CSCI 141

Scott Wehrwein

Binary representation

Goals

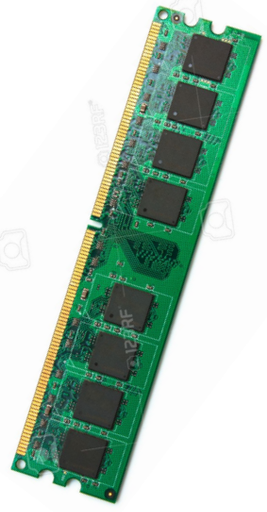
- Know how to convert a decimal integer to binary and vice versa.
- Understand the basic idea behind how strings and floating-point numbers are represented on computers.

Representing Numbers on Computers

- What happens “under the hood” when we execute:

```
result = 5
```

- The value 5 gets stored somewhere in main memory (and we somehow keep track of where it’s stored).



Main Memory

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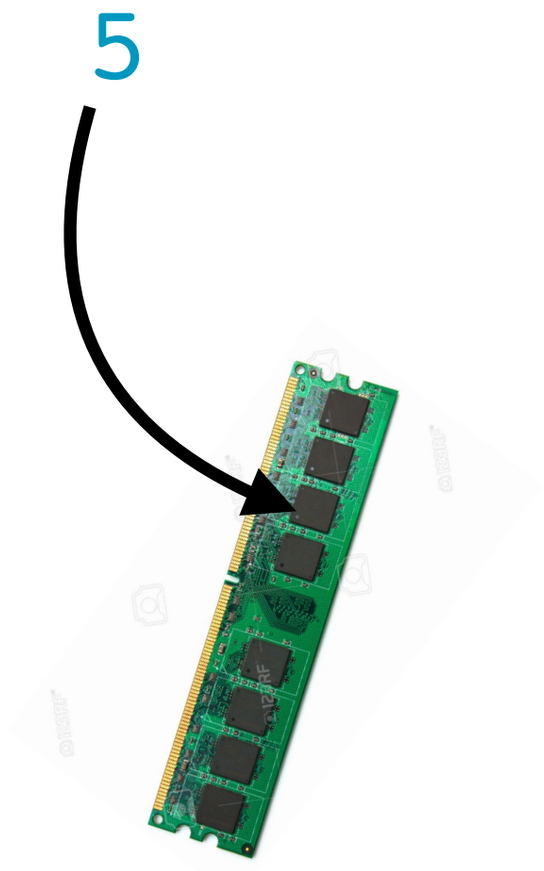
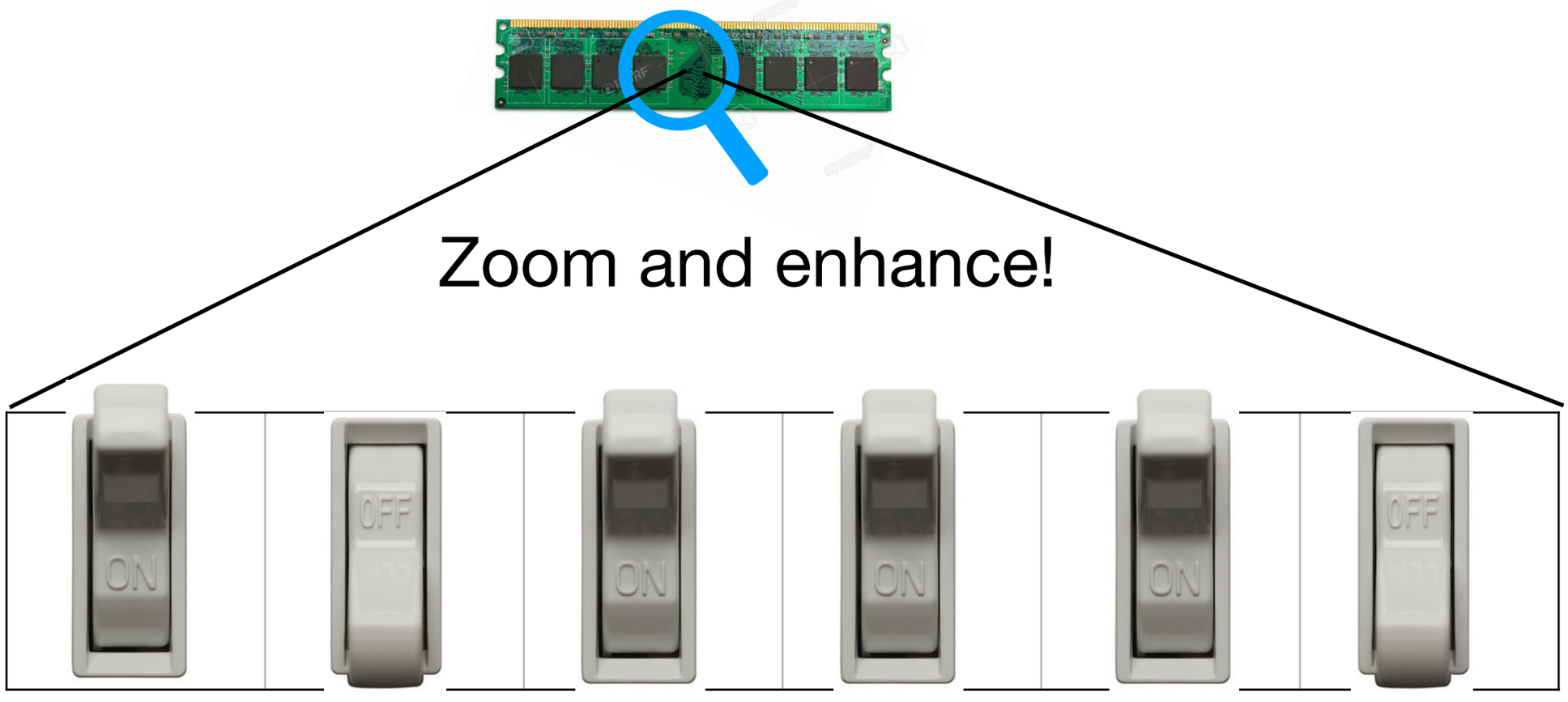


