

Analyzing Protein Flexibility

An Introduction to Combinatorial Rigidity Methods and Their Applications

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Maltose-binding protein engulfing ligands

Flores, Echols, Milburn, Hespenheide, Keating, Lu, Wells, Yu, Thorpe, Gerstein (2006). Nucleic Acids Res. 34:D296-301.

Protein's are ...























Problem: We cannot observe proteins on the atomic level or their motions directly, but we'd like to understand how they flex or bend.







Molecular Dynamics (MD) Simulations





Molecular Dynamics provides the means to solve the equations of particle motion and evaluate these mathematical formulas

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J. Chem. Theory Comput., 4, 435-447 (2008)

Molecular Dynamics Simulation - Demonstration



GROMACS 4: Algorithms for Highly Efficient, Load-Balanced, and Scalable Molecular Simulation Hess, B., Kutzner, C., van der Spoel, D. and Lindahl, E. J. Chem. Theory Comput., 4, 435-447 (2008)

BIBM 2011 Tutorial: Analyzing protein flexibility: an introduction to combinatorial rigidity methods and applications Linkage Laboratory, University of Massachusetts Amherst, Smith College Ileana Streinu, Naomi Fox, Filip Jagodzinski

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- Demonstration of Rigidity Analysis
- Introduction to Rigidity Theory in 2-dimensions
- Introduction to Rigidity Theory in 3-dimensions
- Break
- Molecular Modeling and Rigidity Analysis using KINARI
- Potential Applications and ongoing research using Rigidity Analysis
- Concluding Remarks





Check on Molecular Dynamics Simulation







Part 1: Demonstrations of Using Rigidity Analysis









Horse heart cytochrome c (PDB file 1HRC) is a 105 residue heme protein found loosely associated with the inner membrane of mitochondria

What features of KINARI-Web can we use to determined/investigate its rigidity?

Where are the rigid regions? Hinges? Clusters?

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The same protein can be viewed as a cartoon

A hinge between two bodies is easily seen when the protein is displayed using a barand-stick representation

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KINARI-Web: A Server for Protein Rigidity Analysis Naomi Fox, Filip Jagodzinski, Yang Li, Ileana Streinu Nucleic Acids Research, **39** (Web Server Issue), 2011.





HIV-1 Protease (1HVR) with the ligand (blue)

HIV-1 Protease (1HVR) without the ligand

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Can we use rigidity analysis to infer affect of the ligand on the protease's rigidity?





When the ligand is retained in the molecular model of HIV-1 protease (left), the flaps of the dimer are held rigidly in place. When the ligand is remove (right) the flaps are flexible.

KINARI-Web: A Server for Protein Rigidity Analysis Naomi Fox, Filip Jagodzinski, Yang Li, Ileana Streinu Nucleic Acids Research, **39** (Web Server Issue), 2011.

Lysozyme from bactriophage T4



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Lysozyme is part of the immune response system; It helps to break apart bacterial cellular walls, thus killing the bacteria.

Lysozyme from bactriophage T4 – Rigidity Results UMASSCS



Default Modeling

Addition of two hydrogen bonds causes the protein to rigidify By removing hydrophobic interactions, their effect can be easily seen

KINARI-Web: A Server for Protein Rigidity Analysis Naomi Fox, Filip Jagodzinski, Yang Li, Ileana Streinu Nucleic Acids Research, **39** (Web Server Issue), 2011.



Part 2: Comparative overview of tools for studying protein flexibility





Henzler-Wildman, Katherine and Kern, Dorothee, Dynamic personalities of proteins., Nature, volume 450, number 7172, pages 964-72, 2007.



Necessary for important protein functions such as Enzyme catalysis, Signal transduction, and Protein-protein interactions



The enzyme hexokinase changes conformation when glucose binds to it.

David W. Ball, John W. Hill, Rhonda J. Scott (2011) The Basics of General, Organic, and Biological Chemistry. Flat World Knowledge.



