CSCI 141 - Spring 2021
Assignment 4: Functions

# 1 Overview

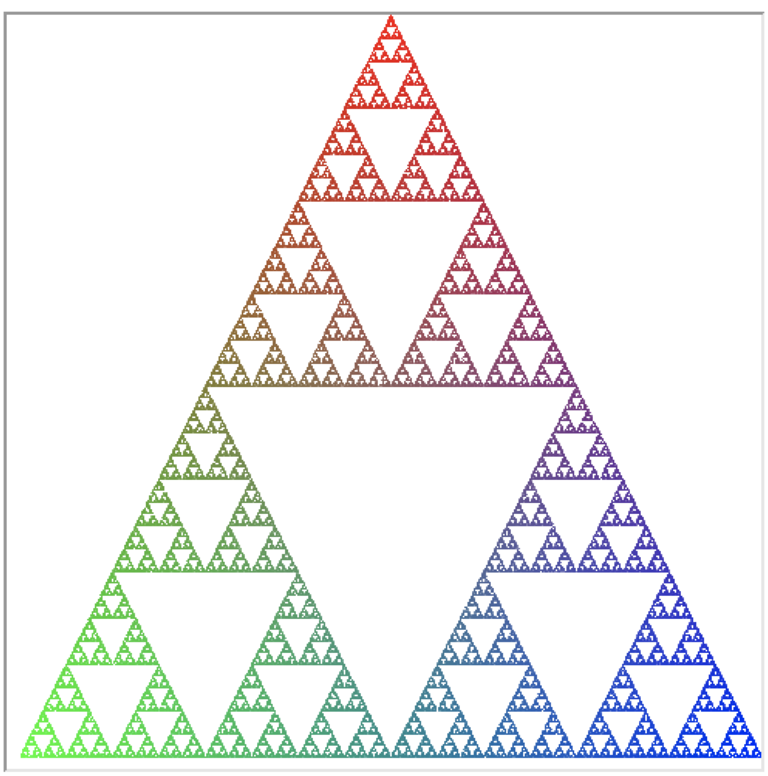
For this assignment you will write a Python program that draws a Sierpinski Triangle using a method called a "chaos game". A chaos game is an example of what's called a **zero-player** game, so called because we set things up a certain way to start, and the "game" unfolds deterministically based on a set of rules - there are no players involved.

The chaos game is played as follows. The user specifies the window size (say 300 by 300 pixels). Denote the three corners of a triangle $1, 2, 3$, where corner 1 is at the top center of the screen, corner 2 is in the lower left of the screen, and corner 3 is in the lower right of the screen.

First, a corner of the triangle is chosen at random. Then, at each step of the game, a dot is drawn at the midpoint between the current location (where the prior dot was drawn) and a randomly-chosen corner of the triangle. Here's some pseudocode to help you understand how the chaos game works:

```
p = a random corner of the triangle
loop 10000 times:
    c = a random corner of the triangle
    m = the midpoint between p and c
    choose a color for m
    color the pixel at m
    p = m
```

This process will generate a Sierpinski Triangle like the one pictured below.



## 2 Details

### Skeleton Code

You are provided with a skeleton code file called `sierpinski.py`. This file contains some code to help get you started, including a function that sets up the Turtle graphics window for our somewhat nontraditional turtle use case. Read through the skeleton code. You need not understand every detail of the code I've written, but you must **carefully read and understand the specification for the `turtle_setup` function**: this takes care of creating a turtle, resizing the window to the desired dimensions, and several other details that I don't want you to have to write the code for. Your code should call `turtle_setup` function before beginning the chaos game iterations, and proceed to use the turtle it returns to do all your pixel coloring.

The `turtle_setup` function changes the window so its coordinate system now has (0,0) at the bottom left corner (instead of the default, with (0,0) in the center. The positive $x$ axis points right and positive $y$ axis points up, so the top left corner is at (`0, canv_height`), the bottom right corner is at (`canv_width, 0`), and the top right is at (`canv_width, canv_height`). This helps to simplify the math when locating corners of the triangle. The setup function also calls `tracer(0, 0)`, which you may recall disables automatic re-drawing of the canvas. This means that to get your picture to show up, **you need to call `turtle.update()`** yourself. For the sake of speed, I recommend re-drawing the picture only every 100 or every 1000 iterations so the drawing doesn't take too long.