Main Memory

Representing Numbers on Computers

How are numbers stored in memory?

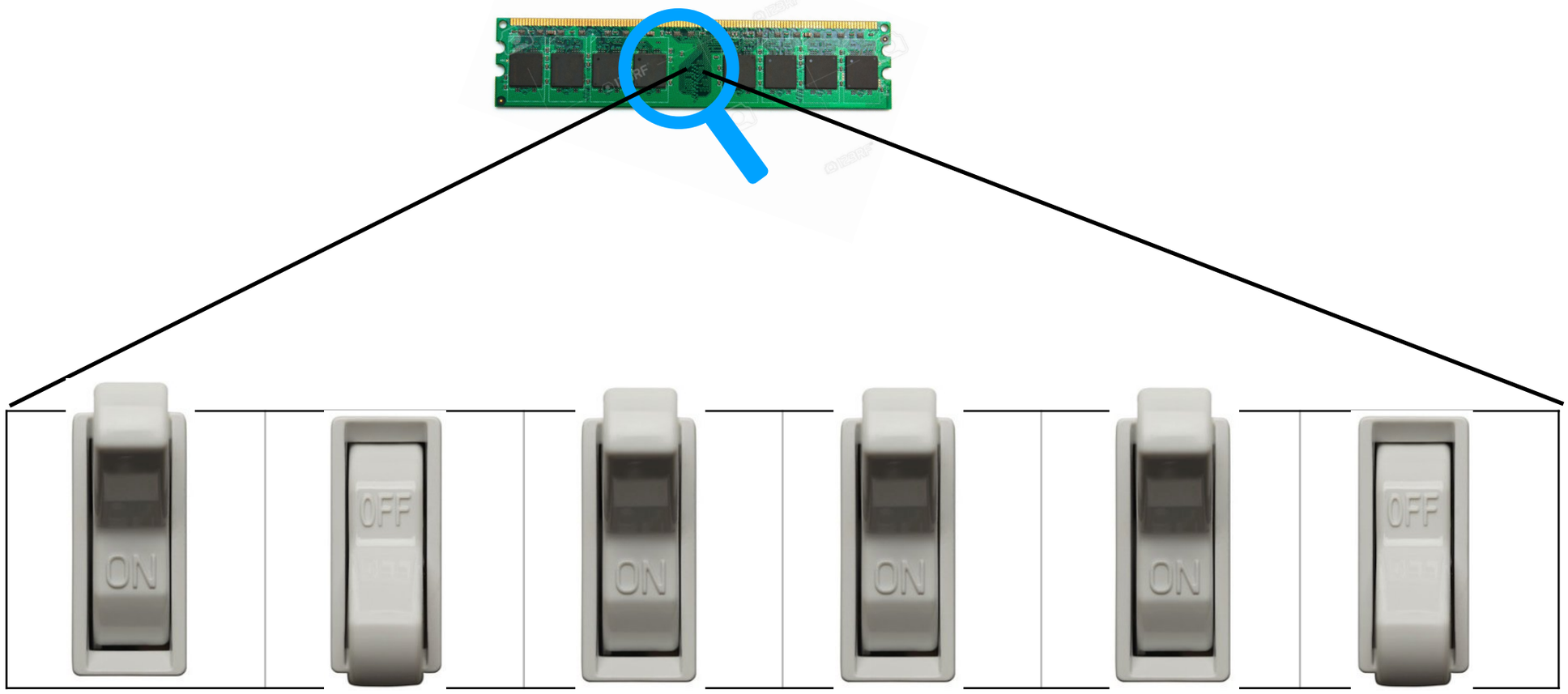


Memory is made of specialized electric circuits that provide cells that can “store” information by being in one of two states: on or off.

(Physics-wise: high or low voltage)

Representing Numbers on Computers

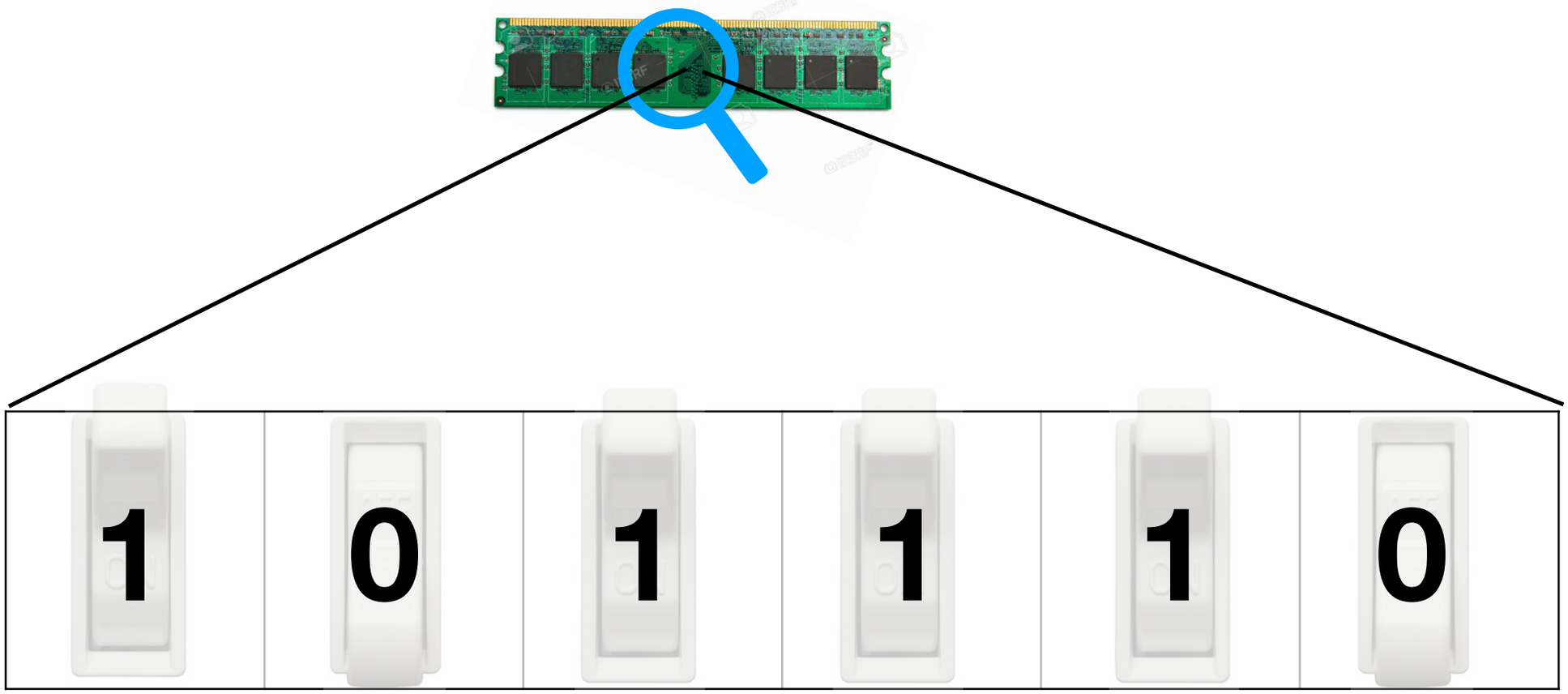
How are numbers stored in memory?