	In Vitro	In Silico
Large Domain, Collective Motions	Fluorescence	ENM and NMA Combinatorial Rigidity Analysis
Thermal fluctuations	X-ray Crystallography NMR Cryo-EM	Molecular Dynamics









Image from Wikimedia courtesy Thomas Splettstoesser



B-factors – describe the displacement from mean coordinate

"Missing" atoms – position could not be determined

>70,000 X-ray data files in PDB





Computes coordinates from set of distance constraints

Multiple conformations produced

~9,000 NMR-resolved structures in PDB



PDB File 1BVE, HIV-1 protease



Rapidly freeze protein in solution and view under electron microscope

< 400 structures in PDB



EM DATA BANK (EMDB) / 5124



Model all the interatomic interactions in a system and simulate with fs time step.



Can only simulate fast timescales

Figure 4.5: Principle of bond stretching (left), and the bond stretching potential (right).

(2010) GROMACS USER MANUAL Version 4.5 http://manual.gromacs.org/

Elastic Network Models and Normal Mode Analysis



mode 7 mode 8 Build an elastic network model of the protein atom. 50 2 atom squar. sound. 2 Determine the normal modes 20 of the model 50 150 200 50 200 100 100 150 residue index in sequence residue index in sequence mode 9 mode 10 displ atom. disp œ 30 ŝ atom. 30 4 squar. squar. e 0 e. É 0 50 100 150 200 50 100 150 200 residue index in sequence residue index in sequence mode 11 mode 12 atom. disp 20 atom. ₽ ø squar. ≌ squar. ÷ 64 ыń 0 150 200 50 100 150 200 0 50 100 0 residue index in sequence residue index in sequence

Fluctuations of 1HVR computed using NMA. SM, Sælensminde G, Reuter N. WEBnm@: a web application for normal mode analysis of proteins BMC Bioinformatics. 2005 Mar 11;6(1):52 Chennubhotla, Chakra, Rader, A. L. and Babar, Lvet, Elastic network models for understanding biomolecular machinery; from enzymes to supramolecul

Chennubhotla, Chakra, Rader, A. J. and Bahar, Ivet, Elastic network models for understanding biomolecular machinery: from enzymes to supramolecular assemblies, in: Physical Biology, volume 2, pages S173-S180, 2005.



Part 3 : Introduction to Rigidity Theory in 2 and 3 Dimensions



A framework G(p) is *rigid* if it has no continuous deformation.



Which frameworks are rigid and which are not?






Exercise: Counting Degrees of Freedom



A 2D framework with n points and no edges has 2n degrees of freedom (DOF).





Adding a bar reduces the number of degrees of freedom by 1.

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Framework has 7 DOF





Adding a second bar again reduces the number of degrees of freedom by 1.

39

Framework has 6 DOF





Adding a third bar again reduces the number of degrees of freedom by 1.

40

Framework has 5 DOF





Adding a fourth bar again reduces the number of degrees of freedom by 1.

41

Framework has 4 DOF





Adding a fifth bar again reduces the number of degrees of freedom by 1.

42

Framework has 3 DOF



Exercise: Counting degrees of freedom in 2D





The Maxwell-Laman theorem

A framework G(p) is generically minimally rigid

iff

- For every subset of vertices, |E'| <= 2|V'| 3
- |E| = 2|V|-3











Rigid Components

Rigid components are maximal sets of vertices which are rigid to each other.





Rigid components

Rigid zcomponent	Maximal set of vertices that are rigid to each other.
(2,3)-component	Maximal set of vertices containing a (2,3)- tight graph.





Can we extend Laman to 3D?

Will 3n-6 edge counts work for 3D?

Why 3n-6?

For each vertex:

Translations along x,y,z axes.

For the entire framework:

Rigid translations along x,y,z axes

Rigid rotations around x,y,z axes



Can we extend Laman to 3d?

Can we rigidify with 3(4) - 6 = 6 edges?





