n)	$O(n^2)$
D	$O(n^2)$	$O(n^2)$

Why is this best-case runtime interesting?

```
insertionSort1(A):
  i = 0;
  while i < A.length:
    j = i;
    while j > 0 and A[j] < A[j-1]:
      swap(A[j], A[j-1])
      j--
    i++
```

```
insertionSort2(A):
  i = 0;
  while i < A.length:
    j = i;
    tmp = A[i];
    while j > 0 and tmp < A[j-1]:
      A[j] = A[j-1]
      j--
    i++
```

ABCD: What's the best and worst-case asymptotic runtime complexity of insertionSort2?

	Best	Worst
A	O(n)	O(n)
B	$O(n^2)$	O(n)
C	O(n)	$O(n^2)$
D	$O(n^2)$	$O(n^2)$

Practice problems

Write a precondition, postcondition, and loop invariant for the *inner* loop of InsertionSort.

Develop SelectionSort using the precondition, loop invariant, and postcondition by completing the four tasks:

Initialization

Termination

Progress

Maintenance