We impose mathematical meaning on these states:
“off” = 0
“on” = 1

Representing Numbers on Computers

How are numbers stored in memory?



We impose mathematical meaning on these states:
“off” = 0
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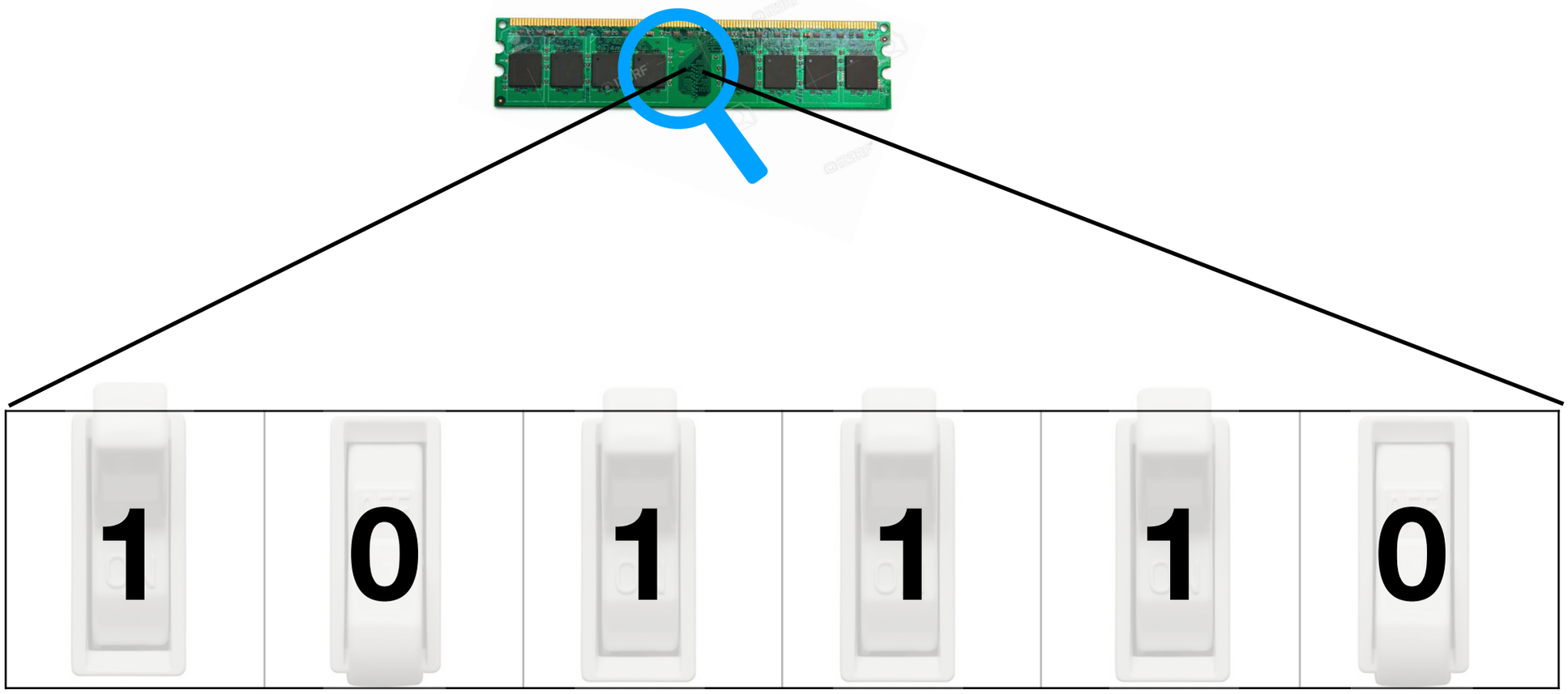
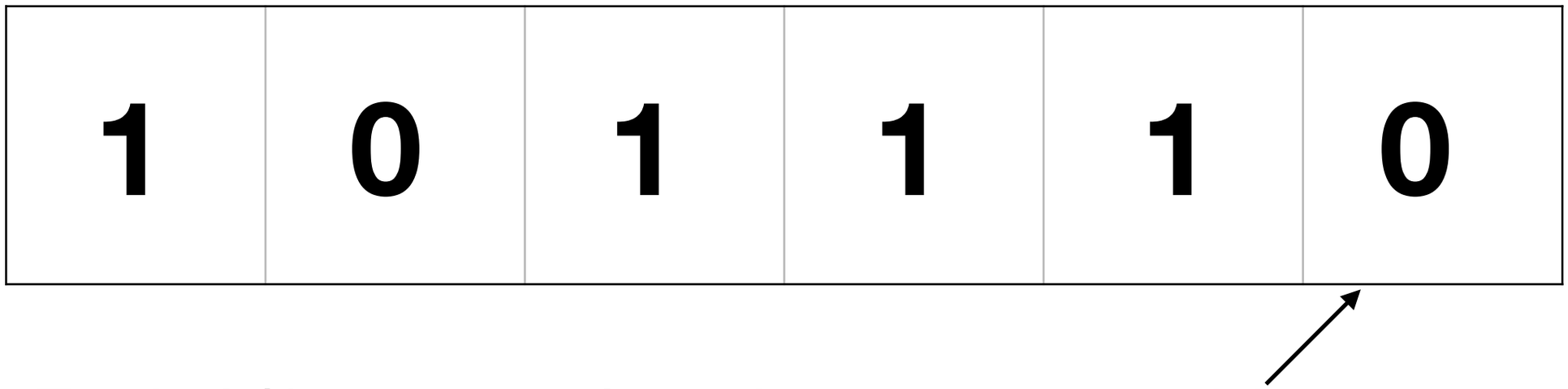
Representing Numbers on Computers