Exercise: Counting Degrees of Freedom in 3D



A 3D framework with n points and no edges has 3n DOF.

Framework has 3(4) = 12 DOF.









Exercise: Counting Degrees of Freedom in 3D





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2D vs 3D generic rigidity

2d	3d
G contains a spanning (2,3)- tight subgraph $\leftarrow \rightarrow$ G is generically rigid.	Sometimes , when G contains a spanning (3,6)- tight subgraph, G is generically rigid.
Rigid components ALWAYS induce spanning (2,3)-tight subgraphs.	Sometimes , rigid components induce spanning (3,6)-tight subgraphs.

Is there a class of frameworks for which:

G contains a spanning (3,6)-tight subgraph

 \leftrightarrow

G is generically rigid in 3D.

Bar and Joint and Proteins



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A body-bar-hinge framework is generically minimally rigid iff:

- its associated (multi) graph has exactly 6n-6 edges
- and every subgraph has at most 6n-6 edges



Tay 84, 89; White and Whiteley 87; Katoh and Tanigawa 09



Educational site and demos http://linkage.cs.umass.edu/pg/pg.html





Part 4: Molecular Modeling and Rigidity Analysis using KINARI



Covalent bonds impose constraints





peptide and double bonds fix dihedral angle

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- An atom and its covalent-bonded neighbors form a rigid body.
- Rotatable covalent bonds act as hinges.





- We know how to determine trivial bodies.
- Would like to find **maximal** bodies.
- A body is maximal if there are no other atoms which are rigidly attached to it.





 A body or *rigid cluster* is a maximal set of atoms and all bonds and interactions that hold the atoms rigidly together.









	ATOM	1	N	PRO	А	1	52.574	58.851	-7.646	1.00	34.60	
	ATOM	2	CA	PRO	A	1	51.842	59.784	-6.815	1.00	34.88	
	ATOM	3	С	PRO	Α	1	52.146	59.438	-5.356	1.00	36.01	
	ATOM	4	0	PRO	Α	1	53.031	58.612	-5.150	1.00	35.05	
I	ATOM	5	CB	PRO	А	1	50.391	59.581	-7.189	1.00	34.07	
	ATOM	6	CG	PRO	А	1	50.353	58.166	-7.724	1.00	32.00	
	ATOM	7	CD	PRO	Α	1	51.621	58.242	-8.540	1.00	31.72	
	ATOM	8	N	GLN	А	2	51.488	60.047	-4.359	1.00	35.47	
	ATOM	9	CA	GLN	А	2	51.577	59.564	-2.997	1.00	35.15	
	ATOM	10	С	GLN	Α	2	50.109	59.273	-2.694	1.00	33.50	
ļ	ATOM	11	0	GLN	Α	2	49.368	60.231	-2.829	1.00	37.45	
	ATOM	12	CB	GLN	Α	2	52.076	60.630	-2.075	1.00	37.17	
	ATOM	13	CG	GLN	Α	2	52.024	59.911	-0.749	1.00	42.02	
	ATOM	14	CD	GLN	Α	2	52.639	60.592	0.439	1.00	43.50	
	ATOM	15	OE1	GLN	Α	2	53.841	60.484	0.675	1.00	44.41	
	ATOM	16	NE2	GLN	Α	2	51.824	61.272	1.246	1.00	43.72	
	ATOM	17	N	ILE	Α	3	49.555	58.119	-2.306	1.00	30.33	
	ATOM	18	CA	ILE	А	3	48.107	57.932	-2.171	1.00	27.03	
	ATOM	19	С	ILE	Α	3	47.924	57.639	-0.692	1.00	26.95	
	ATOM	20	0	ILE	Α	3	48.736	56.921	-0.102	1.00	25.73	
	ATOM	21	CB	ILE	А	3	47.623	56.740	-3.053	1.00	24.36	
	ATOM	22	CG1	ILE	А	3	47.870	57.072	-4.513	1.00	24.07	
	ATOM	23	CG2	ILE	Α	3	46.116	56.491	-2.898	1.00	22.78	
	ATOM	24	CD1	ILE	А	3	47.651	55.899	-5.455	1.00	21.74	
	-											



Includes atoms and coordinates from X-ray crystallography or NMR experiments

0

0

Usually, does not include hydrogen atoms

Curation from PDB File

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Important interactions for holding a protein's 3D shape

- Covalent bonds
- Hydrogen bonds
- Hydrophobic interactions

Main chain and side chains connected by covalent bonds.