### Coloring Pixels with Turtles

In this program, we're not really using turtles for what they were meant for. Instead of drawing lines as the turtle moves, we'll use the turtle to color individual pixels on the canvas. Turtles draw as they move, but they can also stamp shapes, such as circles and dots; we'll make use of the aptly named `dot` method. To fill in a pixel, all you need to do is move the turtle to that pixel, then draw a dot of size 1. If nothing shows up when you use a dot size of 1 (this seems to happen on some Windows systems), using a dot size of 2 is also fine.

When the turtle draws via movement with the pen down, or via other methods such as `dot`, the color it draws is determined by the turtle's current color. You can change the turtle's current color using the (again, aptly named) `color` method. One way to specify colors is using various standard color names (`"red", "green", "purple"`, etc.). A more flexible way is to specify how much red, green, and blue you want: some combination of these three primary colors can represent all colors that your screen can display. When storing images on computers, we typically store each R,G, and B value using a single byte (8 bits). That means a color is represented by three numbers from 0 to 255, which is the maximum number representable using 8 bits. For instance $(255, 0, 0)$ is red, $(0, 255, 0)$ is green and $(0, 0, 255)$ is blue. Furthermore, $(255, 255, 255)$ is white and $(0, 0, 0)$ is black.

In the figure on the first page, you can see that the colors of the pixels are related to their coordinates. If you simply followed the pseudocode at the top of this document, but chose black for the color every time, then you'd have a black and white version of the Sierpinski triangle. Once you have that working, you should figure out how to make the triangle prettier. The color scheme used in the example above chooses each color value based on the distance from one of the corners. In particular, the red color scales from 255 to 0 based on distance from corner 1, the green scales with distance from corner 2, and blue scales with distance from corner 3. You may choose a different scheme, but your colors should have the following properties:

- Colors appear in a smooth gradient across the triangle
- Each corner should have a value of 255 for one of the "base" colors (red, green, and blue)

- The farthest point from a given corner should have a value of 0 for that corner's color
- A corner that is closer than the farthest point does not necessarily have zero. For example, the bottom left corner of the 100x300 sample output does not have blue value 0, because it's a lot closer than the top corner.
- My coloring scheme is based solely on distance from the corner; other approaches are ok too, as long as the above requirements are met.

This is not too difficult in a square window but you need to be a little more careful in a rectangular window (when the width of the window is not equal to the height of the window).

## 3 Suggested Approach

This may seem like a big problem to solve all at once; in fact it is. A good strategy for tackling a big problem is to turn it into a collection of small ones: break the problem down into small pieces, write functions to solve each piece, then put them together into a solution to the full program.

1. In L12B, I wrote a `midpoint` function. This will come in handy, so I've included in the skeleton.

2. I included the pseudocode for the chaos game in the skeleton file. This is a handy way to keep track of your overall program structure: start with pseudocode and piece-by-piece fill in code that accomplishes each of the steps. Because each step has some complexity to it, **I recommend defining a function that takes care of the details of each step**. That way, the code in your main program will end up corresponding fairly closely to the lines of the pseudocode, and it will be easy to understand.

3. Based on the pseudocode, decide what functions you'd like to have in order to make the algorithm easy to implement. In my solution, I have almost one-to-one correspondence between functions and lines in the pseudocode. To give one example, to choose a color for the point `m`, I have a `choose_color` function. It takes a point and the three corners and calculates the RGB color values based on distance of the point from each of the colors. This function in turn makes use of another function that calculates the distance between two points.

4. Instead of immediately starting to code each function you've decided to write, try this instead: write out the specification (docstring) for the function. This means deciding what the function takes as arguments and what it returns. Once you have this, try sketching out the code for the chaos game, using the functions (even though you haven't written them yet!). In doing this, you may discover changes that you want to make to your function specifications—make them now so you don't have to rewrite the code.

5. Now, go implement each of your functions. Start with the ones that will be needed to draw the triangle in black. After finishing one function, **test it**. Use the interactive shell and/or put code in your main program that checks whether the code does what you expect it to. For example, to test my `choose_color` function, I first tried passing in each corner: I made sure the top corner gave me $(255, 0, 0)$ back, and so on for all three corners. Then I tried the bottom middle point on the canvas, because it's easy for me to calculate that its blue and green values should be about 128 (it's equidistant from the green corner and the blue corner). Then test the center point - its RGB values should all be equal because it's equidistant from all three corners.