How are numbers stored in memory?



Each 1/0 memory location is called a bit.

Representing Numbers on Computers



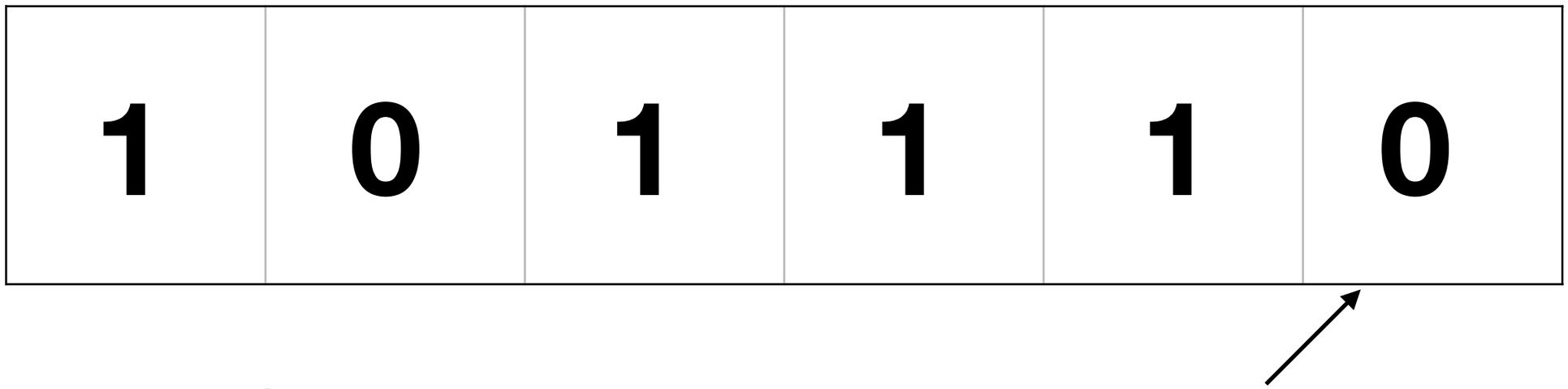
Each 0/1 memory location stores one bit.