Secondary structures held by backbone hydrogen bonds.





3D folded shape held together by hydrogen bonds and hydrophobic interactions.



- Covalent bonds modeled as hinges, remove 5 degrees of freedom.
- Hydrogen bonds include only strong hydrogen bonds. Model as hinges.
- Hydrophobic interactions weaker interactions. Model with 2 'bars' removing 2 degrees of freedom.

Differences in Systems







- Body-bar-hinge file (xml file)
 - Set of bodies lists of atoms
 - Set of bars pairs of bodies, pairs of atoms
 - Set of hinges pairs of bodies which share a hinge along the axis between pair of atoms



Part 5: Applications of Protein Rigidity Analysis



- Dilution Analysis (Hespenheide, et al)
- Redundancy Analysis (Fox, Streinu)
- FRODA Geometric Simulation (Wells, et al)
- Probabilistic Roadmaps (Thomas, et al)
- Thermophile stability (Gohlke, Radestock)
- Flexibility of RNA (Fulle, Gohlke)
- KINARI Mutagen (Jagodzinski, Streinu)

Dilution Analysis

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Observing unfolding pathway



Hespenheide, B. M, Rader, A. J., Thorpe, Michael F. and Kuhn, Leslie A., *Identifying Protein Folding Cores: Observing the Evolution of Rigid and Flexible Regions During unfolding*, in: J Mol Graph Model, volume 21, number 3, pages 195-20, 2002.


Motivation: Contributions of noncovalent interactions:

- Responsible for holding a protein's 3D shape.
- break and form during normal fluctuations

Our contribution: Classify each interaction in terms of its contribution to the rigidity.



Fox, Naomi and Streinu, Ileana, Redundant Interactions in Protein Rigid Cluster Analysis, in: 1st IEEE International Conference on Computational Advances in Bio and medical Sciences (ICCABS). Feb. 3-5, 2011, Orlando, FL, 2011.







Cluster	Atoms	H-Bonds	HPh-inter.	Redun.
		redun / all	redun / all	Score
orange	251	0 / 23	6 / 10	0.089
left yellow	88	1/5	6/9	0.40
$\frac{5*0+2*6}{5*23+2*10} = \frac{12}{135} \approx .089$				

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Fox, Naomi and Streinu, Ileana, Redundant Interactions in Protein Rigid Cluster Analysis, in: 1st IEEE International Conference on Computational Advances in Bio and medical Sciences (ICCABS). Feb. 3-5, 2011, Orlando, FL, 2011.

Survey Results

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Atoms

1648

203

95

903

62

32

79

1302

H-Bonds

0 / 121

0/16

1 / 10

0/74

0/4

0/2

0/44

0/8

redun / all

HPh-inter.

redun / all

59 / 91

0/6

0/0

3/4

3/4

1/1

24 / 45

159 / 197

Redun.

Score

0.15

0.00

0.10

0.10

0.21

0.33

0.52

0.05

PDB

1FTO

1A2P

1J4P

Cluster

1

2

3

1

2

3

1

2

Almost all hydrogen bonds were critical.

The majority of hydrophobic interactions were not critical.





Use rigid clusters to generate motion for Barnase (110 residue)



simulation of diffusive motion in proteins. Physical Biology 2:S127-S136, 2005.

FRODA Directional Motion





closed to occluded

The enzymatic protein has three very similar conformations (open, closed, and occluded), most significant differences in residues 14-24 (M-20 loop)/Mobile loop in closed (red) and occluded (blue) positions.