6. Finally, turn your sketch of the overall chaos game algorithm into real code that uses your functions to draw the Sierpinski triangle. Make sure it works with different square window sizes first (e.g., 200 by 200, 300 by 300). Then try testing it with unequal width and height (e.g., 200 by 300).

## 4 Testing

In past assignments, we've mostly been testing our entire program at once. As our programs get more and more sophisticated, it becomes increasingly difficult to test and debug an entire program at once, because there are simply too many places a bug could be. It's always a good idea to test as small a piece of code as possible so bugs are easier to find. Now that we're breaking our program into smaller functions that solve individual pieces of the problem, we can test each function independently of the rest of the program.

You have been provided with a second skeleton file called `sierpinski_test.py`. In this file, I've written two functions so far. The `check_equal` function checks two values for equality and prints a message saying whether the test case passed (if the values are equal) or failed (if they are not equal). The `test_midpoint` function tests whether the `midpoint` function, imported from `sierpinski.py`, works correctly given several different sets of inputs[1]

For this assignment, I'd like you to write a function similar to `test_midpoint` that thoroughly tests one of the functions you write in `sierpinski.py`. You may write tests for a function of your choice, or for multiple functions if you wish. In my solution, some of my functions do not return values but have effects, such as drawing something with a turtle. These functions are not so well suited to this style of testing. You can still test such functions using the interactive shell and/or by writing small programs—it's just less straightforward to write code that tells you whether the test passed or failed. Your tests should follow these guidelines:

- The test function's name should match the name of the `sierpinski.py` function it tests, but prefixed with `test_`.
- Your test function should make use of the `check_equal` function for all of its output.
- The tests should be as thorough as possible. The easiest way to come up with test cases is to work out (on paper, or in your head) the value you expected for a given set of inputs, then then verify that your function gives that value.
- You should add a call to your test function at the bottom of `sierpinski_test.py` inside the main guard.

## 5 Hints

1. I defined three variables in my main program that hold the coordinates of the three corners, since the corner coordinates are needed in several places. The functions that do calculations involving corners need to take the relevant corners as parameters.
2. Drawing a black and white triangle is a great first step. I recommend choosing black as the color for all pixels to ensure the geometry is all working correctly before getting into the color logic.
3. The midpoint function we wrote in class used tuples to pack the coordinates of points together into a single argument / return value. This is a design decision, and you may choose to use this approach in your functions or not. For example, my function that colors a certain pixel a given color has the following header:

```
color_pixel(turt, point, color)
```

[1] I'm assuming you're using the version we developed in class, so if you've modified it (e.g., to take individual arguments instead of tuples), you'll need to modify my test code to call it correctly.

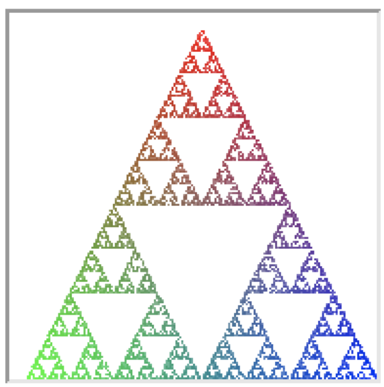
where point is expected to be a 2-tuple point = (x, y), and color = (r, g, b) is a 3-tuple of the RGB color values. I could also have written it

```
color_pixel(turt, px, py, r, g, b)
```

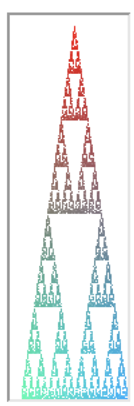
but I think it's slightly cleaner to pass points and colors to functions as tuples.

4. How solidly filled in your triangle is depends on how many iterations of the chaos game you run and how large your canvas is. A smaller canvas has fewer pixels to fill in, so fewer iterations will make a more solid picture, but it will have lower resolution. A large picture requires more iterations but has higher definition. Feel free to experiment with running more iterations to get larger, higher-definition triangles, but **please turn in code that runs 10000 iterations and runs in less than 5 seconds.** To keep things fast, remember that you can choose how often to call `turtle.update()`; for maximum speed, call it once after all your iterations are complete. The images below show what you can expect your drawing to look like with 10000 iterations for a few different canvas sizes.