8 bits is called a byte.

Metric prefixes are used to represent numbers of bytes, e.g. **kilo**, **mega**, **giga**, etc.

In computer science, the prefixes have slightly different meaning:
kilo is not 1000, it's **1024**.

Representing Numbers on Computers



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Usual SI prefixes:

- kilo = 10^3 = 1000
- mega = 10^6 = 1 million
- giga = 10^9 = 1 billion
- tera = 10^{12} = 1 trillion

Base 2 prefixes:

- kilobyte = 2^{10} = 1,024 bytes
- megabyte = 2^{20} = 1,048,576 bytes
- gigabyte = 2^{30} = 1,073,741,824 bytes
- terabyte = 2^{40} = 1,099,511,627,776 bytes

Binary Representation

If all we can store is 0's and 1's, how do we represent other numbers (e.g., 23?)

- By representing numbers in base 2 (binary) instead of base 10 (decimal).
- Observation:

In decimal:

$$104 = 1 * 10^2 + 0 * 10^1 + 4 * 10^0$$

(hundreds place) (tens place) (ones place)

- The decimal representation of a number is a sum of multiples of the powers of ten.

Binary Representation

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(hundreds place) (tens place) (ones place)

- Key idea: use 2 here instead of 10.

Binary to Decimal

1	0	1	1	1	1
2^5	2^4	2^3	2^2	2^1	2^0

- In binary, each digit represents a multiple of a power of **2**

Binary to Decimal

1	0	1	1	1	1
2^5	2^4	2^3	2^2	2^1	2^0

32 + 8 + 4 + 2 + 1 **= 47**

- In binary, each digit represents a multiple of a power of **2**
- 101111 in binary is 47 in decimal.

Decimal to Binary

Converting decimal to binary: go the other way.

Strategy: similar to making change.
Algorithm:

- Find the largest power of two less than the number.
- Put a 1 in that place and subtract the power of two
- Repeat with the remaining number until it's 0

Problem: write 23 as a sum of powers of 2

$$
\begin{aligned}
23 = {} & \boxed{?} * 2^4 \ (16) \\
+ \quad & ? * 2^3 \ \ (8) \\
+ \quad & ? * 2^2 \ \ (4) \\
+ \quad & ? * 2^1 \ \ (2) \\
+ \quad & ? * 2^0 \ \ (1)
\end{aligned}
$$

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$$
\begin{aligned}
23 = \ & \mathbf{1} * 2^4 \ (16) \quad (23 - \mathbf{1}*16 = 7 \text{ left}) \\
+ \ & ? * 2^3 \ (8) \\
+ \ & ? * 2^2 \ (4) \\
+ \ & ? * 2^1 \ (2) \\
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\begin{aligned}
23 = {} & \mathbf{1} * 2^4 \ (16) && (23 - \mathbf{1}*16 = 7 \text{ left}) \\
+ {} & \mathbf{0} * 2^3 \ (8) && (7 - \mathbf{0} * 8 = 7 \text{ left}) \\
+ {} & ? * 2^2 \ (4) \\
+ {} & ? * 2^1 \ (2) \\
+ {} & ? * 2^0 \ (1)
\end{aligned}
$$