Stephen Wells, Scott Menor, Brandon Hespenheide, and MF Thorpe. Constrained geometric simulation of diffusive motion in proteins. Physical Biology 2:S127-S136, 2005.



- Extension of Probabilistic Roadmap algorithm
- Models protein backbone as a linkage, and samples different conformations
- Uses rigidity analysis to restrict conformers sampled



Thomas, Shawna, Tang, Xinyu, Tapia, Lydia and Amato, Nancy M., Simulating Protein Motions with Rigidity Analysis, in: Journal Of Computational Biology, volume 14, number 6, pages 839-855, 2007.



Do mesophile/thermophile homologs have corresponding states during unfolding?



Gohlke, Holger and Radestock, S., Exploiting the link between protein rigidity and thermostability for data-driven protein engineering, in: Eng. Life Science, volume 8, pages 507-522, 2008.



- Different parameterization required for calculating hydrophobics
- Found correspondence between B-values and flexibility index



Colored by Bvalue

Fulle, Simone and Gohlke, Holger, Analyzing the Flexibility of RNA Structures by Constraint Counting, in: Biophysics Journal, volume 94, pages 4202-4219, 2008.





Crambin (PDB file 1CRN)

Crambin is a 46 amino acid plant protein, whose crystals diffract to ultra-high resolution.

Can we use Rigidity Analysis to infer critical residues of crambin?

The insight: "mutate" different residues to generate *in-silico* mutants, and perform rigidity analysis on them, to infer which residues, IF mutated, would destabilize the protein, and hence are important

Filip Jagodzinski, Jeanne Hardy, Ileana Streinu, Using Rigidity Analysis To Probe Mutation-Induced Structural Changes in Proteins, CSBW 2011.

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To simulate a substitution to a glycine, all hydrogen bonds and hydrophobic interactions for a residue are removed from the molecular model; this effectively removes the side chain from the molecular model because it cannot contribute stabilizing interactions.



Filip Jagodzinski, Jeanne Hardy, Ileana Streinu, Using Rigidity Analysis To Probe Mutation-Induced Structural Changes in Proteins, CSBW 2011.

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We can compare the rigidity results of mutants to infer which mutations disrupt the protein's rigidity, and hence which residues are critical



The while type of the crambin contains one large rigid cluster (purple) Mutating residue 4 affects the rigid bodies of crambin Mutating Residue 40 causes the largest rigid body to break down, but not as much as when residue 4 is mutated

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Filip Jagodzinski, Jeanne Hardy, Ileana Streinu, Using Rigidity Analysis To Probe Mutation-Induced Structural Changes in Proteins, CSBW 2011. Rigidity results for different mutants can be compared to infer which residue(s) is (are) critical.

Distribution of Rigid Bodies, By Residue



Filip Jagodzinski, Jeanne Hardy, Ileana Streinu, Using Rigidity Analysis To Probe Mutation-Induced Structural Changes in Proteins, CSBW 2011.

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KINARI Mutagen – Correlating results to Web-Lab Experiments and Other Metrics



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Filip Jagodzinski, Jeanne Hardy, Ileana Streinu, Using Rigidity Analysis To Probe Mutation-Induced Structural Changes in Proteins, CSBW 2011.

KINARI Mutagen – Correlating results to Web-Lab Experiments



Largest Rigid Cluster and SASA vs Excised Residue Largest Rigid Body (Num Atoms) Largest Rigid Body SASA SASA (Å squared

Residue on Which Excision Was Performed

Residues 3, 4, 40, and 41 (among others), have been found to be identical among Crambin and two homologous plant toxins viscotoxin A3 and α1 purothioni M.M. Teeter, J.A. Mazer, and J.J. L'Italien. Primary structure of the hydrophobic plant protein crambin. Biochemistry, 20(19):5437–5443, 1981.





KINARI Project Team, 2010



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