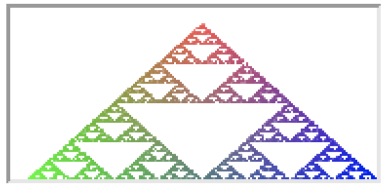
Here's a 200x200 output:



Here's a 100x300 output:



Here's a 200x100 output:



# 6 Guidelines

Please make sure your program follows these guidelines:

- Your code should run 10000 iterations of the chaos game and run in under 5 seconds.
- Your functions should not directly make use of (refer to) any global variables. Any information a function needs to do its job should be passed into the function as a parameter.
- Your code should do all the drawing (i.e., color all pixels) with the `Turtle` object returned by the setup function. Don't create any additional turtles.
- Each of your functions, and the main program, should not be too long. Not counting comments, docstrings, and blank lines, my main program (the part in the `if __name__ == "__main__":` block) is just under 20 lines and each of my functions is less than 10 lines. If you find yourself writing a continuous block of code that's longer than about 30 lines (not counting comments and blank lines), think about how you could break it up into logical subtasks and write functions to accomplish each one.
- Your functions and variable names should be descriptive but not overly long. For example, your corner 1 variable should probably not be called `c1`, nor should it be called `the_top_middle_corner_of_the_triangle`. Somewhere in between is best.

# Submission

Take a screenshot of the drawing produced on a canvas with **width = 120, height = 300**, and name it `triangle.png`. Submit `triangle.png`, `sierpinski.py`, and `sierpinski_test.py` to the A4 assignment on Canvas. As usual, please fill out the A4 Survey.

## Rubric

| | |
|---|---|
| **Submission Mechanics (8 points)** | |
| `triangle.png` shows a screenshot of your program's result on a 120x300 canvas. | 4 |
| Your `sierpinski.py` program runs in under 5 seconds. | 4 |
| **Code Style and Clarity (32 points)** | |
| Both files have a comment at the top stating author, date, and purpose | 2 |
| Program has comments throughout explaining anything non-obvious in the code. | 2 |
| Program defines at least two additional functions beyond the provided setup and midpoint functions. | 8 |
| Each function has a docstring containing a clear function specification. | 8 |
| Functions do not make reference to any global variables. | 4 |
| Main program and each individual function is not excessively long. | 4 |
| Variable and function names are descriptive but not too verbose. | 4 |
| **Correctness (30 points)** | |
| The triangle is drawn correctly for a square window | 10 |
| The triangle is drawn correctly for a non-square window | 10 |
| Each corner is colored one of red, green and blue as described above | 5 |
| The colors gradually blend according to their distance from each corner | 5 |
| **Testing (10 points)** | |
| `sierpinski_test.py` contains a test function for at least one of your functions, named as prescribed. | 2 |
| Your test function calls the function it tests with several different inputs to verify that its behavior is correct | 3 |
| Your test function uses `check_equal` to print the result of each of your test cases. | 3 |
| The test function is called in the main program at the bottom of of `sierpinski_test.py`. | 2 |
| Total | 80 points |

## 7 Challenge Problem

Take a look at the following web page: `http://mathworld.wolfram.com/ChaosGame.html`. There you can see how what we're doing here is just one specific case of a general idea. The general idea is you can have triangles, squares, pentagons, hexagons, etc. And you when you choose a random corner and find the midpoint you could instead find the point that is 1/3 of the way to the corner, or 3/8 of the way to the corner, etc.Make a copy of your main assignment program in a file named `chaos.py`. In this file, implement the following function:

```
def chaos_game(canv_width, canv_height, poly_sides, ratio):
    """ Run a chaos game on a canvas with size (canv_width, canv_height)
        with n = poly_sides (i.e., a poly_sides-sided polygon)
        and  r = ratio (i.e., fraction of distance from the corner)
    """
```

This challenge may require usage of material we haven't covered in detail (for example, lists will likely come in handy to store the corners of the polygon). If you are trying to tackle this and encounter any problems, come talk to me and I'd be happy to help. Successful completion of the challenge problem is worth 5 points of extra credit. Submit `chaos.py` on Canvas.