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+ {} & \mathbf{0} * 2^3 \ (8) && (7 - \mathbf{0} * 8 = 7 \text{ left}) \\
+ {} & \mathbf{1} * 2^2 \ (4) && (7 - \mathbf{1} * 4 = 3 \text{ left}) \\
+ {} & ? * 2^1 \ (2) \\
+ {} & ? * 2^0 \ (1)
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+ \ & \mathbf{1} * 2^1 \ (2) && (3 - \mathbf{1} * 2 = 1 \text{ left}) \\
+ \ & ? * 2^0 \ (1)
\end{aligned}
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- Repeat with the remaining number until it's 0

Problem: write 23 as a sum of powers of 2

Answer: **10111**

$$
\begin{aligned}
23 = {} & \mathbf{1} * 2^4\ (16) && (23 - \mathbf{1}*16 = 7 \text{ left}) \\
+ {} & \mathbf{0} * 2^3\ (8) && (7 - \mathbf{0} * 8 = 7 \text{ left}) \\
+ {} & \mathbf{1} * 2^2\ (4) && (7 - \mathbf{1} * 4 = 3 \text{ left}) \\
+ {} & \mathbf{1} * 2^1\ (2) && (3 - \mathbf{1} * 2 = 1 \text{ left}) \\
+ {} & \mathbf{1} * 2^0\ (1) && (1 - \mathbf{1} * 1 = 0 \text{ left})
\end{aligned}
$$

That's how `int` is stored.

What about `str` and `float`?

How do you store strings?

Various conventions exist:
ASCII, Unicode

A `str` is a sequence of characters (letters, numbers, symbols)

1. Agree by convention on a number that represents each character.
2. Convert that number to binary.
3. Store a sequence of those numbers to form a string.

How do you store strings?

ASCII TABLE

Decimal	Hex	Char	Decimal	Hex	Char	Decimal	Hex	Char	Decimal	Hex	Char
0	0	[NULL]	32	20	[SPACE]	64	40	@	96	60	`
1	1	[START OF HEADING]	33	21	!	65	41	A	97	61	a
2	2	[START OF TEXT]	34	22	"	66	42	B	98	62	b
3	3	[END OF TEXT]	35	23	#	67	43	C	99	63	c
4	4	[END OF TRANSMISSION]	36	24	$	68	44	D	100	64	d
5	5	[ENQUIRY]	37	25	%	69	45	E	101	65	e
6	6	[ACKNOWLEDGE]	38	26	&	70	46	F	102	66	f
7	7	[BELL]	39	27	'	71	47	G	103	67	g
8	8	[BACKSPACE]	40	28	(	72	48	H	104	68	h
9	9	[HORIZONTAL TAB]	41	29	)	73	49	I	105	69	i
10	A	[LINE FEED]	42	2A	*	74	4A	J	106	6A	j
11	B	[VERTICAL TAB]	43	2B	+	75	4B	K	107	6B	k
12	C	[FORM FEED]	44	2C	,	76	4C	L	108	6C	l
13	D	[CARRIAGE RETURN]	45	2D	-	77	4D	M	109	6D	m
14	E	[SHIFT OUT]	46	2E	.	78	4E	N	110	6E	n
15	F	[SHIFT IN]	47	2F	/	79	4F	O	111	6F	o
16	10	[DATA LINK ESCAPE]	48	30	0	80	50	P	112	70	p
17	11	[DEVICE CONTROL 1]	49	31	1	81	51	Q	113	71	q
18	12	[DEVICE CONTROL 2]	50	32	2	82	52	R	114	72	r
19	13	[DEVICE CONTROL 3]	51	33	3	83	53	S	115	73	s
20	14	[DEVICE CONTROL 4]	52	34	4	84	54	T	116	74	t
21	15	[NEGATIVE ACKNOWLEDGE]	53	35	5	85	55	U	117	75	u
22	16	[SYNCHRONOUS IDLE]	54	36	6	86	56	V	118	76	v
23	17	[ENG OF TRANS. BLOCK]	55	37	7	87	57	W	119	77	w
24	18	[CANCEL]	56	38	8	88	58	X	120	78	x
25	19	[END OF MEDIUM]	57	39	9	89	59	Y	121	79	y
26	1A	[SUBSTITUTE]	58	3A	:	90	5A	Z	122	7A	z
27	1B	[ESCAPE]	59	3B	;	91	5B	[	123	7B	{
28	1C	[FILE SEPARATOR]	60	3C	<	92	5C	\	124	7C	|
29	1D	[GROUP SEPARATOR]	61	3D	=	93	5D	]	125	7D	}
30	1E	[RECORD SEPARATOR]	62	3E	>	94	5E	^	126	7E	~
31	1F	[UNIT SEPARATOR]	63	3F	?	95	5F	_	127	7F	[DEL]

Decimal	Hex	Char	Decimal	Hex	Char	Decimal	Hex	Char
32	20	[SPACE]	64	40	@	96	60	`
33	21	!	65	41	A	97	61	a
34	22	"	66	42	B	98	62	b
35	23	#	67	43	C	99	63	c
36	24	$	68	44	D	100	64	d
37	25	%	69	45	E	101	65	e
38	26	&	70	46	F	102	66	f
39	27	'	71	47	G	103	67	g
40	28	(	72	48	H	104	68	h
41	29	)	73	49	I	105	69	i
42	2A	*	74	4A	J	106	6A	j
43	2B	+	75	4B	K	107	6B	k
44	2C	,	76	4C	L	108	6C	l
45	2D	-	77	4D	M	109	6D	m
46	2E	.	78	4E	N	110	6E	n
47	2F	/	79	4F	O	111	6F	o
48	30	0	80	50	P	112	70	p
49	31	1	81	51	Q	113	71	q
50	32	2	82	52	R	114	72	r
51	33	3	83	53	S	115	73	s
52	34	4	84	54	T	116	74	t
53	35	5	85	55	U	117	75	u
54	36	6	86	56	V	118	76	v
55	37	7	87	57	W	119	77	w
56	38	8	88	58	X	120	78	x
57	39	9	89	59	Y	121	79	y
58	3A	:	90	5A	Z	122	7A	z
59	3B	;	91	5B	[	123	7B	{
60	3C	<	92	5C	\	124	7C	|
61	3D	=	93	5D	]	125	7D	}

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15	F	[SHIFT IN]	47	2F	/	79
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19	13	[DEVICE CONTROL 3]	51	33	3	83
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23	17	[ENG OF TRANS. BLOCK]	55	37	7	87
24	18	[CANCEL]	56	38	8	88
25	19	[END OF MEDIUM]	57	39	9	89

this is '\n': it's just another character!

That's how `str` works.

- What about `float`?
- It's harder to write 4.3752 as a sum of powers of two.

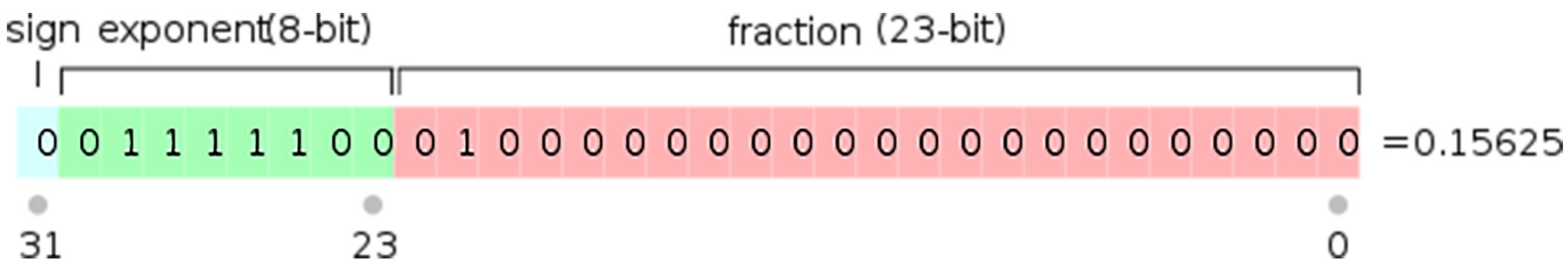
What about `float`?

- Floating-point numbers are stored similarly to scientific notation:

  1399.94 = +0.39994 * 10^3

- Need to store the sign, the base **and** the exponent.

- In memory, it looks something like this:



- Base and exponent are represented as base-2 integers

- Finite precision: not all numbers can be represented!